

Memristor – New Electronic Device

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Classical Circuits

Four Fundamental Circuit Variables:

1. Charge – q
2. Voltage – v
3. Current – I
4. Magnetic Flux – ϕ

Three Classical Fundamental Circuit Elements:

1. Capacitor: $dq = C dv$
2. Resistor: $dv = R di$
3. Inductor: $d\phi = L di$

Two Additional Relationships:

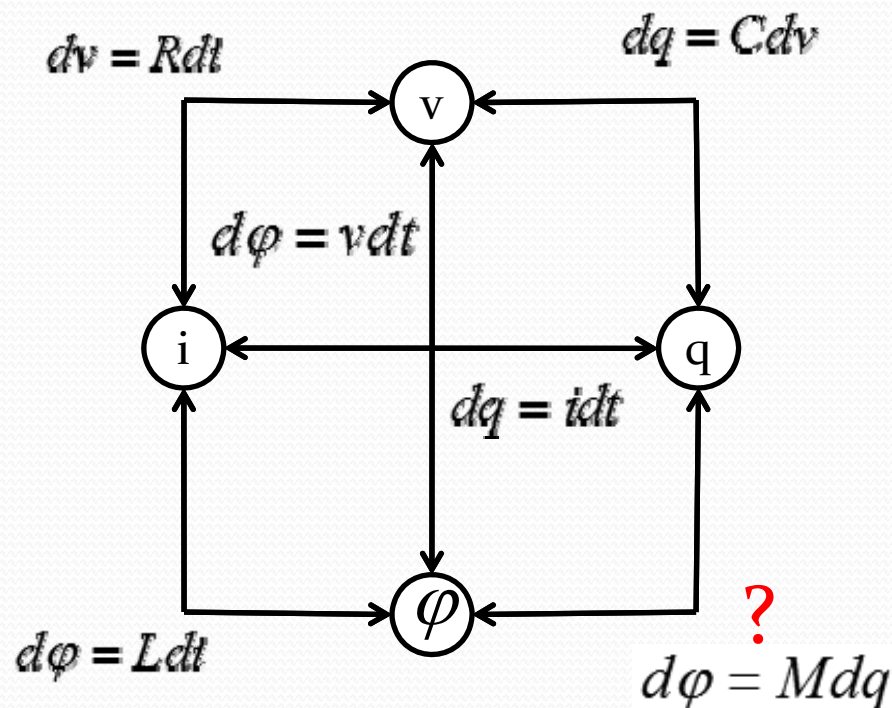
1. $dq = i dt$
2. $d\phi = v dt$

Modeling of RLC Circuit with Applied Voltage

$$L Q'' + R Q' + Q/C = V(t)$$

Existence of the Memory Resistor

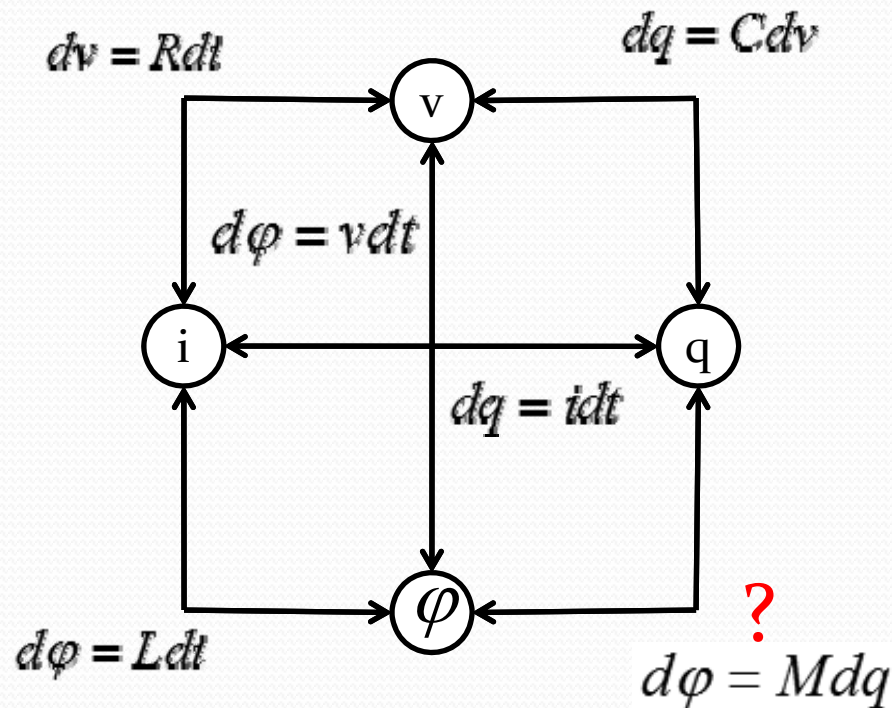
$$v = \int M(q) dq$$



- An element that is:
 - Passive
 - Dissipative
 - 2-terminal
- Effectively a resistor with resistance that depends on charge that has passed through - memory

Existence of the Memory Resistor

$$v = \int M(q) dq$$



- Applications
 - Higher density circuits
 - Unique modeling possibilities
 - Generation of special waveforms
 - Signal processing

Memristor - The Fourth Element

- Proposed by Leon Chua in 1971 using an argument based on symmetry.
- Described by the sixth relationship between the four fundamental circuit variables: $d\phi = M dq$
- Faraday's Law of Induction states the induced EMF or voltage in a closed circuit is equal to the time rate of change in magnetic flux. Therefore, the memristor equation can be expressed as the following:

