



Competing Species

Coexistence and Chaos in Complex Ecologies

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University of Arizona, March 25, 2010

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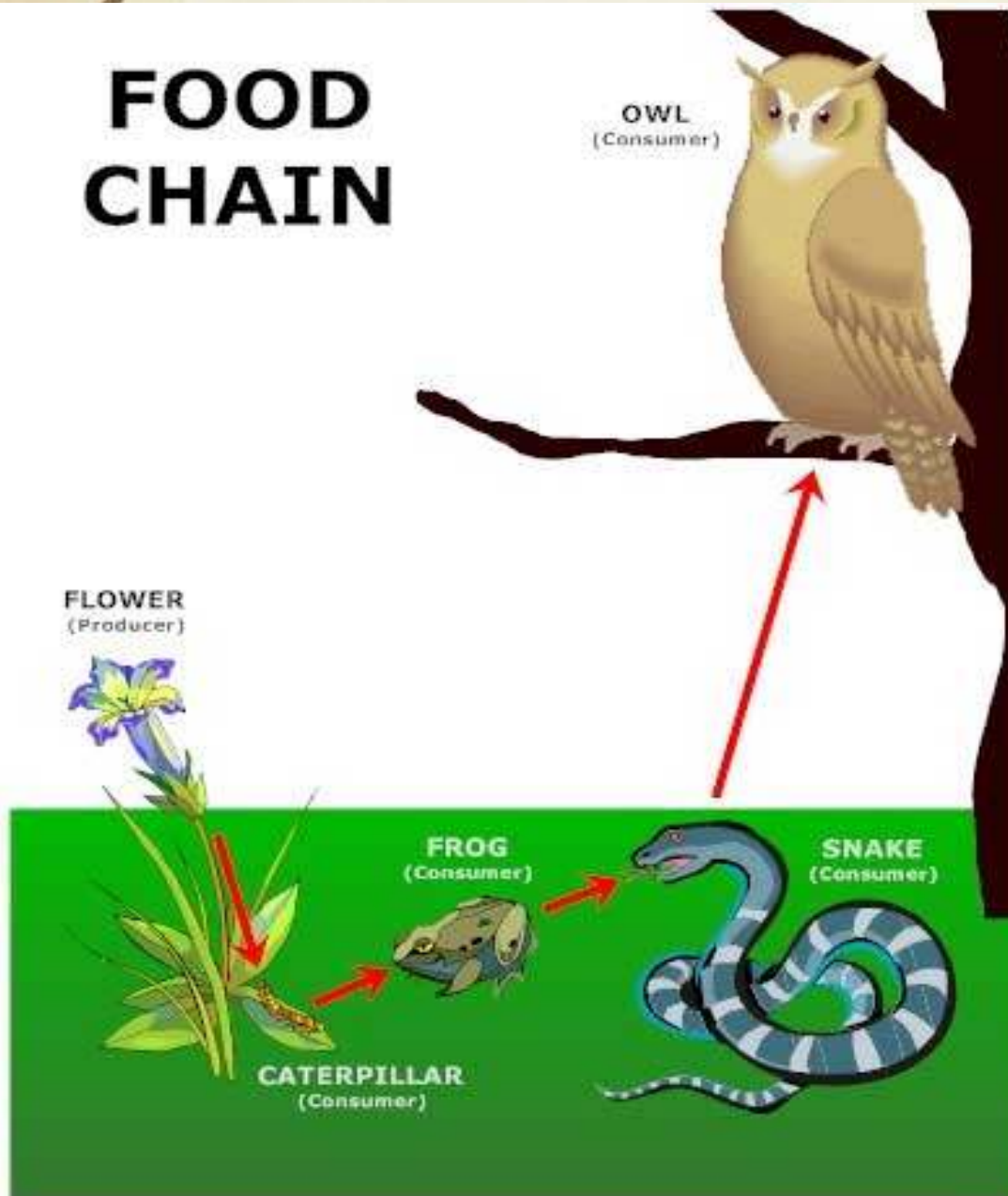
- Mentor: Sarah Mann

Modeling Species



- Competition in the Real World
- Why Use Models?
 - Predict instability
 - Parameters are chosen in a variety of ways

