THE UNIVERSITY OF ARIZONA

# MATH 585
# HOW INSECTS FLY

by

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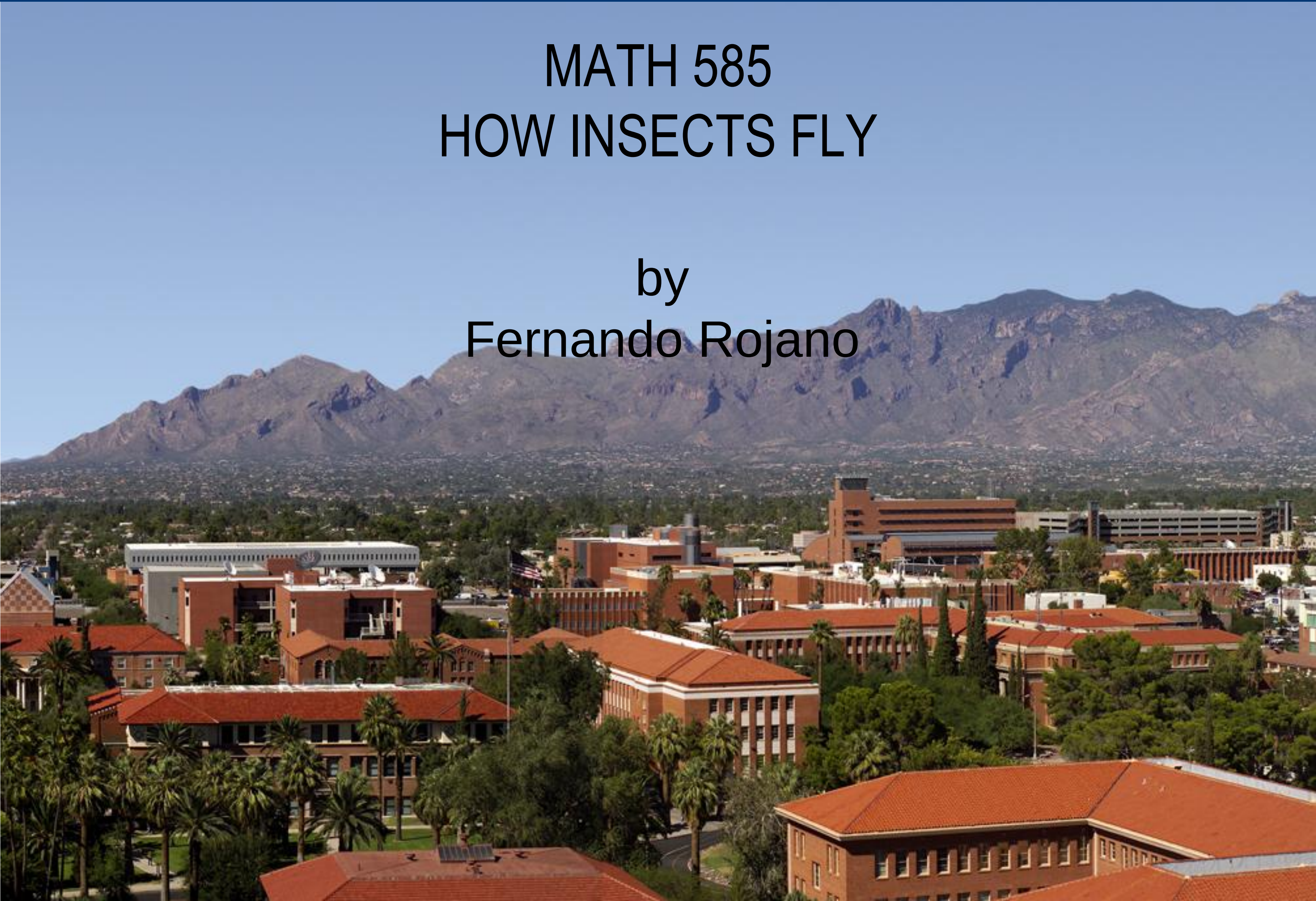

- Acknowledgements to

- Dr. Ildar Gabitov (University of Arizona)
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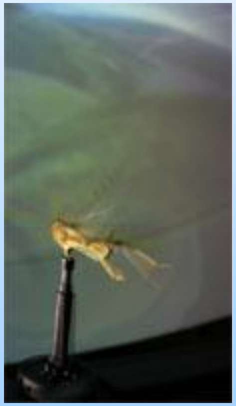
- Dr. Sergey V Shkarayev (University of Arizona)

- Project based in the paper: “Nonlinear time-periodic models of the longitudinal flight dynamics of the desert *Schistocerca gregaria*” by Graham K Taylor and Rafal Zbikowski

- Introduction

- Approaches to study aerodynamics:
  - Flow visualization
  - Computational fluids dynamics
  - Aircraft stability

- How to model flight of insects



- Approach used for helicopters or airplanes can be a time invariant system:
- $\dot{x} = f(x)$
- A better representation by time variant systems:
- $\dot{x} = f(x, t)$

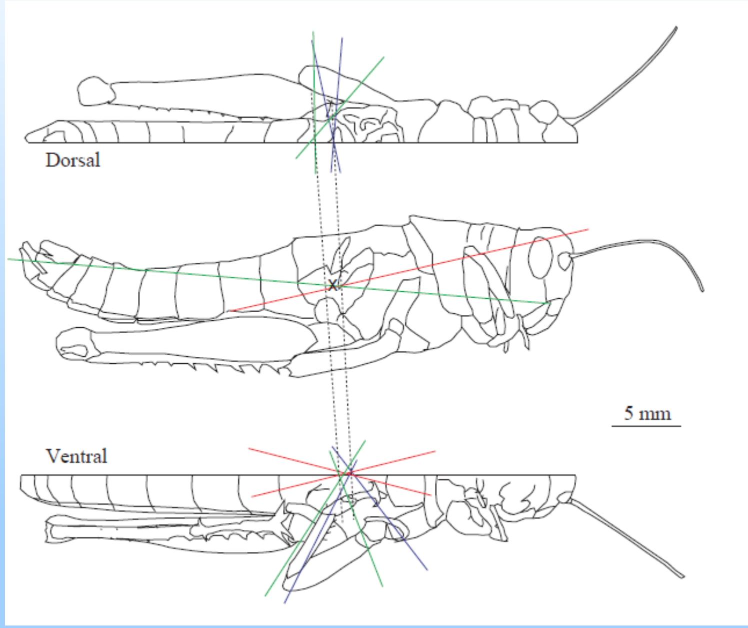
- Since Taylor and Thomas (2003) suggest a linearized framework, but linearized fails to explain stability.