$$v = M(q)i$$

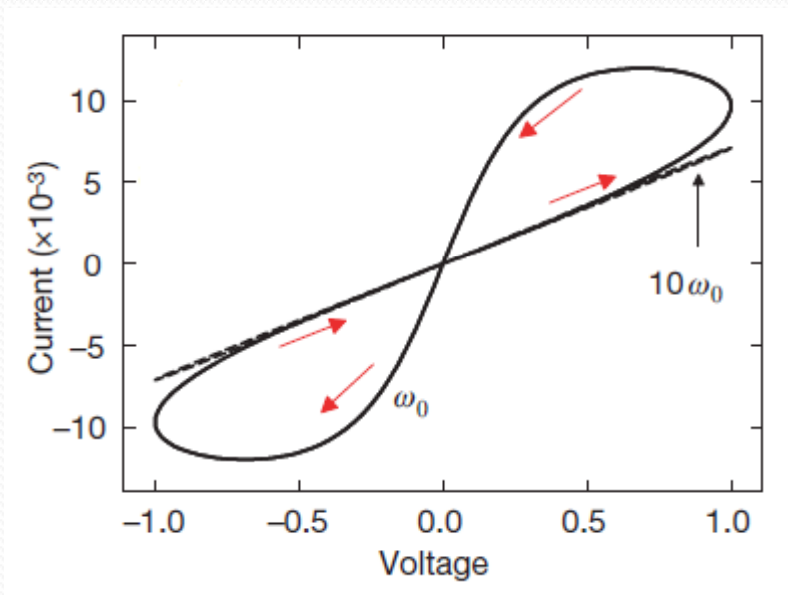
- Similar equation to resistors described by Ohm's Law ($v = R i$). Memristance can reduce to resistance if certain conditions are met.
- Memristor combination of "memory" and "resistor".
- Symbol for memristor:



Properties of Memristors

- Non-linear relationship between current and voltage.
- Reduces to resistor for large frequencies as evident in the i-v characteristic curve. May also reduce to a resistor based on defined state variables.
- Memory capacities based on different resistances produced by the memristor.
- Non-volatile memory possible if the magnetic flux and charge through the memristor have a positive relationship ($M > 0$).
- Does not store energy.
- Similar to classical circuit elements, a system of memristors can also be described as a single memristor.

i-v Characteristic Curve



- Pinched hysteresis unique to memristors.
- No combination of other fundamental circuit elements makes Lissajous figure.
- Proof by Chua and Kang published in 1976.

Strukov, et. al. Nature Vol 453 (2008) 80-83

Chua, LO, Kang SM Proceedings of the IEEE 64 Issue 2 (1976)

Generalization of Memristance

Recall the derived equation for memristance:

$$v = M(q)i$$

This can be generalized further by considering a set of state variables $\mathbf{x} = (x_1, x_2, \dots, x_n)$. These state variables are dependant on the specific implementation of the memristor. We can use the state variables to make a substitution in our memristance equation.

$$v = M(\mathbf{x})i \quad \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, i)$$

The state variables must be related to the current. This generalization leads to a unique set of equations for different memristors and memristive systems.

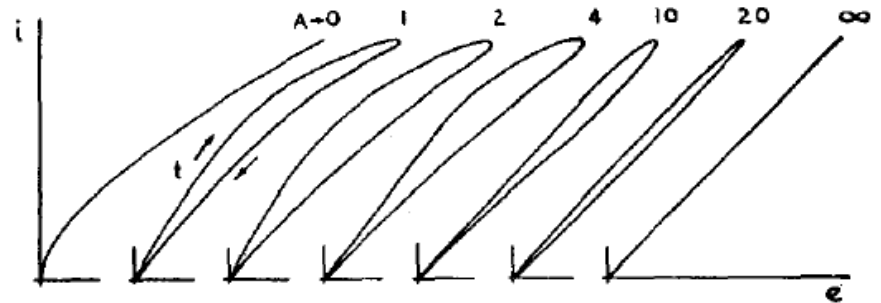
Example of a Memristive System

Consider a light bulb. In general, a light bulb can be thought of as a resistor. However, as the filament heats up, the resistance of the bulb increases. This behavior creates a non-linear resistance which can be described with the following temperature-dependant equations:

$$V = (R_0 T)I \equiv M(T)i$$

$$dT/dt = aTi^2 - b(t^4 - t_0^4) \equiv f(T,i)$$

where R_0 , T_0 , a , and b are constants. These equations satisfy our conditions for a memristive system.



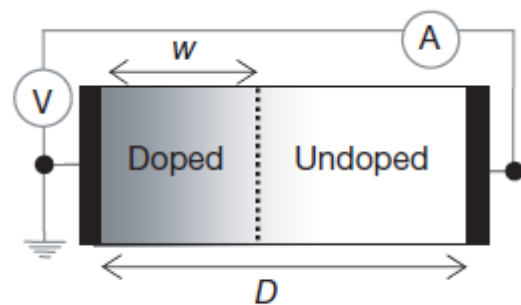
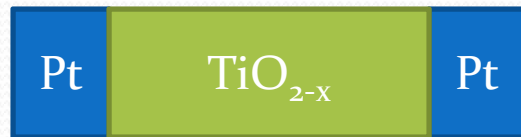
Difficulties in Finding Memristors

- Why has it taken almost four decades to find a memristor?
- Memristors are not new. Memristive properties have been observed by researchers for more than three decades.
- Pinched hysteresis curve is a unique property of a memristor. However, researchers did not made the connection with their observations.
- Often described as anomalous inductance/resistance and disregarded in certain practical applications (e.g. Josephson Junction).
- Direct link between charge and magnetic flux not necessary.

Formation and Applications

- Physical model
- Electroforming
- Light emitting memristor
- Memristors in logic gates
- Modeling simple learning

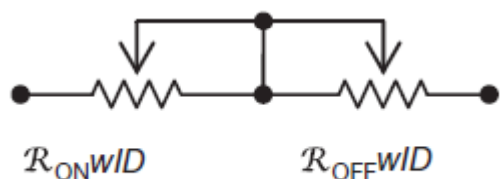
Memristor Found



Undoped:



Doped:



$$R_{Total} = R_{ON} \frac{w(t)}{D} + R_{OFF} \left(1 - \frac{w(t)}{D} \right)$$

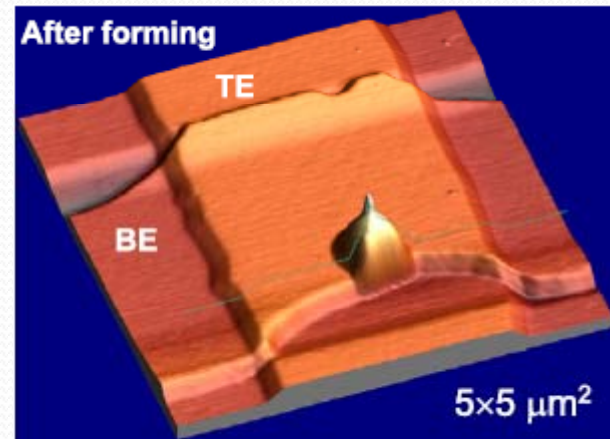
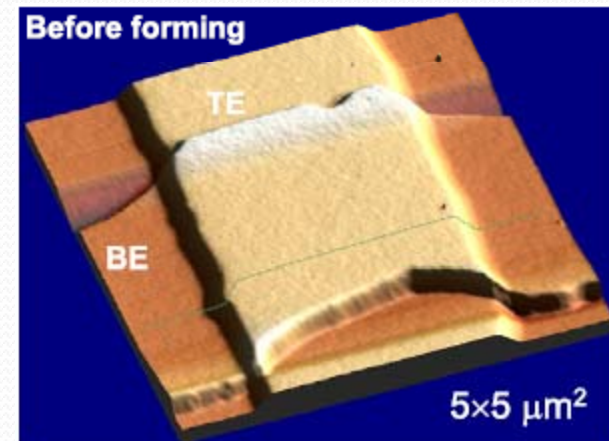
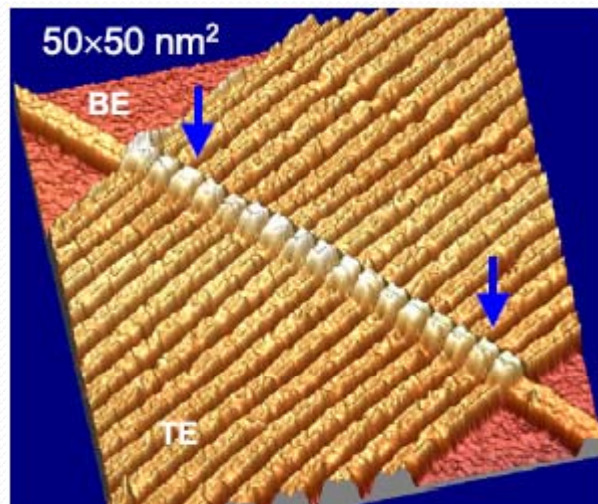
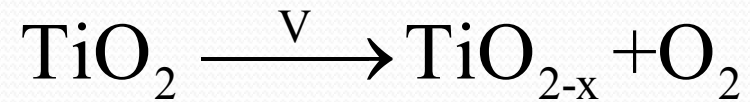
$$v(t) = \left(R_{ON} \frac{w(t)}{D} + R_{OFF} \left(1 - \frac{w(t)}{D} \right) \right) i(t)$$

$$\frac{dw(t)}{dt} = \mu_V \frac{R_{ON}}{D} i(t)$$

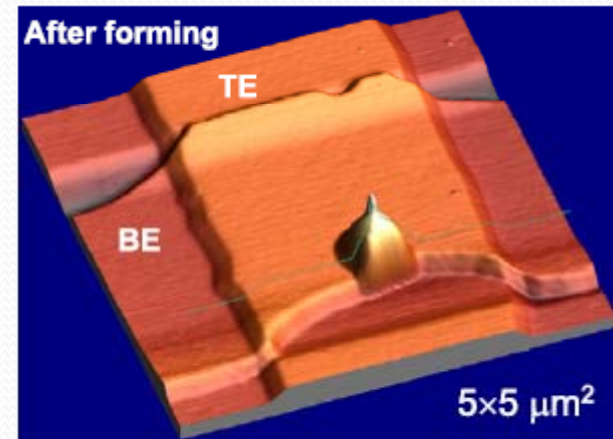
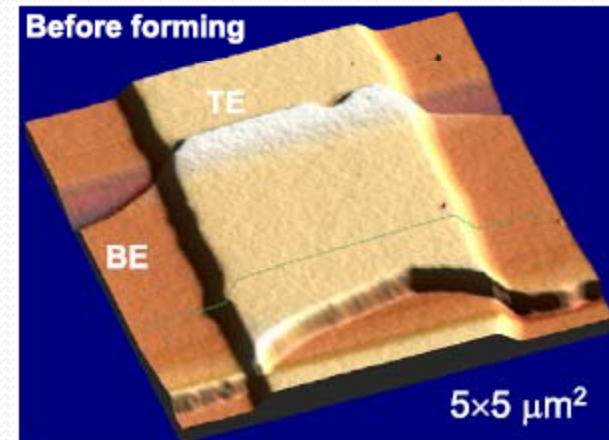
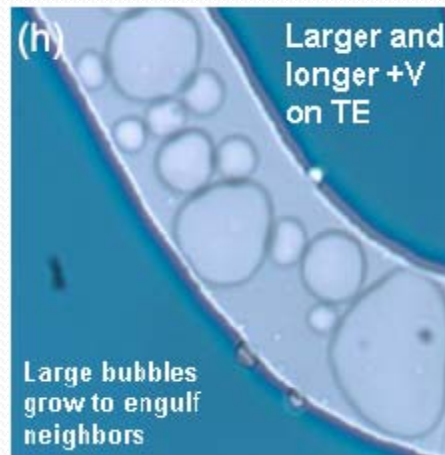
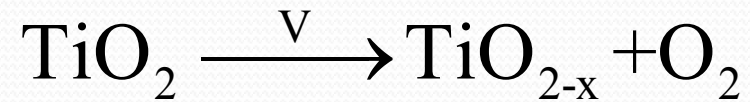
$$w(t) = \mu_V \frac{R_{ON}}{D} q(t)$$