FOOD CHAIN

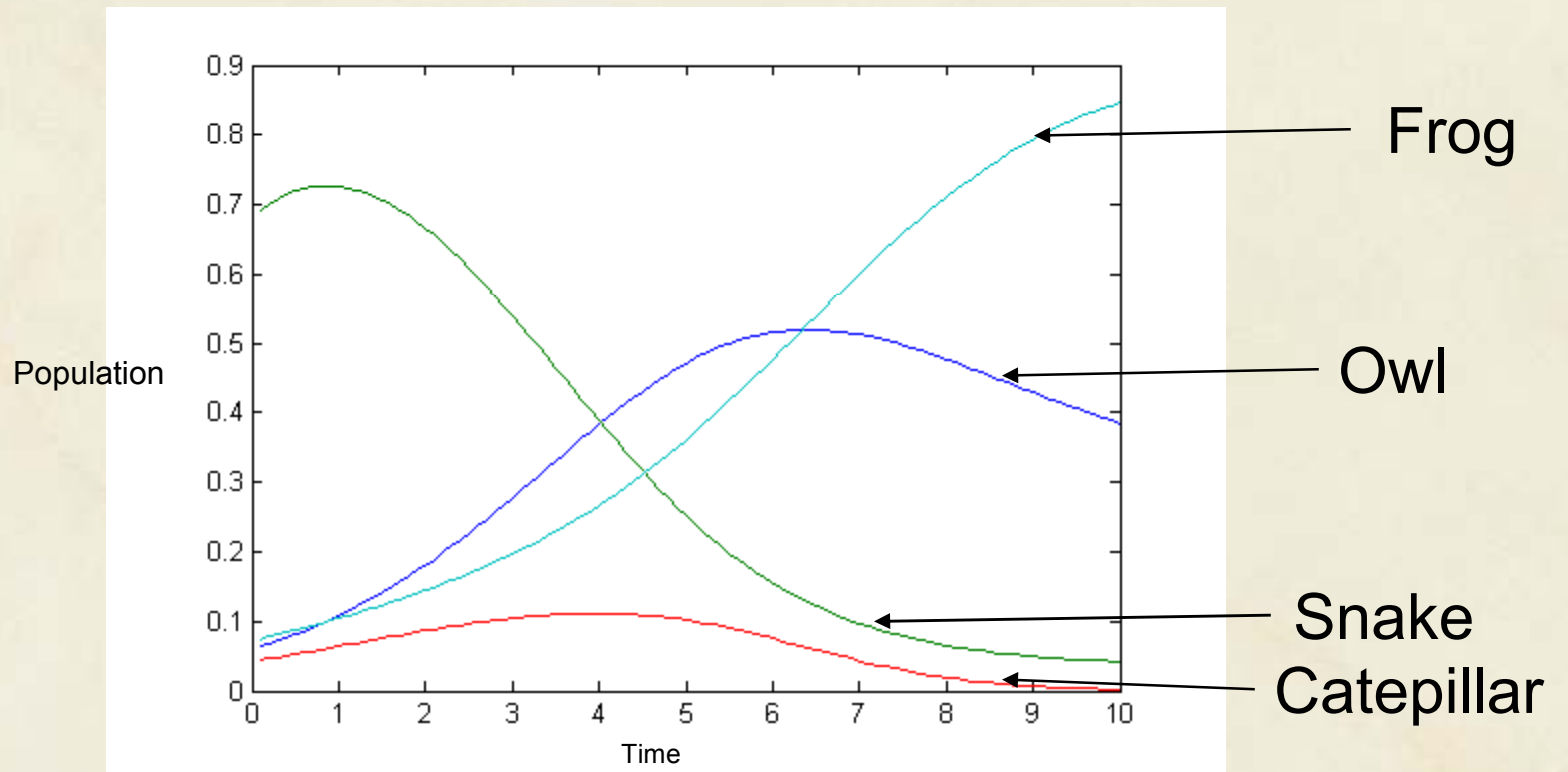


IN THE WILD...

Can model relations with equations:

For Example:
Owl, Snake, Frog, Caterpillar

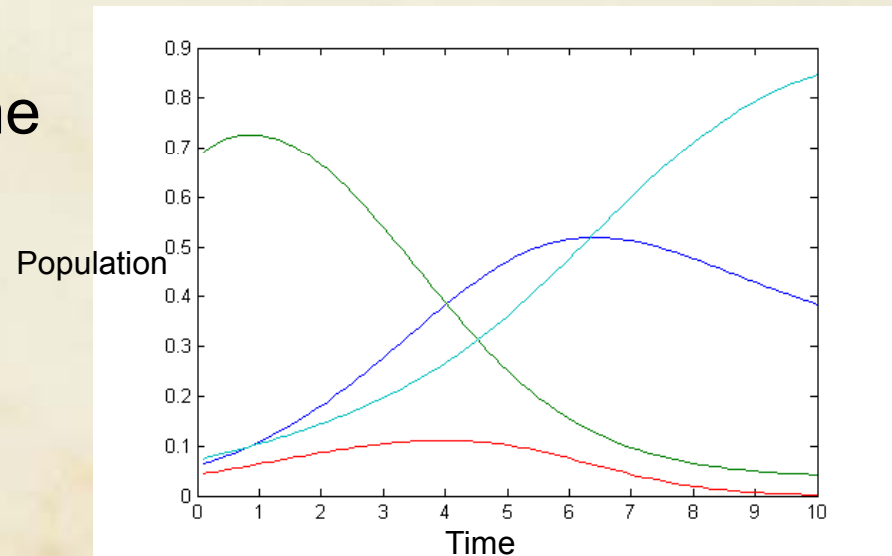
Population Graph



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What You Can't See...

- Adaptation
 - Occurs every 20 time steps
- Clamping
 - Occurs at 10^{-6} to prevent extinction





Lotka-Volterra model

- Lotka-Volterra equations:

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = -\gamma y + \delta xy$$

x = prey, y = predator, t = time



Variation of Lotka-Volterra equations

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \sum_{j=1}^N a_{ij} x_j \right)$$

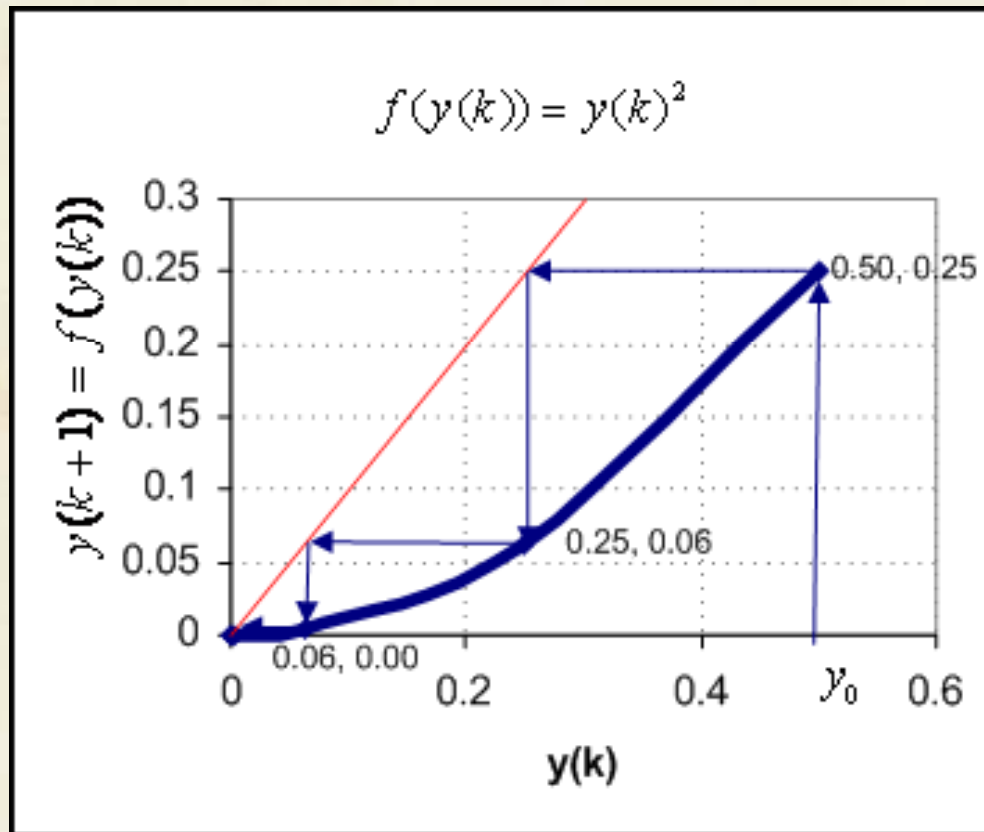
- x_i = Population size of species i
- dx_i/dt = Rate of change in size of population i
- r_i = Growth rate
- a_{ij} = Competition matrix



The Numerical Method

- Discretize
- Develop difference equation (Forward Euler Method)
- Implement in Matlab

Difference Equations



$$y_n = f(y_{n-1})$$

y would represent an animal population

y_0 would represent the initial conditions

Forward Euler

- Approximation of time derivative of $x(t)$:
$$dx/dt \approx (x_n - x_{n-1}) / \Delta t$$
- Exact time derivative of $x(t)$ from DE:

$$f = \frac{dx_i}{dt} = r_i x_i \left(1 - \sum_{j=1}^N a_{ij} x_j \right)$$

- The iterative method:

$$x_n \approx x_{n-1} + f\Delta t$$



Matlab Implementation

- Initialization of population vector and competition matrix
- Clamping at 10^{-6}
- Adaptation
- Step size
- Why Forward Euler?