- It is based in Newton-Euler rigid body equations of motion

- Issues on flight of insects

- ❑ It is different to the approach used for helicopters or airplanes
- ❑ How to model instantaneous force production from wings
- ❑ Information from experiments:
- ❑ - Measure forces and moments from wings
- ❑ - Obtain weight and moment of inertia

# What else to know from locusts




- All locusts are significantly different in sizes
- A typical locust flies in cruise speed of 4 m/s
- The span is around 0.1m
- It is flapping at 20Hz
- As any insect has 4 wings
- 2 forewings which sweeps 110°
- 2 hindwings which sweeps 70°
- A locust use its antennae and hair on the head to sense the air speed
- Center of mass is fixed (because 4% of the total weight is in the wings)
- Body is symmetric in the longitudinal axis

## What to know to model a flight of a locust

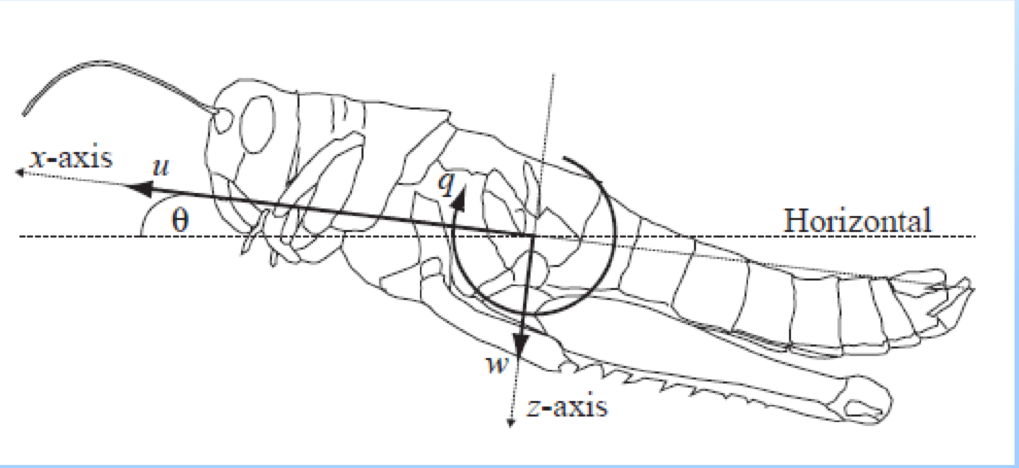
- Newton-Euler Equations

$$\dot{u} = -wq + \frac{X}{m} - g\sin\theta$$

$$\dot{w} = uq + \frac{Z}{m} + g\cos\theta$$

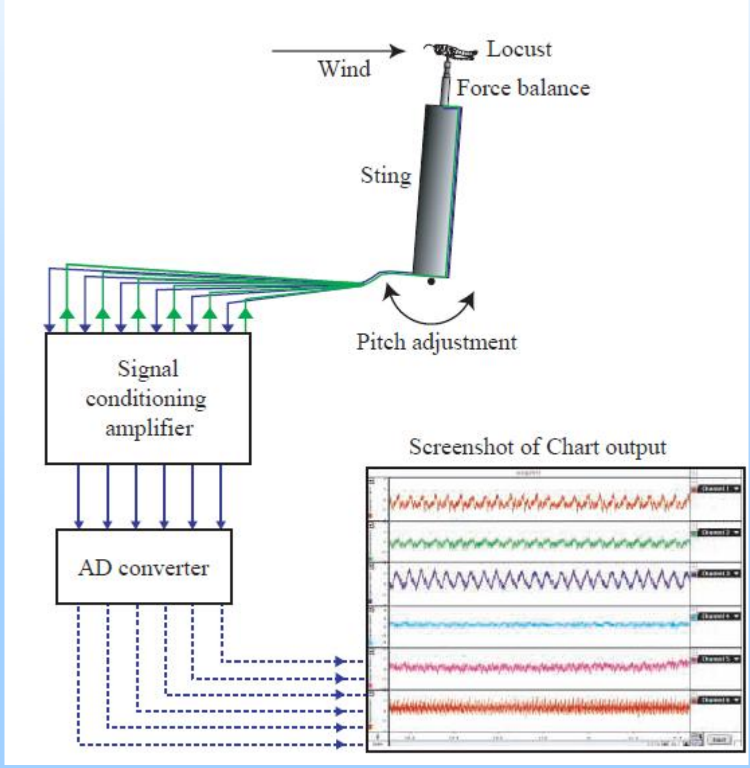
$$\dot{q} = \frac{M}{I_{yy}}$$

$$\dot{\theta} = q$$




- From these equations implies that we have:
- Center of gravity is fixed
- Body is symmetric in the longitudinal axis
- Four state variables
- Constant mass, moment of inertia and gravity

