Competing Species
Coexistence and Chaos in Complex Ecologies

J.C. Sprott, J.A. Vano, J.C. Wildenberg, M.C. Anderson, J.K. Noel

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Group Members

- David DeCesari
- Jennifer Kanemaru
- Daniel Weiss
- Carolyn Wise

Mentor: Sarah Mann

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Modeling Species

- Competition in the Real World

- Why Use Models?
  - Predict instability
  - Parameters are chosen in a variety of ways

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Can model relations with equations:

For Example: Owl, Snake, Frog, Caterpillar
Population Graph

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What You Can’t See...

- Adaptation
  - Occurs every 20 time steps
- Clamping
  - Occurs at $10^{-6}$ to prevent extinction

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Lotka-Volterra model

Lotka-Volterra equations:

\[
\frac{dx}{dt} = ax - \beta xy
\]

\[
\frac{dy}{dt} = -\gamma y + \delta xy
\]

x = prey, y = predator, t = time

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Variation of Lotka-Volterra equations

\[
\frac{dx_i}{dt} = r_i x_i \left( 1 - \sum_{j=1}^{N} a_{ij} x_j \right)
\]

- \(x_i\) = Population size of species i
- \(dx_i/dt\) = Rate of change in size of population i
- \(r_i\) = Growth rate
- \(a_{ij}\) = Competition matrix

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The Numerical Method

- Discretize
- Develop difference equation (Forward Euler Method)
- Implement in Matlab
Difference Equations

\[ f(y(k)) = y(k)^2 \]

\[ y_n = f(y_{n-1}) \]

- \( y \) would represent an animal population
- \( y_0 \) would represent the initial conditions

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Forward Euler

- Approximation of time derivative of $x(t)$:
  $\frac{dx}{dt} \approx \frac{x_n - x_{n-1}}{\Delta t}$

- Exact time derivative of $x(t)$ from DE:

  $f = \frac{dx_i}{dt} = r_i x_i \left( 1 - \sum_{j=1}^{N} a_{ij} x_j \right)$

- The iterative method:
  $x_n \approx x_{n-1} + f \Delta t$

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Matlab Implementation

- Initialization of population vector and competition matrix
- Clamping at $10^{-6}$
- Adaptation
- Step size
- Why Forward Euler?
Biomass and Biodiversity

• Biomass – The total mass of living organisms in a certain ecosystem

\[ M = \frac{1}{N} \sum_{i=1}^{N} x_i \]

• Biodiversity - The diversity of plant and animal life in a specific habitat

\[ D = 1 - \frac{1}{2(N-1)} \sum_{i=1}^{N} \left| \frac{x_i}{M} - 1 \right| \]
Biomass (with adaption)

Our Graph

Their Graph

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Biodiversity (with adaption)

Our Graph

Their Graph

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Biodiversity vs Biomass

Theirs (without adaptation)  Ours (with adaptation)

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Biomass

With Adaptation:

Without Adaptation:

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Biodiversity

With adaptation: $2 \times 10^6$

Without adaptation: $2 \times 10^4$

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Results

- What effects do the following have on Biomass/Biodiversity?
  - Clamping
  - Adaptation

- What does this all mean?
- Why are our results relevant?

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Modifications

- Different changes in adaptation
- Changes in mutation (different number of time steps to implement mutation)
- Changes in clamp size
Conclusion

• Current Research

• Applications of models:
  • Competition for resources
  • Objects prone to crashes
Acknowledgments


  

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QUESTIONS?