Biomass and Biodiversity

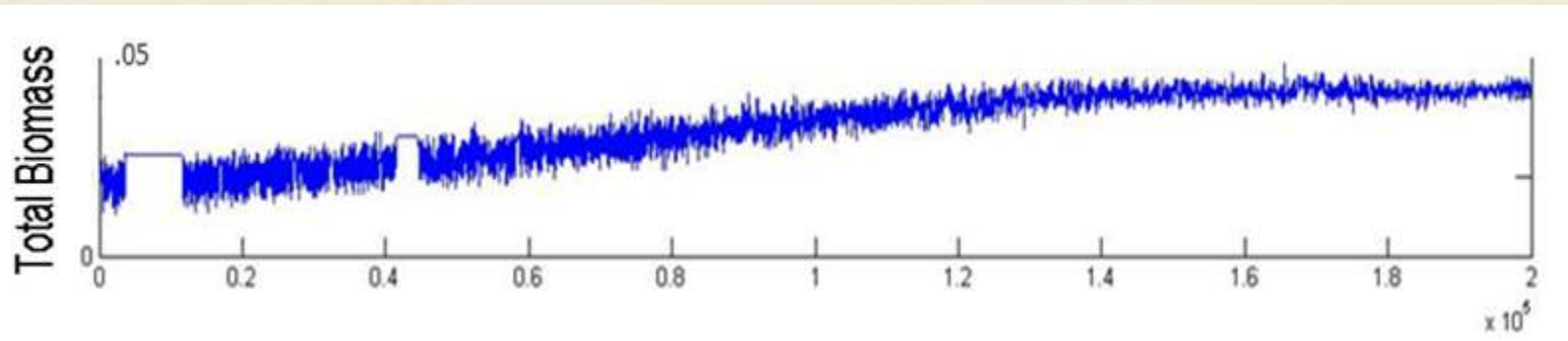
- Biomass – The total mass of living organisms in a certain ecosystem

$$M = \frac{1}{N} \sum_{i=1}^N x_i$$

- Biodiversity - The diversity of plant and animal life in a specific habitat

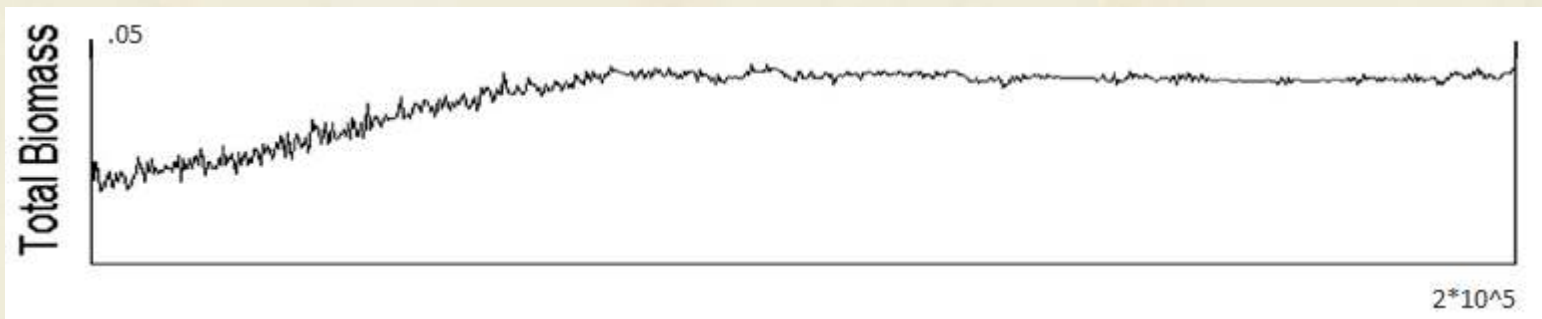
$$D = 1 - \frac{1}{2(N-1)} \sum_{i=1}^N \left| \frac{x_i}{M} - 1 \right|$$