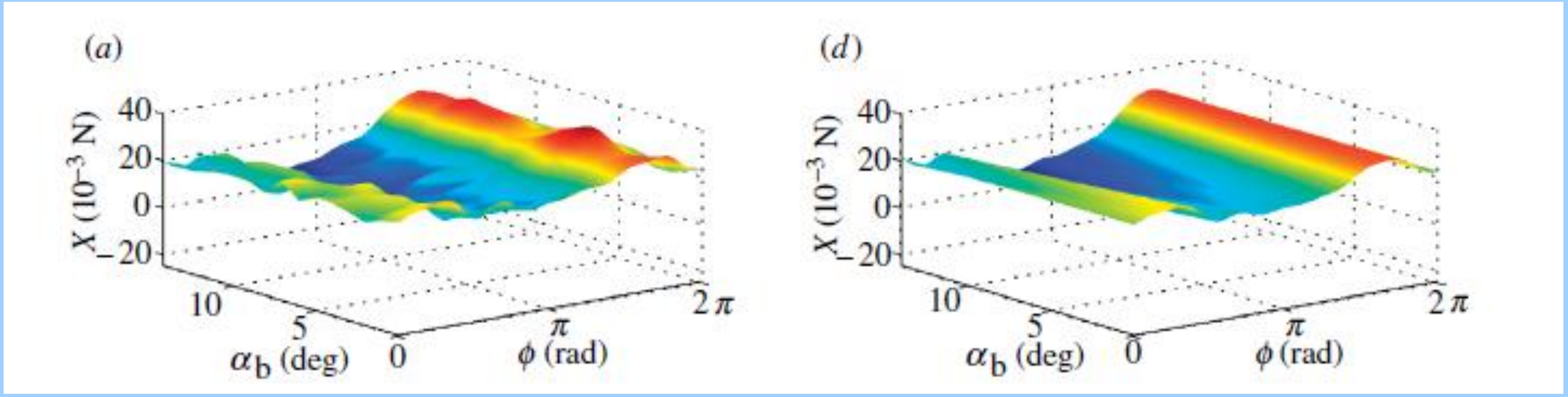
# What is needed from experiments



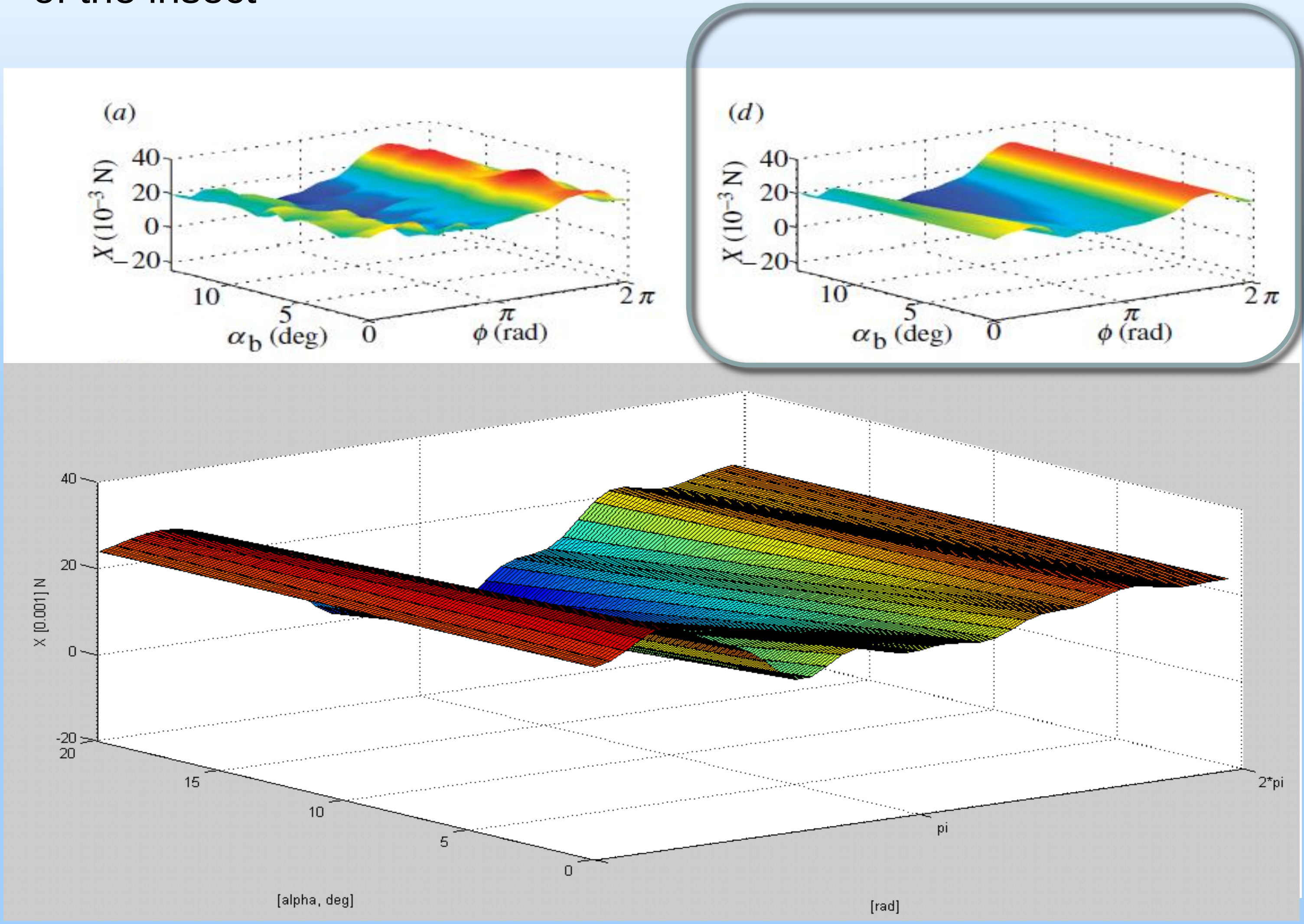

- Three type of locusts
- It is used a wind tunnel
- Data collected under different angles (from 0 to 14 degrees)
- Data collected under different velocities of the wind tunnel (from 2 to 5.5 m/s)
- It was taking between 2-3h for collecting data for each locust

We also need to know:

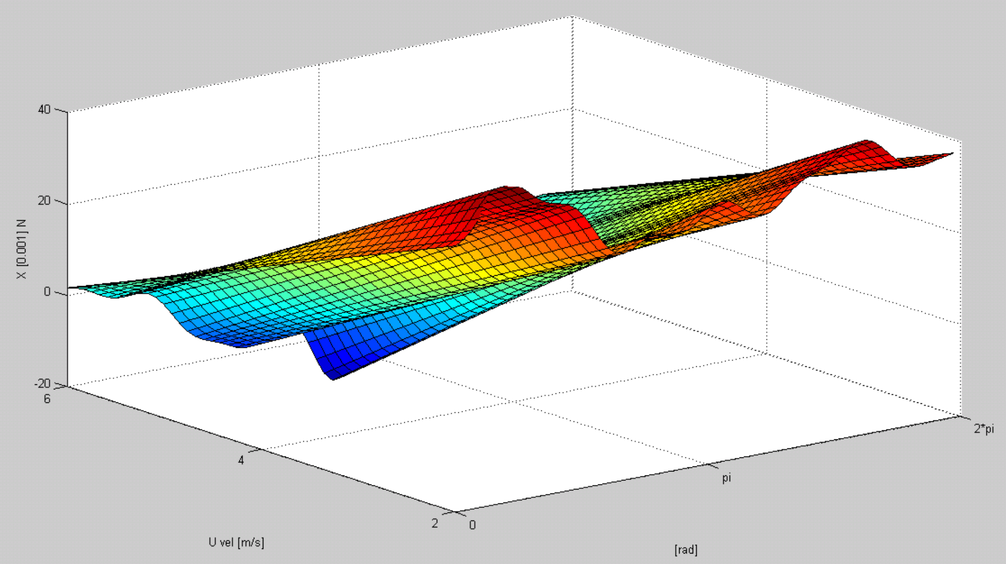
- Incomplete data in locust that failed in the experiment were discarded
- A systematic variation in wing beat frequency will alter the dynamics
- Data from experiments can be well fitted in Fourier series using until the eight harmonics order.