$$M(q) = R_{off} \left(1 - \frac{\mu_V R_{on}}{D^2} q(t) \right)$$

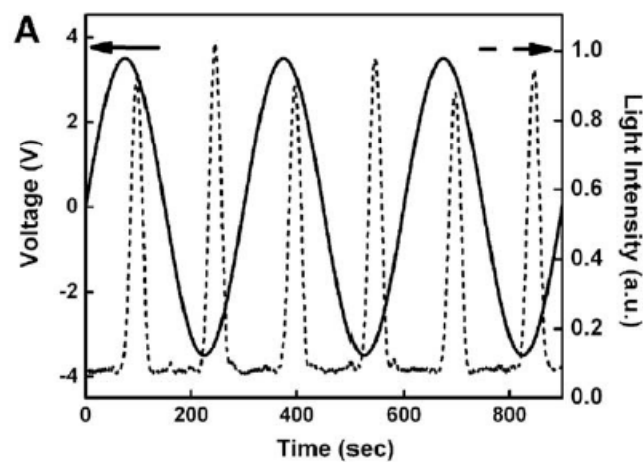
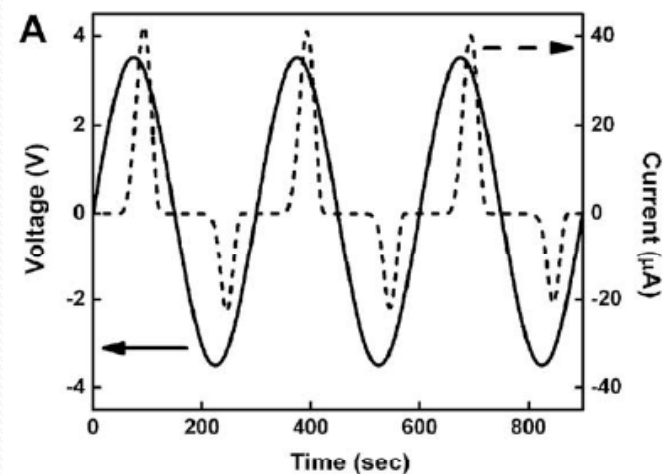
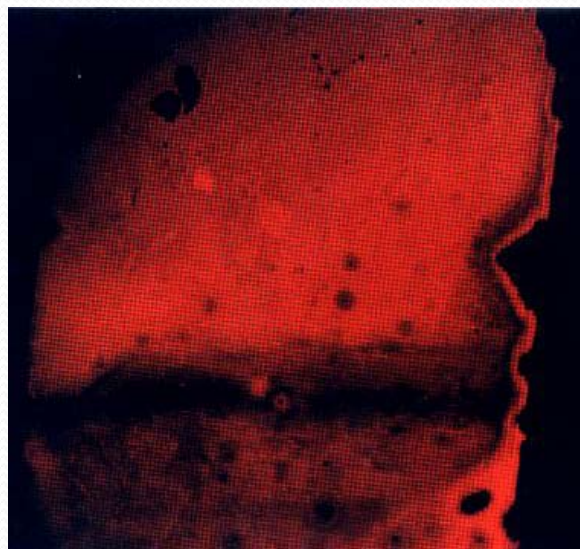
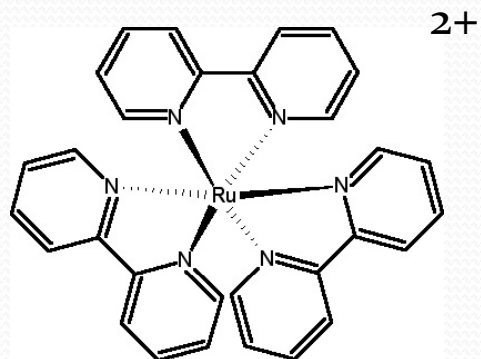
Formation of TiO₂ Memristor



Formation of TiO₂ Memristor

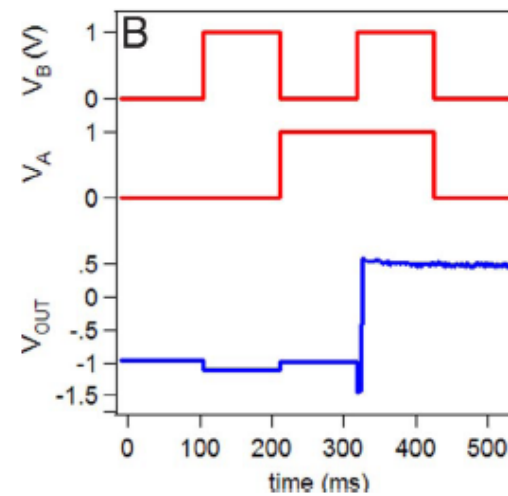
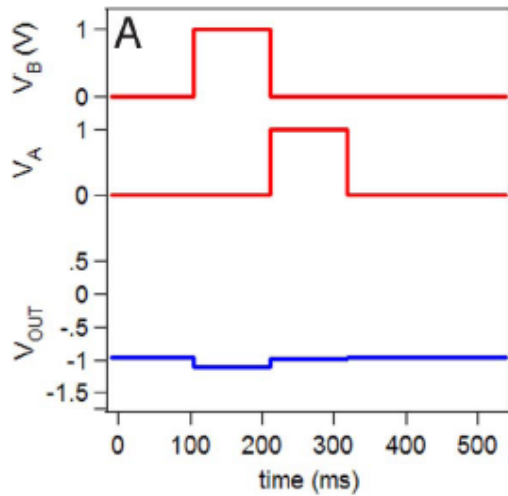
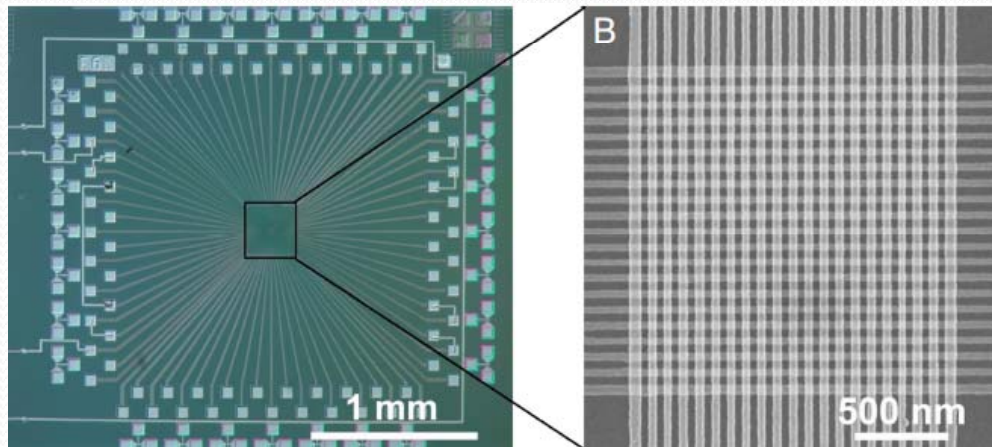