Biomass (with adaption) Our Graph



Time

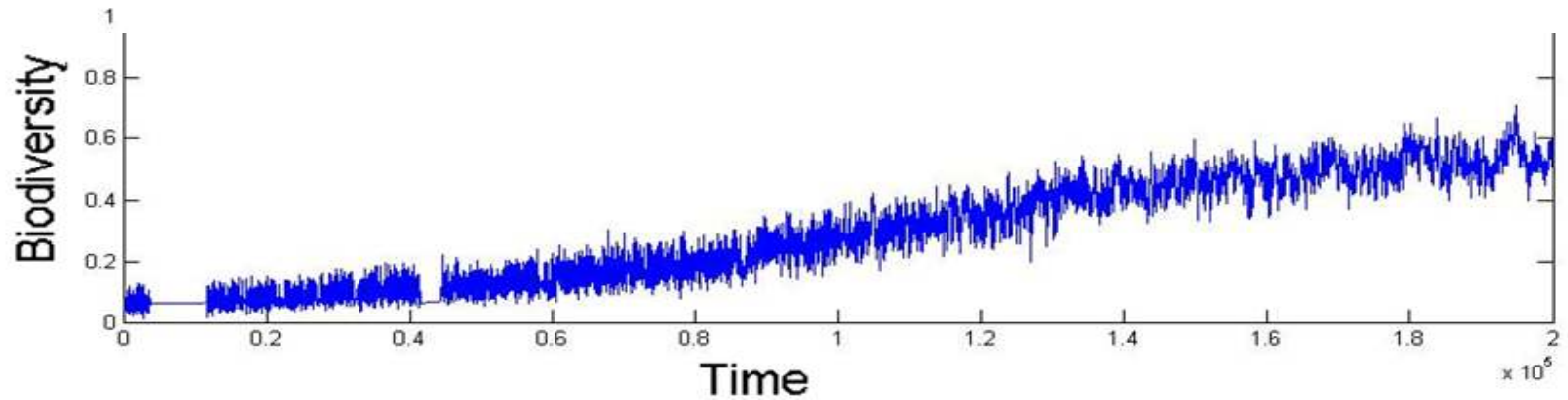
Their Graph



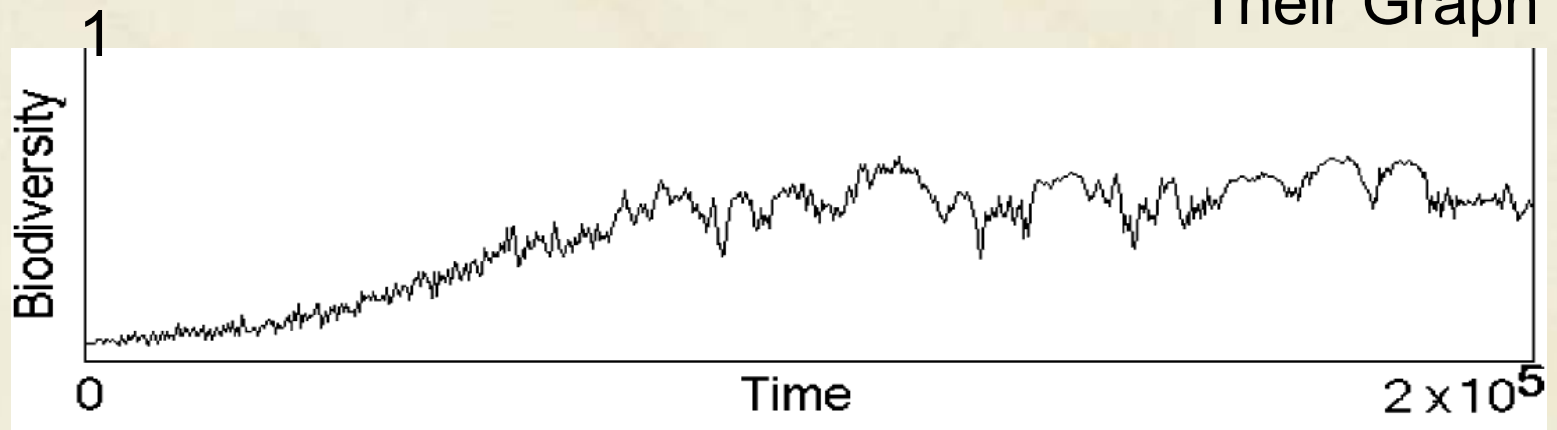
Time

Biodiversity (with adaption)