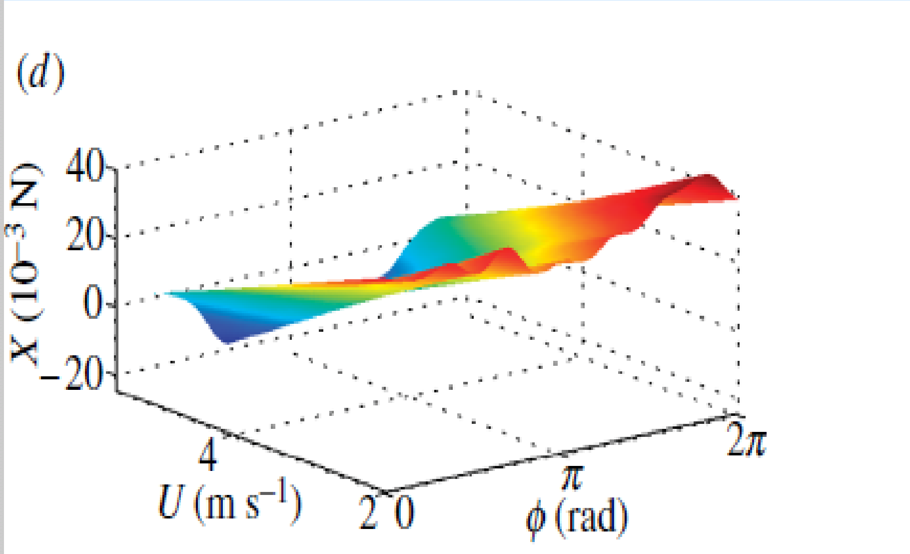


Plot reproduced for the X force for one wing beat period at different angles of the insect

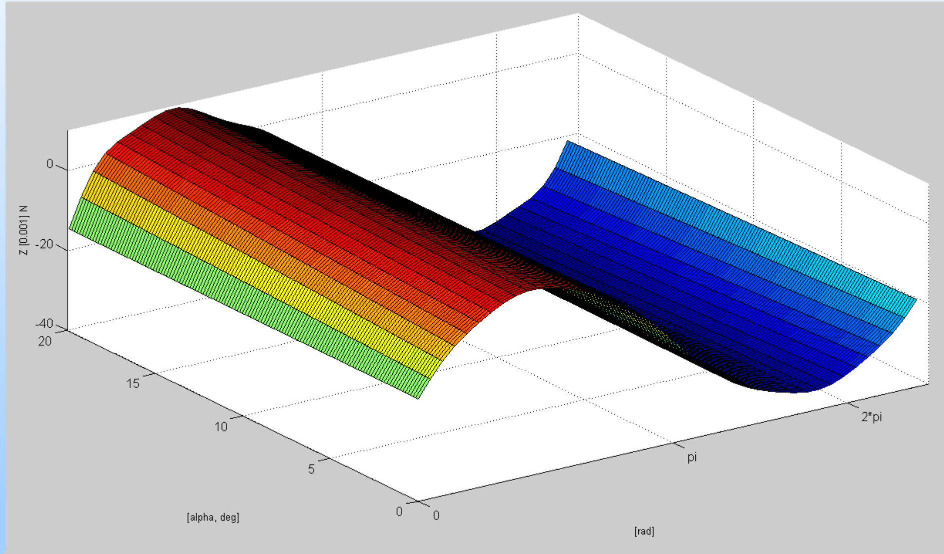


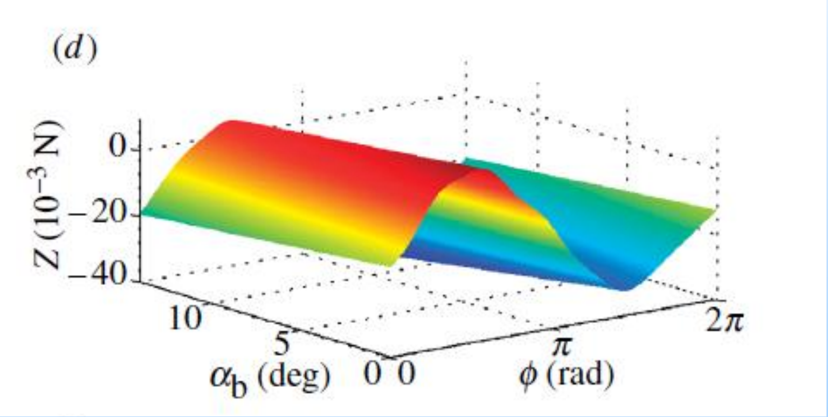
- Plot reproduced for X force
- For one wing beat period
- At different values of U velocity.