Light Emitting Memristor

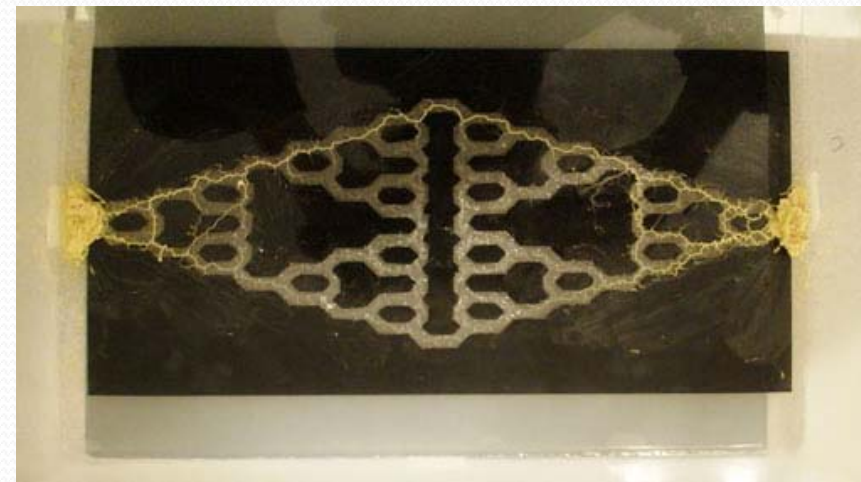
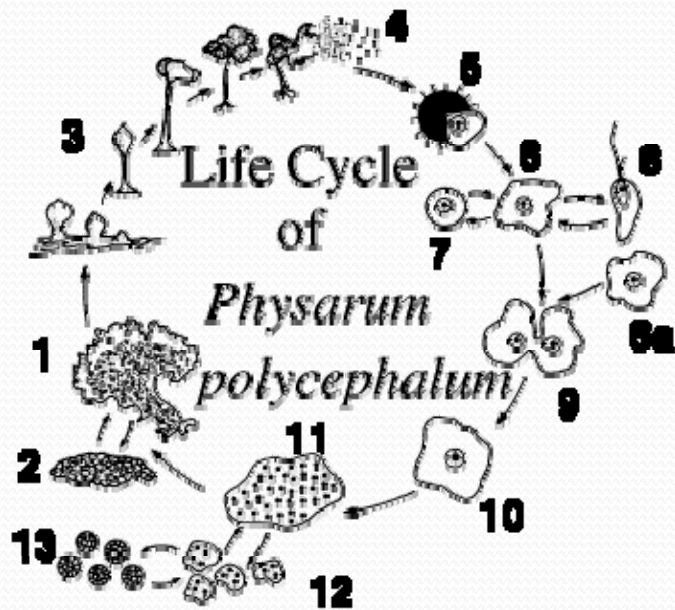


Zakhidov, et. al. Organic Electronics 11 (2010) 150-153

Memristive Logic Gates

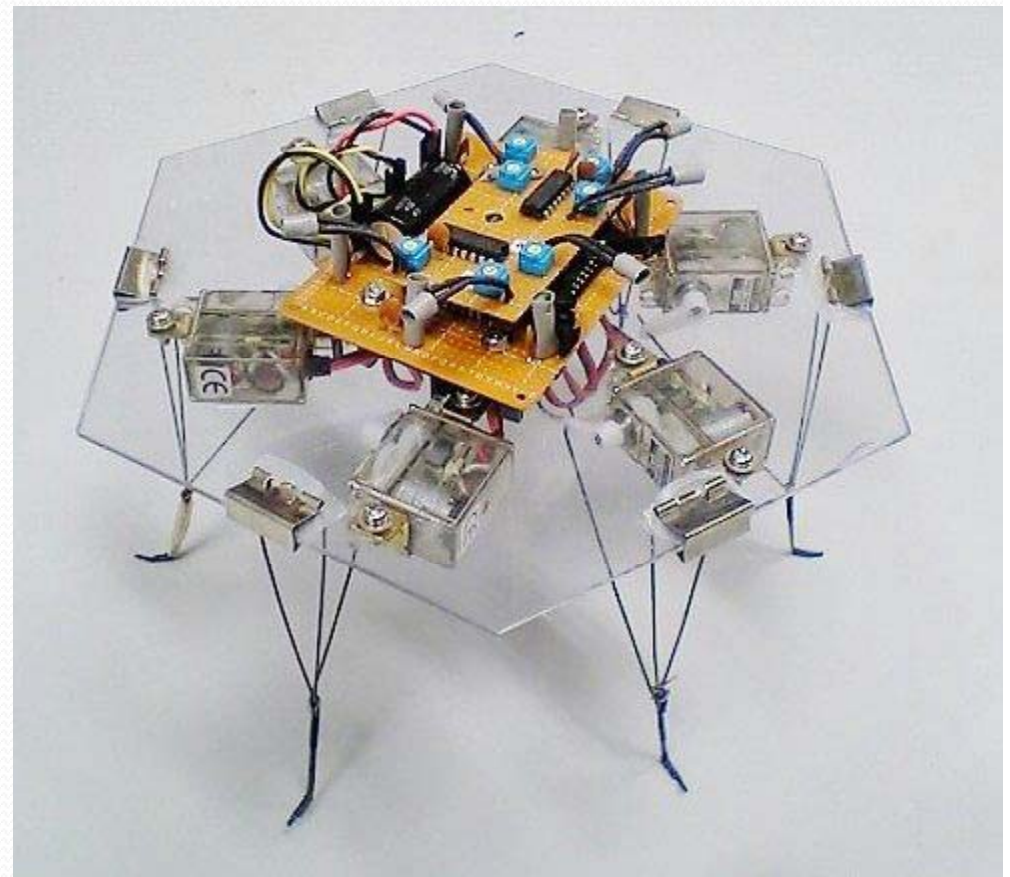
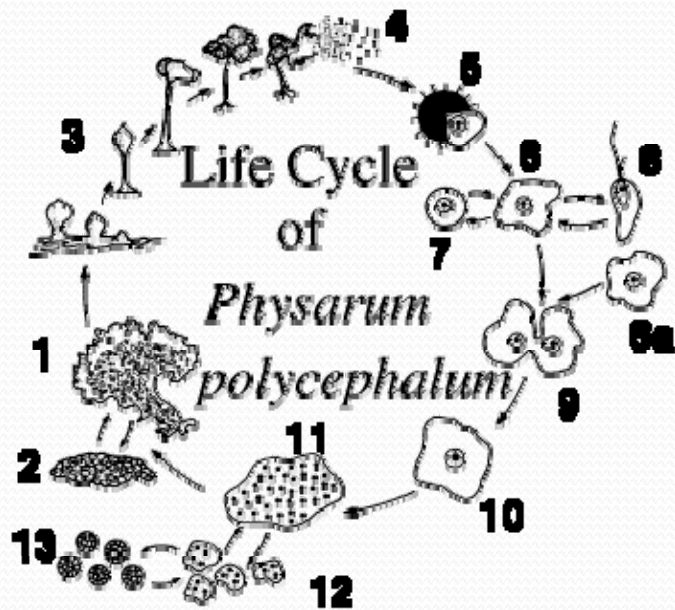