Our Graph



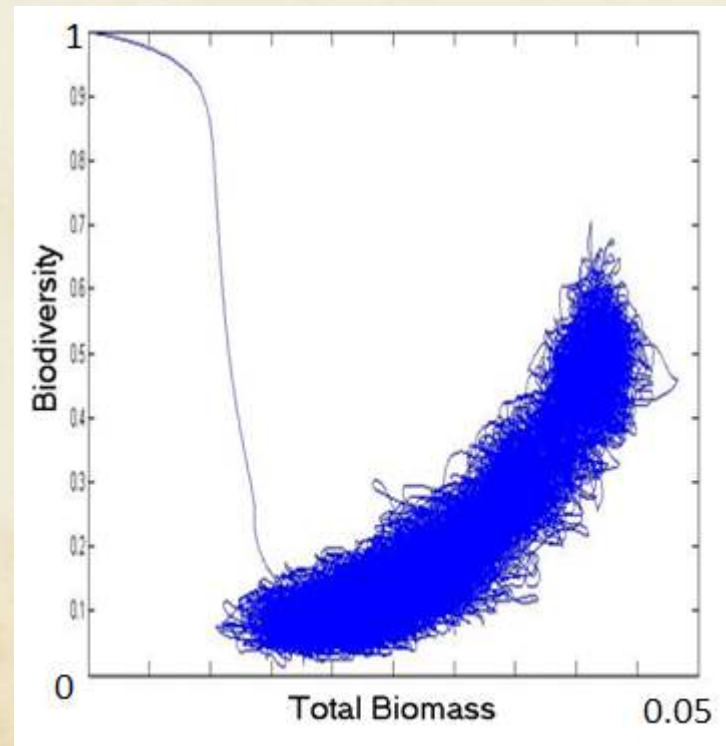
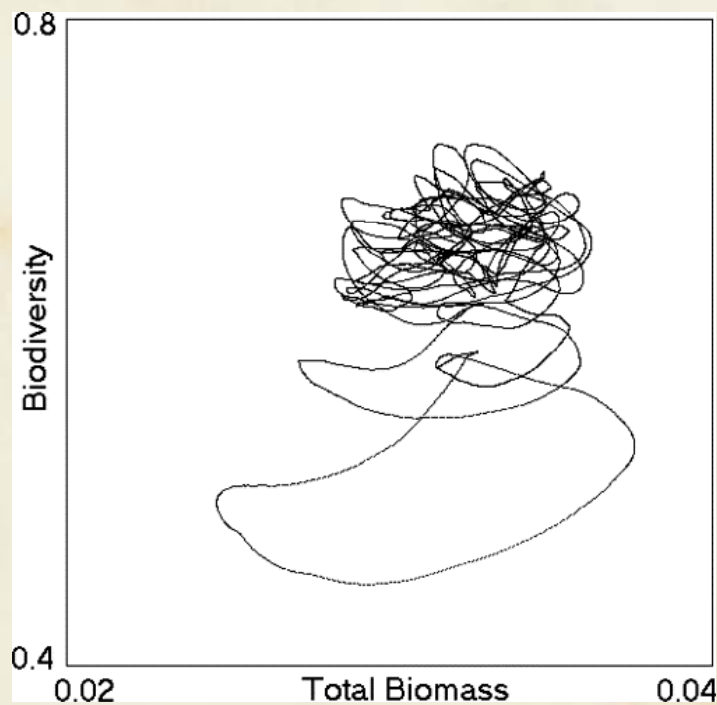
Their Graph



Biodiversity vs Biomass

Theirs (without adaptation)