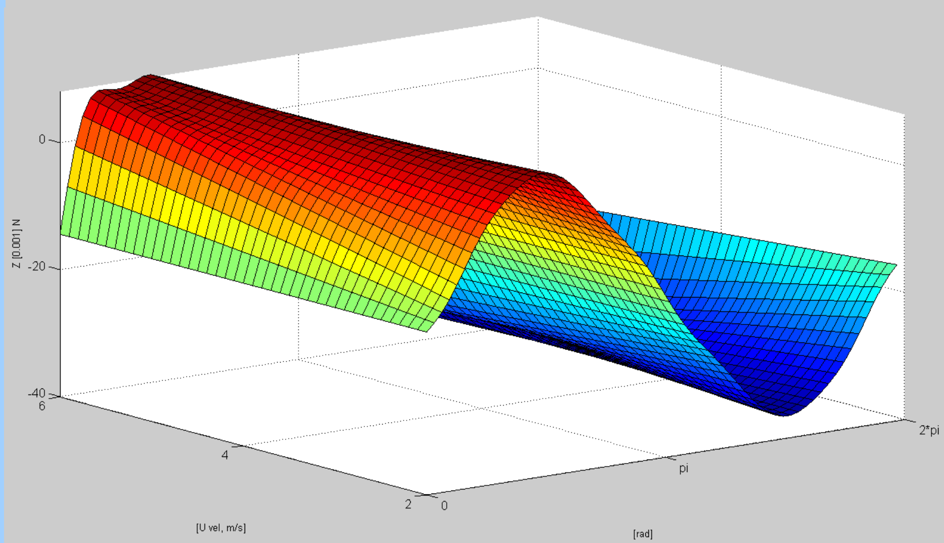


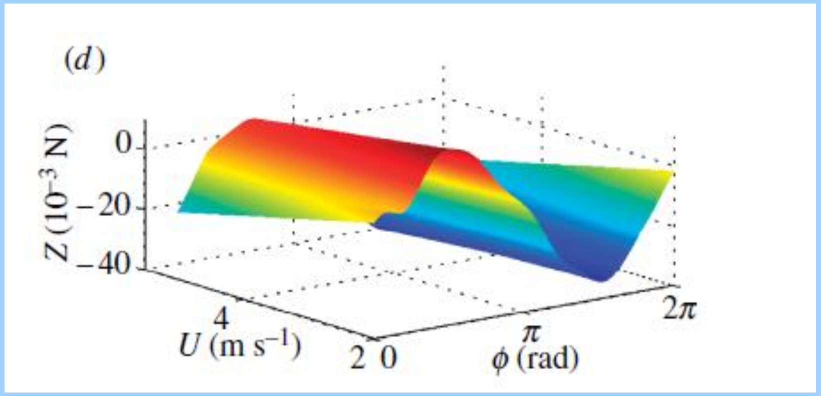



- Z force for one wing beat period

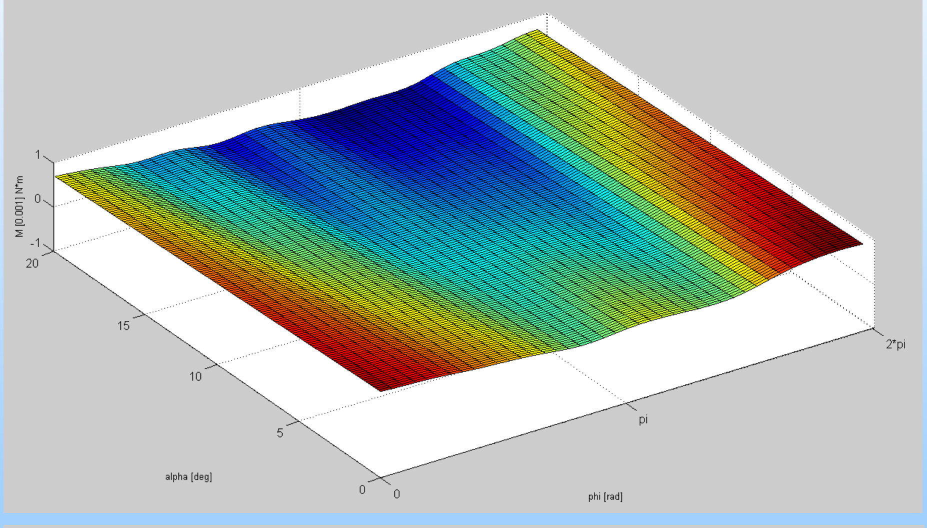


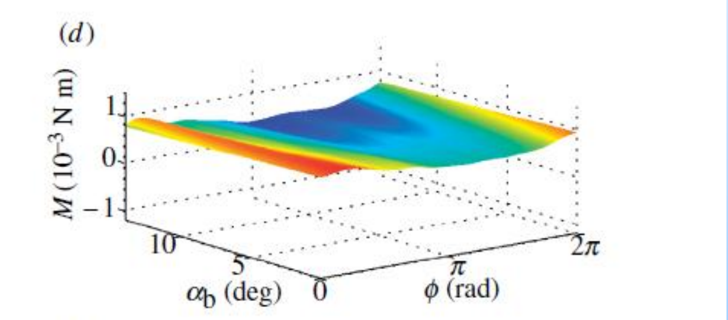



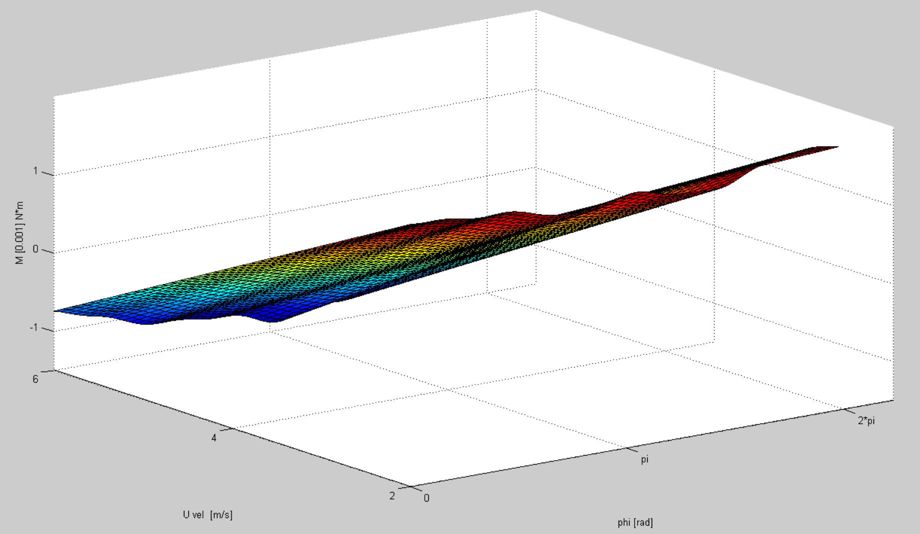



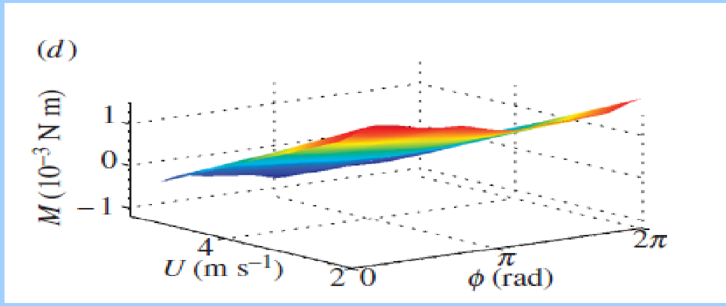



- M moment for one wing beat period



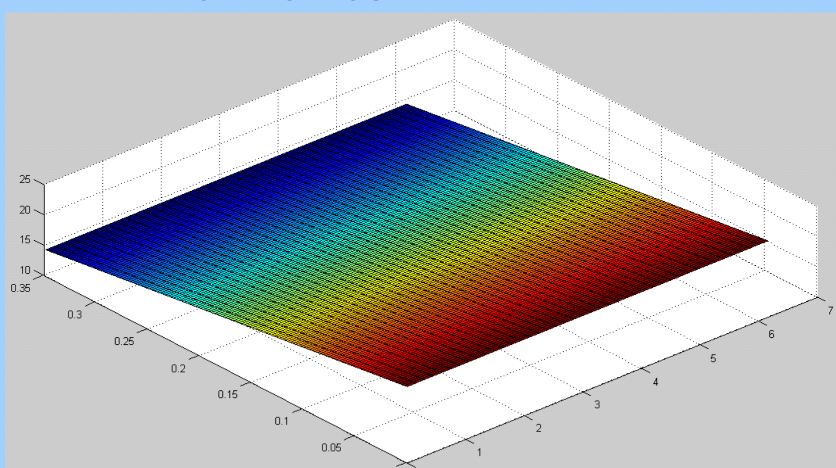



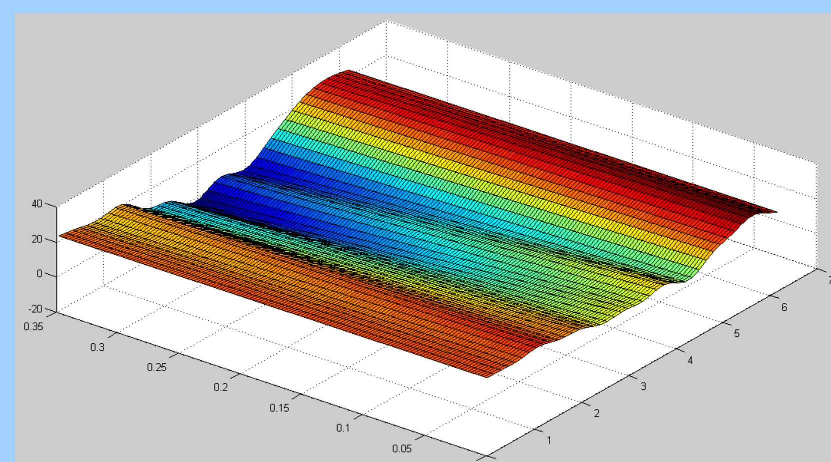
We also need to know:

- Fourier series equation:

$$P(t) = \sum_{n=0}^{h} (a_n \cos n\omega t + b_n \sin n\omega t)$$

- From the Fourier series we can have two types of models:
- Nonlinear time invariant (NLTI) model just considering the zero harmonics
- Nonlinear time periodic model (NLTP) considering until the eight harmonics



- Equations used to represent forces and moment

$$X(\alpha,U,t) = \sum_{n=0}^{8}\left(a_{1,n}\cos n\omega t + b_{1,n}\sin n\omega t\right) + \left(\alpha - \alpha_{ref}\right)\sum_{n=0}^{8}\left(a_{2,n}\cos n\omega t + b_{2,n}\sin n\omega t\right) + (U - U_{ref})\sum_{n=0}^{8}(a_{3,n}\cos n\omega t + b_{3,n}\sin n\omega t)$$

$$Z(\alpha,U,t) = \sum_{n=0}^{8}\left(a_{1,n}\cos n\omega t + b_{1,n}\sin n\omega t\right) + \left(\alpha - \alpha_{ref}\right)\sum_{n=0}^{8}\left(a_{2,n}\cos n\omega t + b_{2,n}\sin n\omega t\right) + (U - U_{ref})\sum_{n=0}^{8}(a_{3,n}\cos n\omega t + b_{3,n}\sin n\omega t)$$

$$M(\alpha,U,t) = \sum_{n=0}^{8}\left(a_{1,n}\cos n\omega t + b_{1,n}\sin n\omega t\right) + \left(\alpha - \alpha_{ref}\right)\sum_{n=0}^{8}\left(a_{2,n}\cos n\omega t + b_{2,n}\sin n\omega t\right) + (U - U_{ref})\sum_{n=0}^{8}(a_{3,n}\cos n\omega t + b_{3,n}\sin n\omega t)$$

- In summary, previous equations follows:

$$P(\alpha,U,t) = P_{ref}(t) + P_{\alpha}(t)(\alpha - \alpha_{ref}) + P_U(t)(U - U_{ref})$$

- Where P involves a vector with the forces:

$$P = [X, \quad Z, \quad M]$$

- From the original Newton-Euler equations:

$$\dot{u} = -wq + \frac{X}{m} - g\sin\theta$$

$$\dot{w} = uq + \frac{Z}{m} + g\cos\theta$$

$$\dot{q} = \frac{M}{I_{yy}}$$

$$\dot{\theta} = q$$

- It's going to look as:

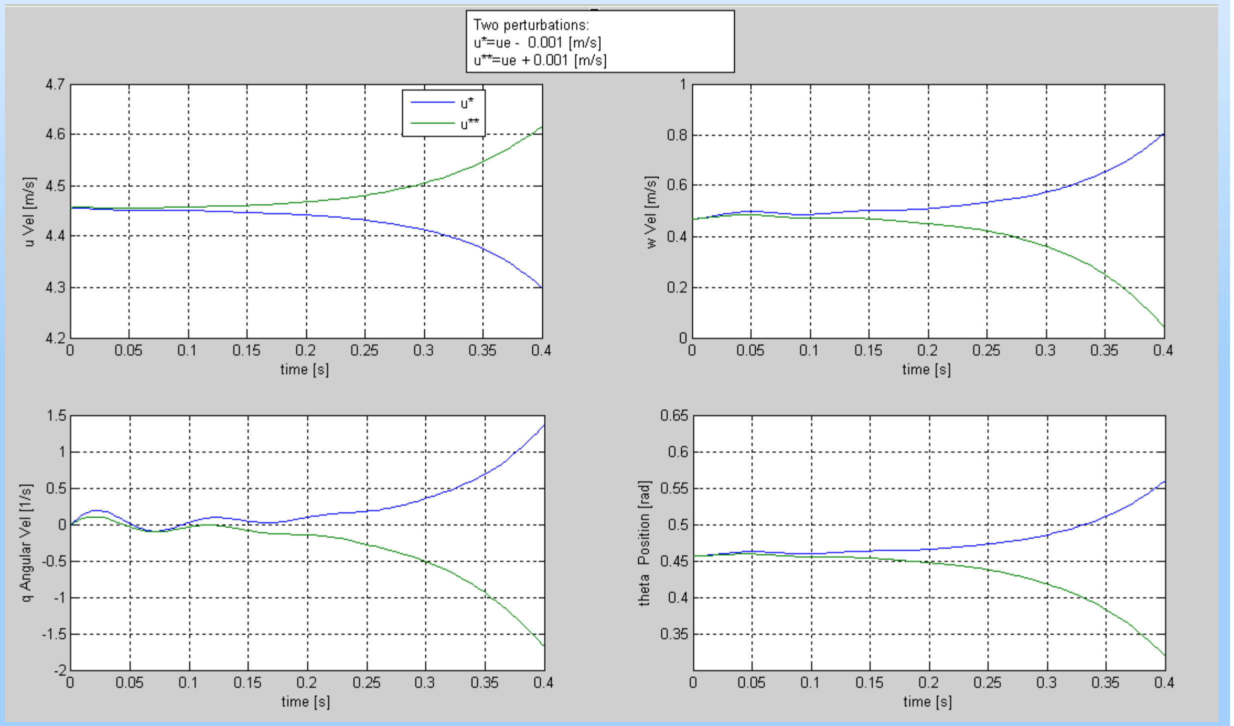
$$\dot{u} = -wq + \frac{X_{ref}(t)}{m} + \frac{X_{\propto}(t)}{m}\left(tan^{-1}\frac{w}{u} - \propto_{ref}\right) + \frac{X_U(t)}{m}\left(\sqrt{u^2 + w^2} - U_{ref}\right) - g\sin\theta$$