Simple “Learning” Circuit



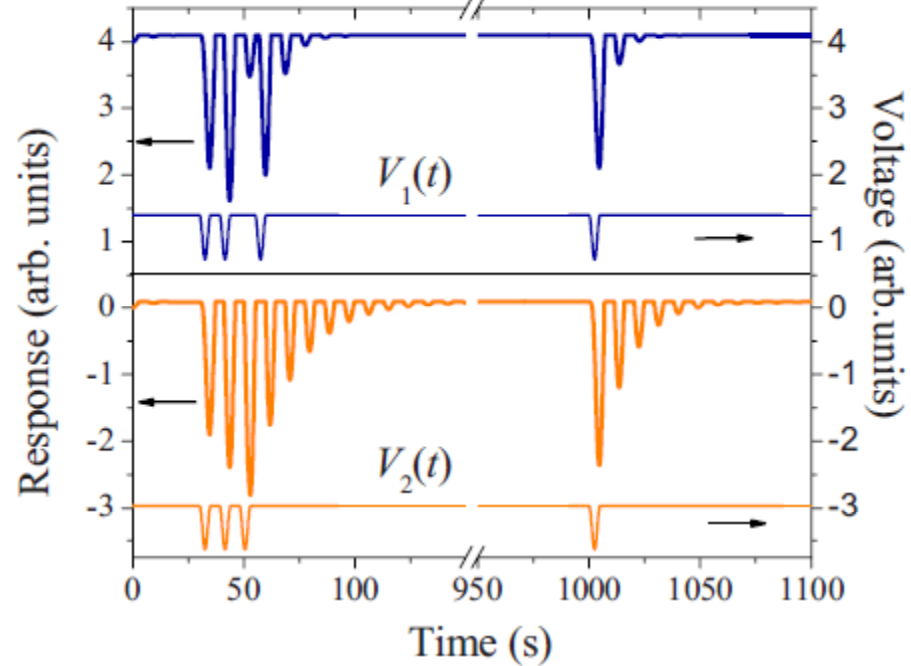
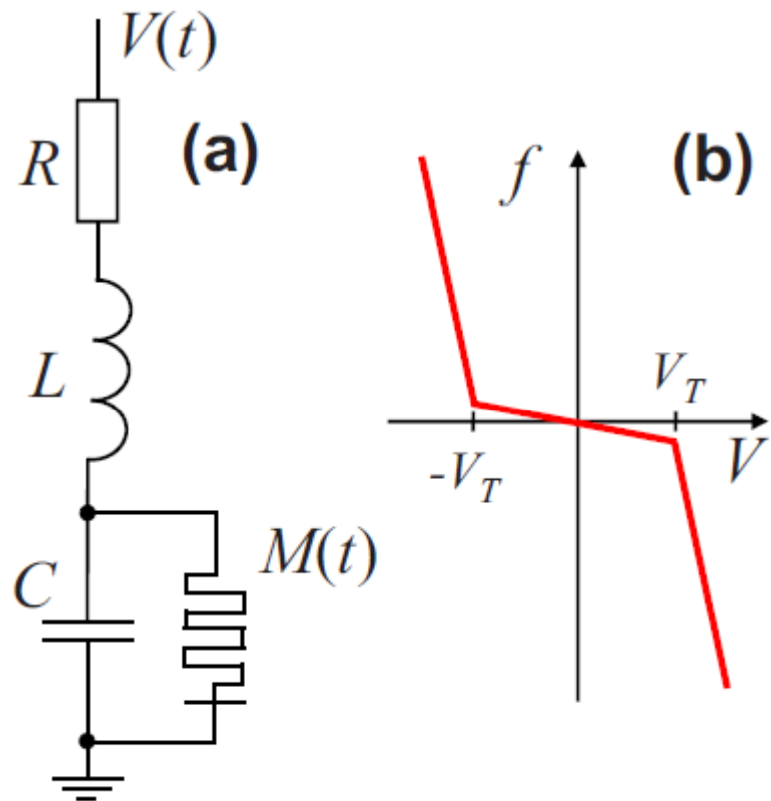
Pershin, et. al. Physical Review E **80** (2009) 021926

Simple “Learning” Circuit



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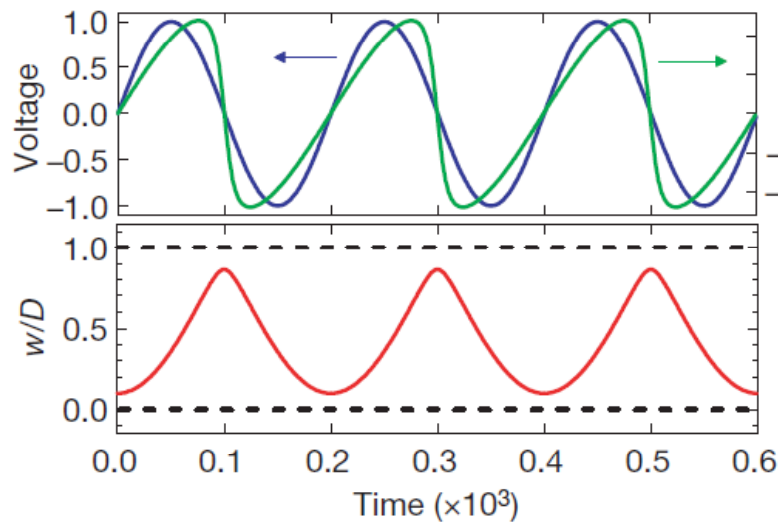
Simple “Learning” Circuit