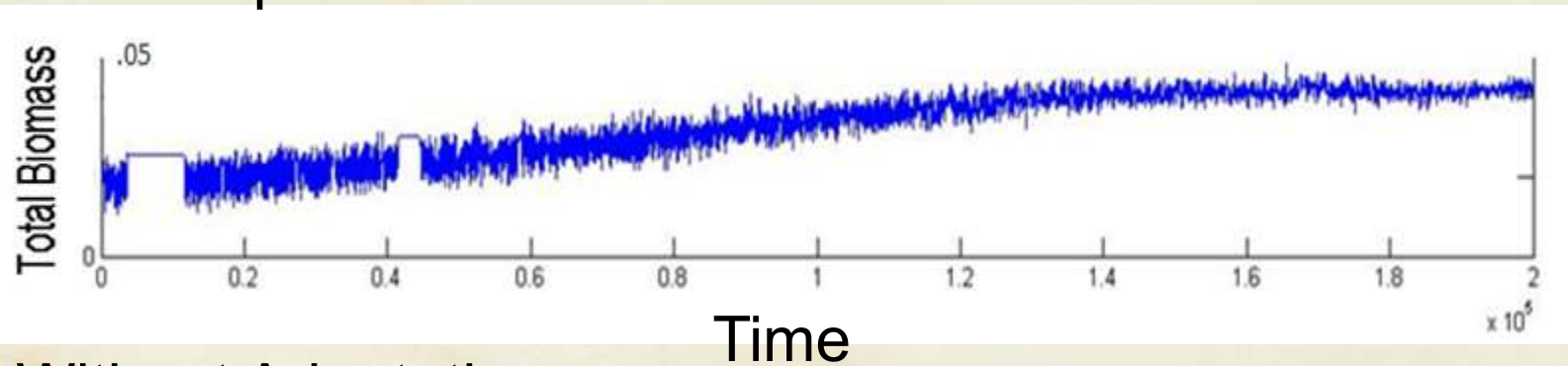
Ours (with adaptation)



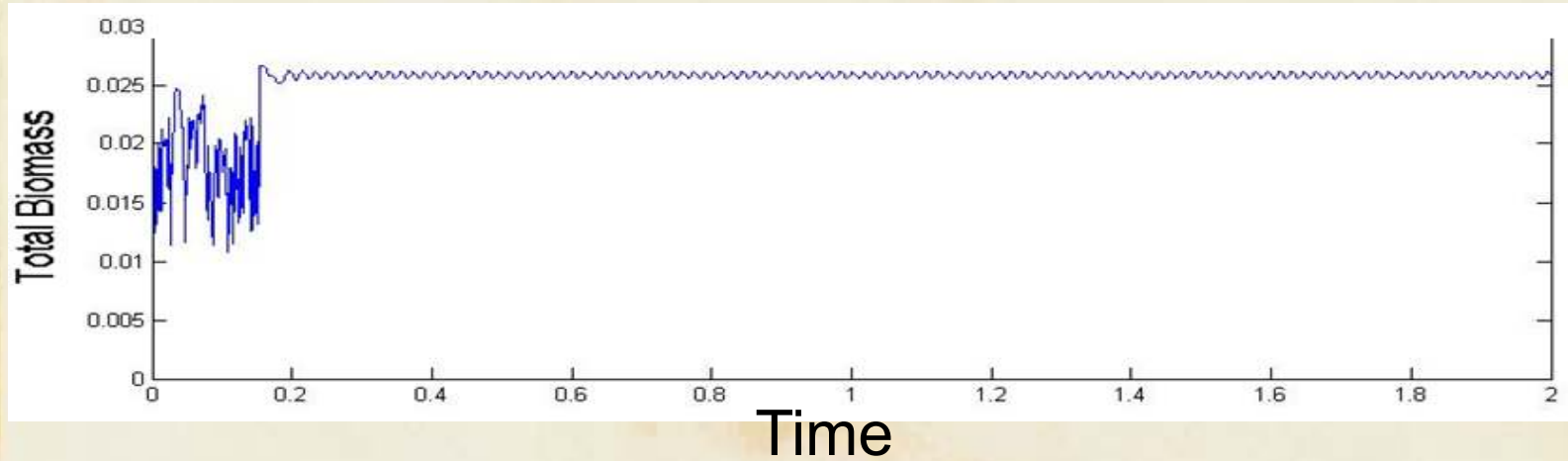
Biomass

2×10^6

With Adaptation:



Without Adaptation:

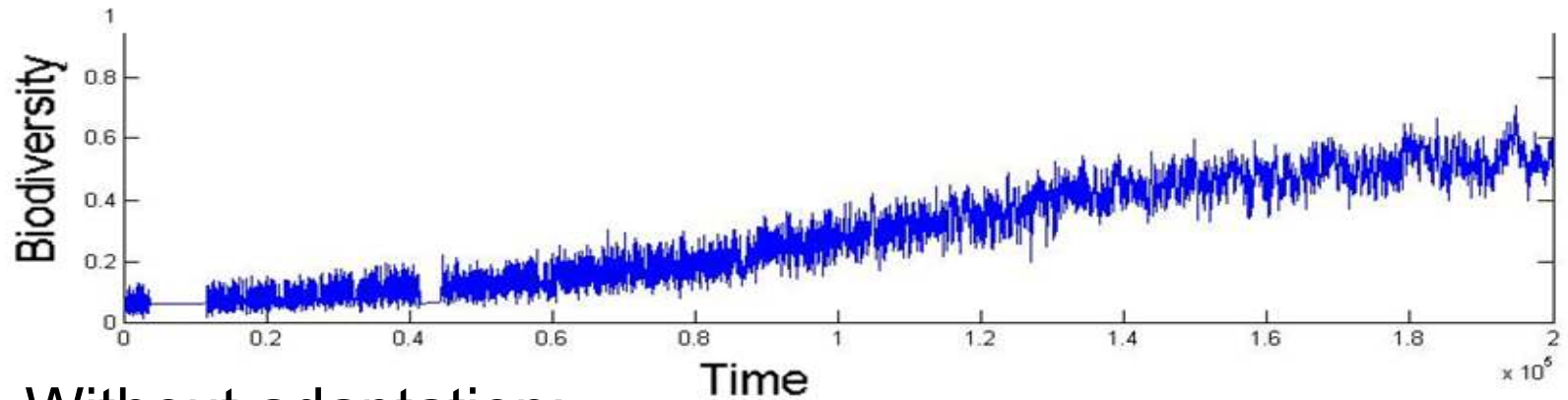


2×10^4

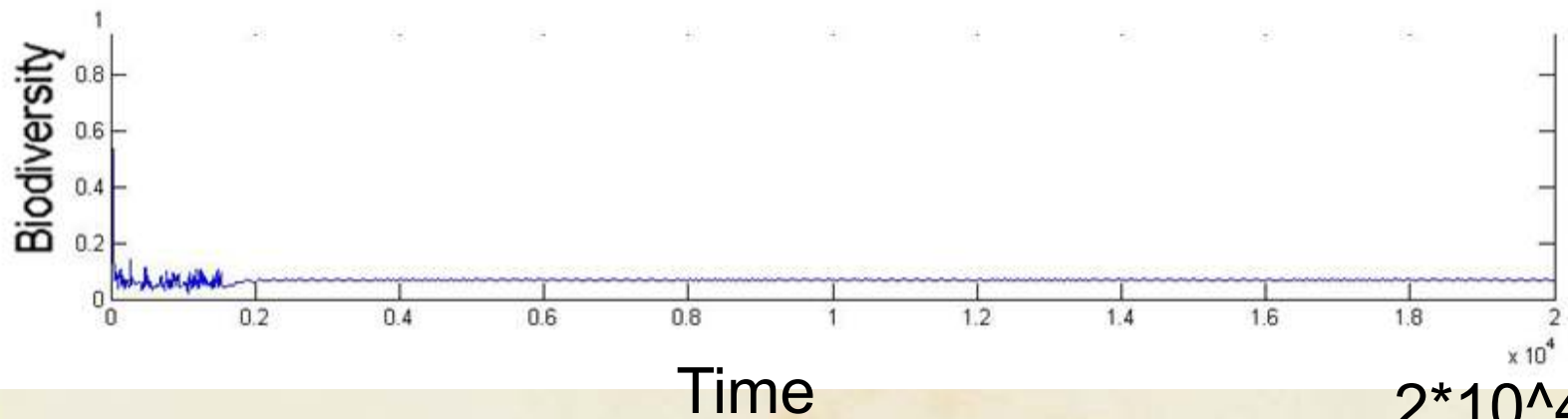
Biodiversity

With adaptation:

2×10^6



Without adaptation:



2×10^4

Results

- What effects do the following have on Biomass/Biodiversity?
 - Clamping
 - Adaptation



- What does this all mean?
- Why are our results relevant?





Modifications

- Different changes in adaptation
- Changes in mutation (different number of time steps to implement mutation)
- Changes in clamp size



Conclusion

- Current Research
- Applications of models:
 - Competition for resources
 - Objects prone to crashes





Acknowledgments

- Sprott, J.C., J.C. Vano, J.C. Wildenberg, M.B. Anderson, and J.K. Noel. "Coexistence and chaos in complex ecologies." *Physics Letters A* 335. (2005): 207-12. Web. 23 Feb 2010.
- Fox, J. W., and D. A. Vasseur. 2008. Character convergence under competition for nutritionally-essential resources. *American Naturalist* 172:667-680
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QUESTIONS?



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