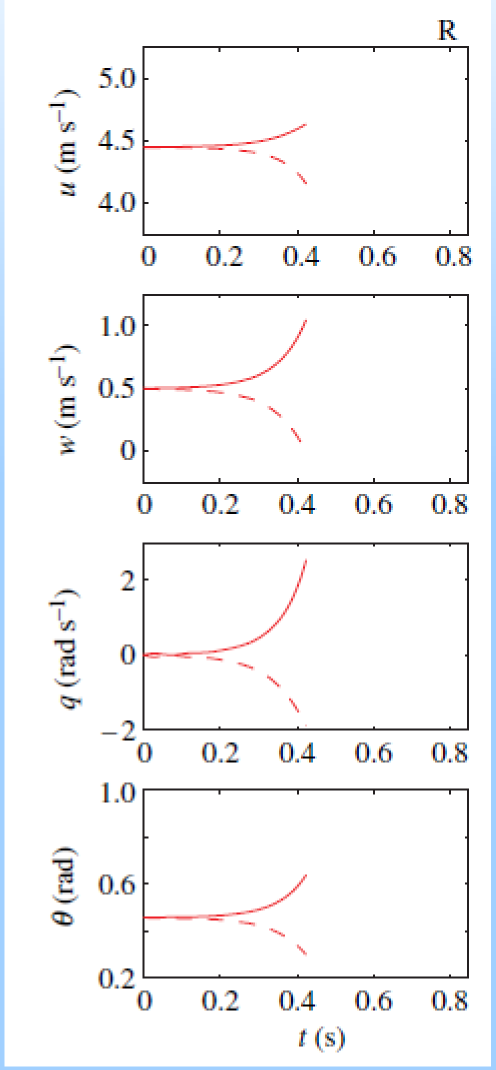
$$\dot{w} = uq + \frac{Z_{ref}(t)}{m} + \frac{Z_{\propto}(t)}{m}\left(tan^{-1}\frac{w}{u} - \propto_{ref}\right) + \frac{Z_U(t)}{m}\left(\sqrt{u^2 + w^2} - U_{ref}\right) + g\cos\theta$$

$$\dot{q} = \frac{M_{ref}(t)}{I_{yy}} + \frac{M_{\propto}(t)}{I_{yy}}\left(tan^{-1}\frac{w}{u} - \propto_{ref}\right) + \frac{M_U(t)}{I_{yy}}\left(\sqrt{u^2 + w^2} - U_{ref}\right)$$