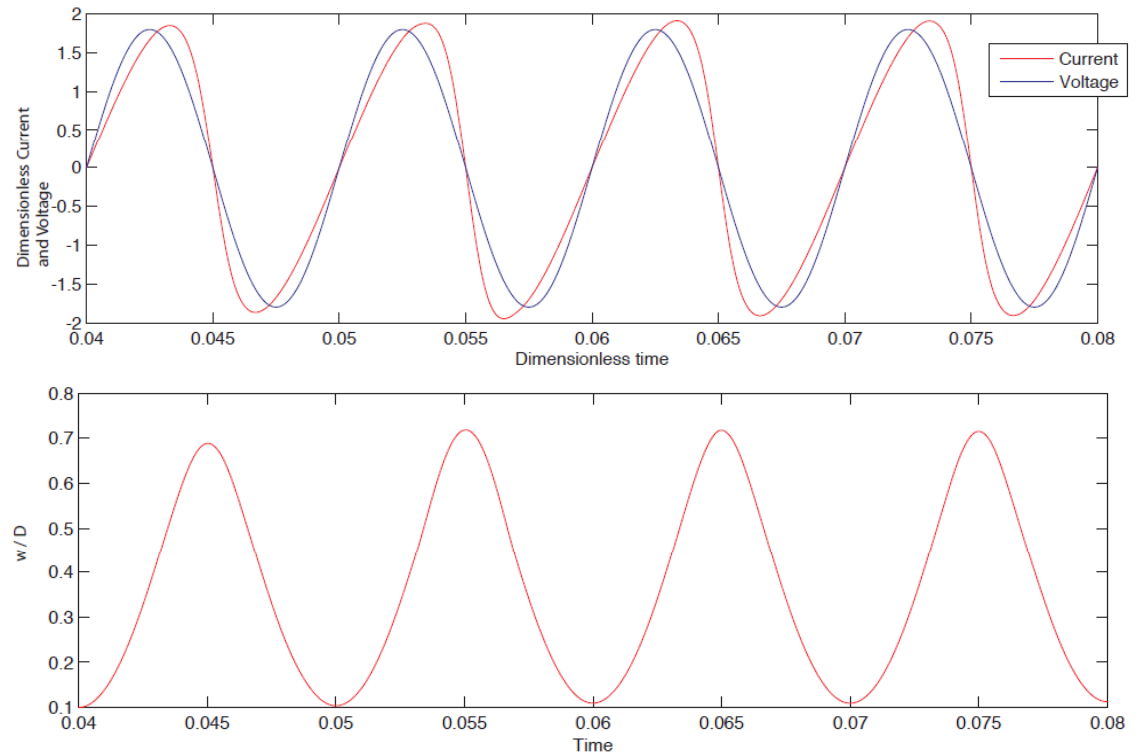
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HP Simulations