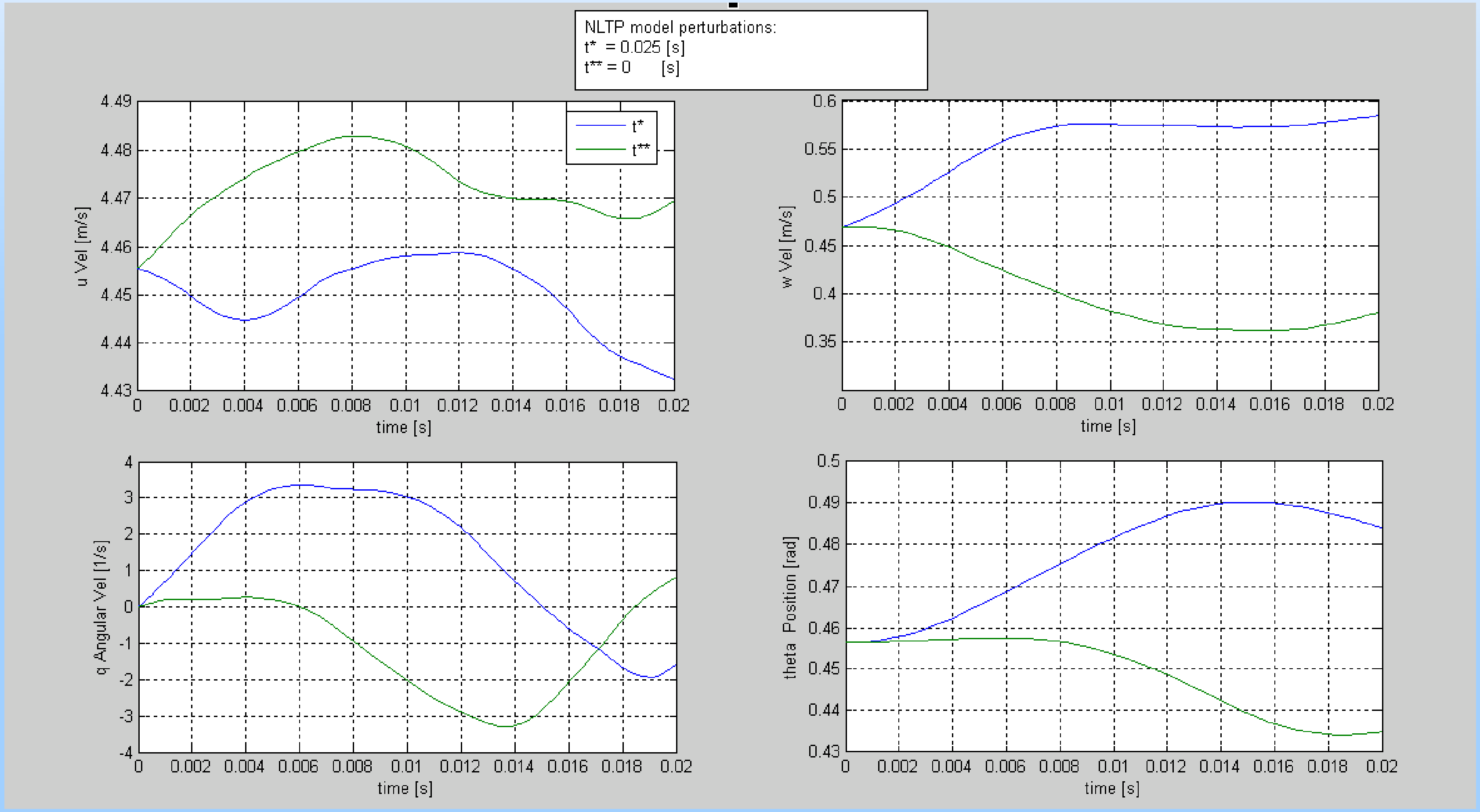
$$\dot{\theta} = q$$

# Response to minimum changes in the horizontal velocity for the Nonlinear time invariant model (NLTI)

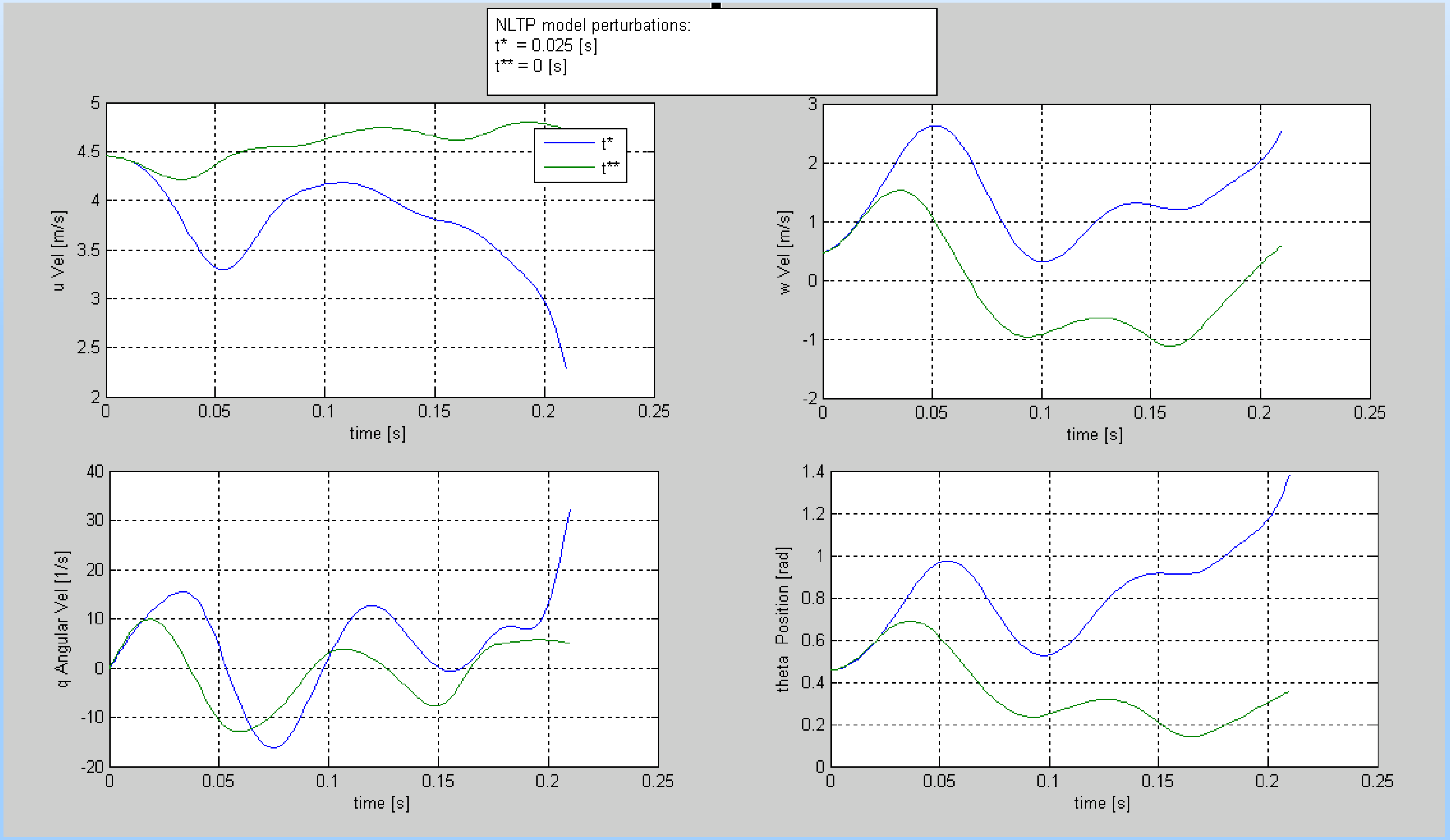


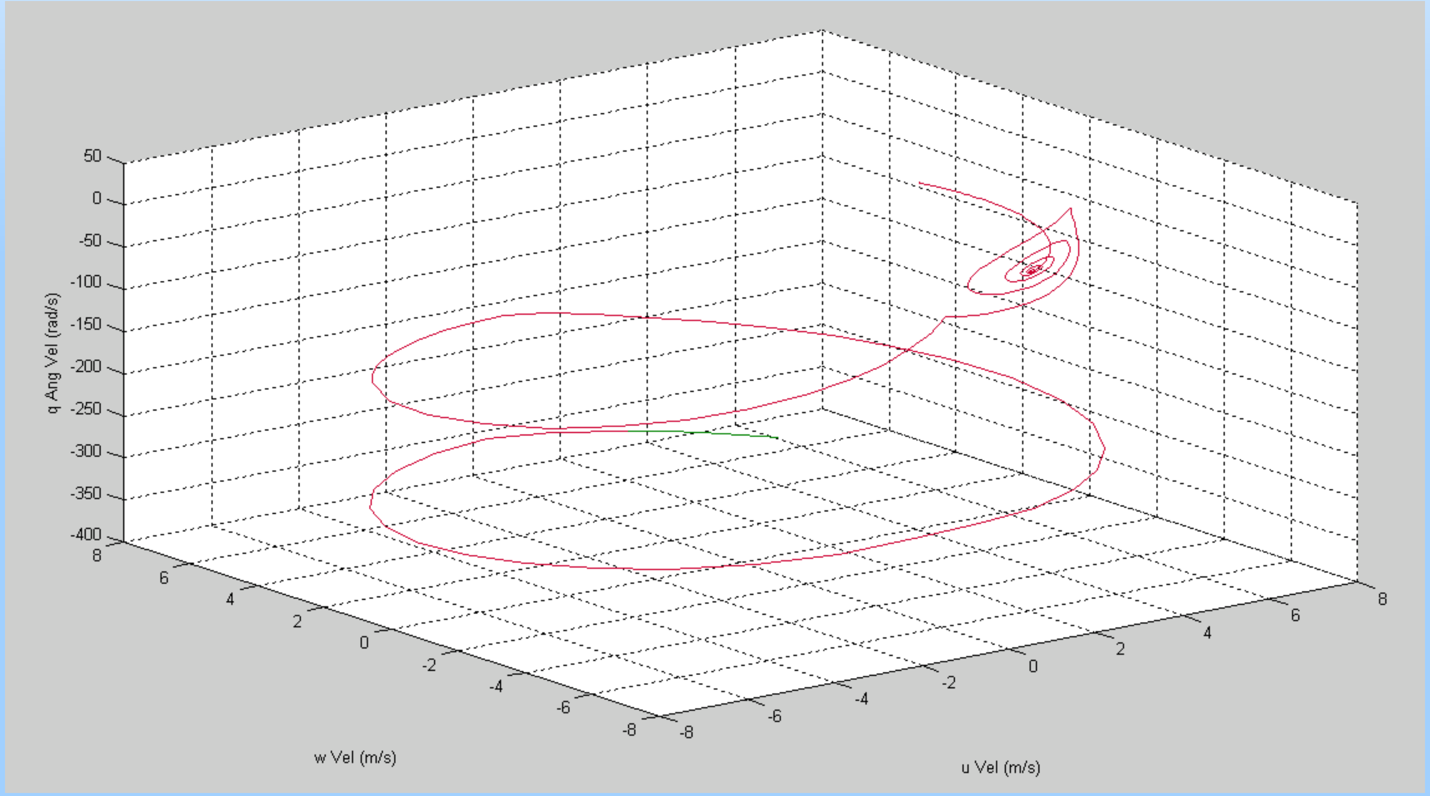
# Response to minimum changes in the initial value of time for the Nonlinear time periodic(NLTP) model.



# Response to minimum changes in the initial value of time for the Nonlinear time periodic(NLTP) model.



# How the value θ is changing with respect to the other variables (u, w and q):



## DISCUSSION AND DRAWBACKS

- Unstable model because
- - Experimental data may be not representative of a free flight
- - Data may be altered because of instruments used
- - Center of mass is constant and the moment of inertia is time invariant
- - There are still some limitations in the data collection

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# FUTURE WORK

- Show results for the other two type of locust
- Obtain plot for longer time of simulation
- Include any possible technique to control the flight (Variable coefficients in the Fourier Series)

# QUESTIONS?