HP Nature Paper



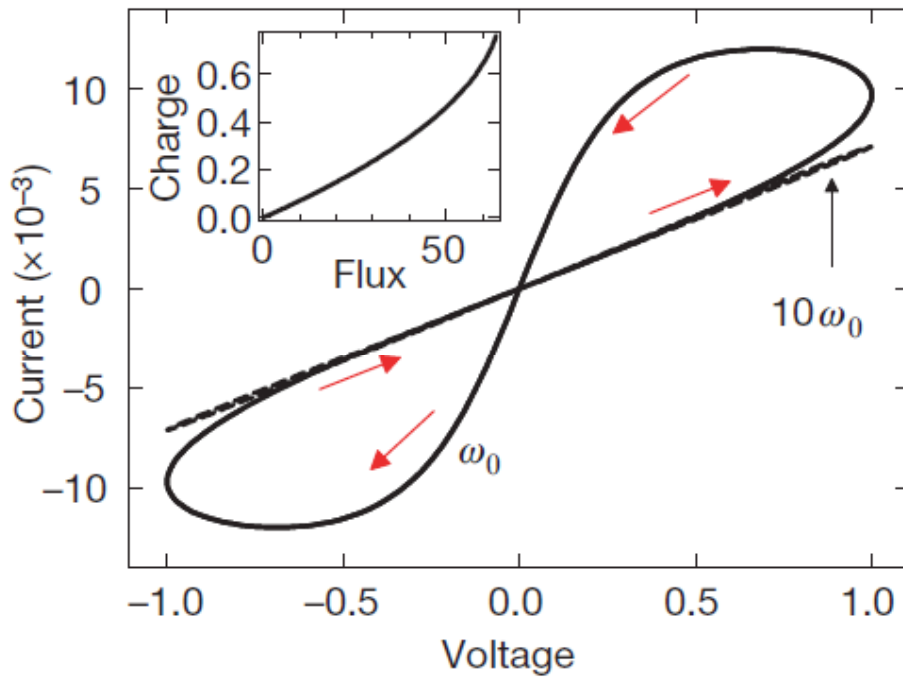
Our Simulation



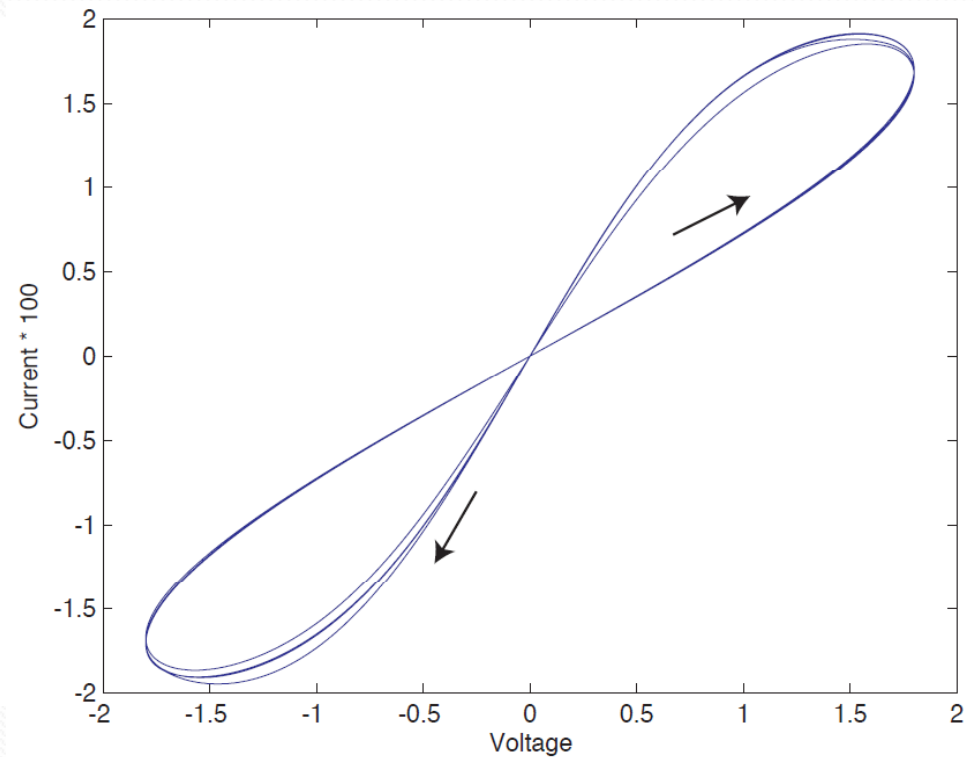
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HP Simulations Cont.

HP Nature Paper



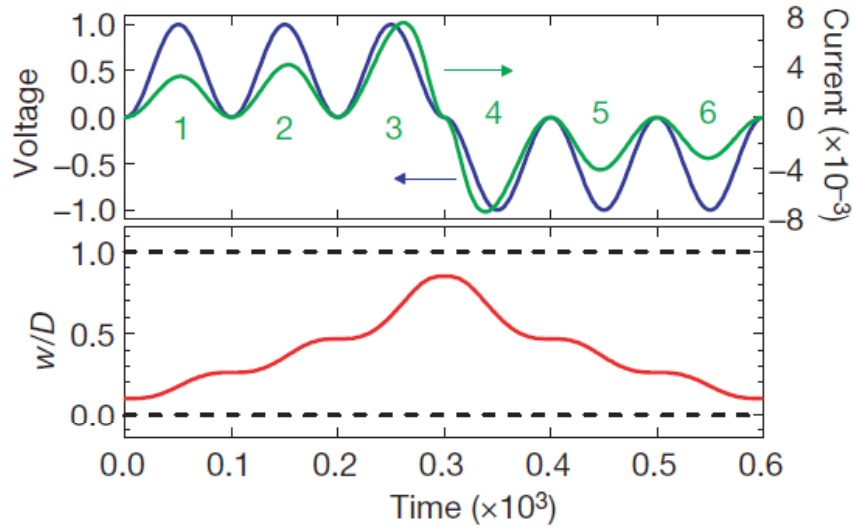
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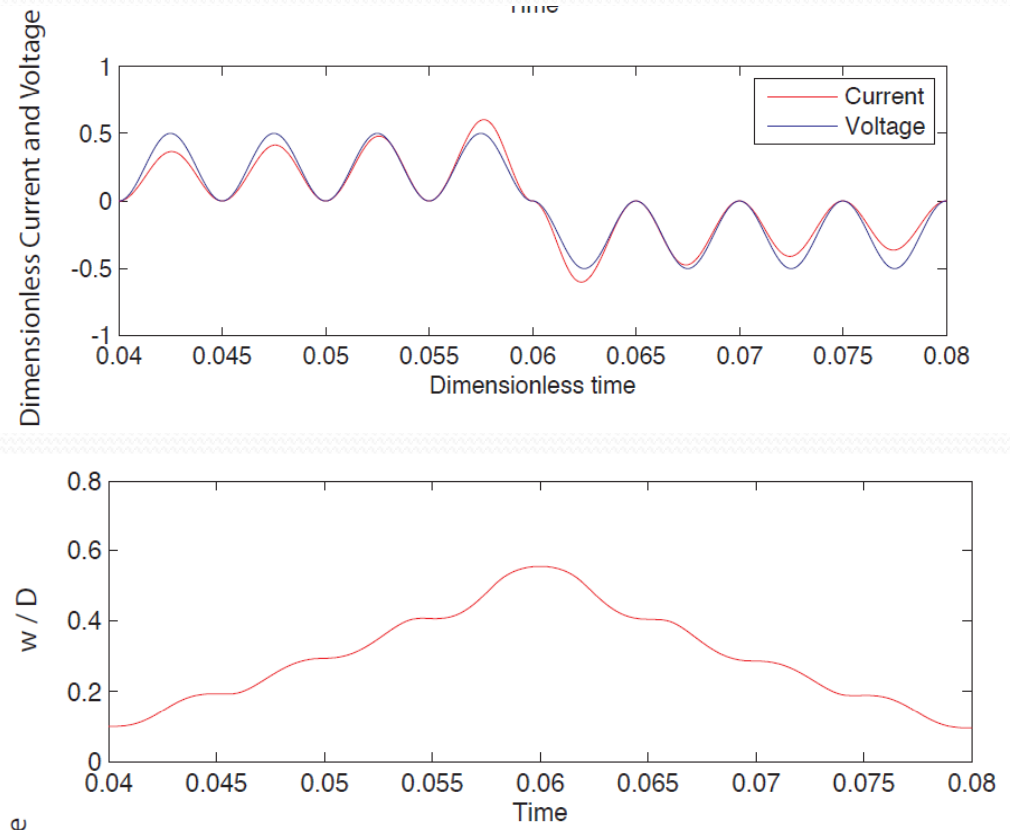
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HP Simulations Cont.

HP Nature Paper



Our Simulation



Strukov, et. al. Nature Vol 453
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Future Directions

- More precise modeling
- Learning circuits in parallel – signal processing
- Additional voltage waveforms

References

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