Modeling language competition

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Competition between languages for speakers has become an important political and sociological issues in recent decades. Mathematical models for two geographically equivalent competing languages have demonstrated situations where coexistence of two languages is possible, but more frequently show the death of one of the languages.

Introduction

In this project we have explored the dynamics of language competition. Our goal was to understand the modeling that has been accomplished and then choose an aspect of language competition to study further. There are many examples of dual language interaction in today's world, such as Quechua and Spanish in Peru, and Welsh and English in Wales. More familiar to this group is the competition between French and English in Canada where language issues dominate not only in social interactions but also affect economic exchanges. At times, the competition between French and English can be so strong that economic competitions become not a matter of region or actual economics, but a competition simply between languages [1]. This imbalance in the desirability of a language is causing many languages to go extinct relatively quickly, "with 90% of them being expected to disappear with the current generation" [2].

Loss of a language is the loss of communication and therefore essentially the loss of culture. When a child can no longer communicate with his or her grandparents, information cannot be passed on. This is especially crippling when the culture's histories are passed down verbally. Because of how integral language is in the human condition, understanding how languages interact is of extreme interest to many. Therefore, by looking at two models presented in three source papers, we attempted to understand the dynamics of two languages that share the same region and interact continuously and more specifically, if those two languages can coexist.

Abrams and Strogatz, 2003

One of the first papers written to analyze the dynamics of language competition was, "Modeling the dynamics of language death," written by Daniel M. Abrams and Steven H. Strogatz in 2003 [2]. Inspired by the question of whether or not humans that share the same geographical area would eventually become a monolingual society, Abrams and Strogatz developed a differential equation to model real world scenarios. Like most models, their representation of language competition requries some key assumptions.

One of the most crucial assumptions made is that this competition exists only between two languages sharing the same geographical plot and similar resources. As a result, the speakers of these languages are presumed to be in constant interaction with one another. Having to communicate constantly with people that speak a different language helps to define a prestige term, or a *status* of a language. This status implies that each language carries its own social and economic opportunities afforded to its speakers that either attract speakers of the other language to switch over and learn this foreign language or the language is positioned less appealingly than their mother tongue, in which case they are unlikely to make the switch. For simplicity, the authors of this paper assume that there is a highly interconnected population existing within both languages such that there is no social structure and the speakers are monolingual. This last assumption allows us to presume that there exists no dialectal difference in one language. Everyone from the commoners to the wealthiest class speaks the language identically.

The model Abrams and Strogatz developed is as follows:

$$\frac{dx}{dt} = yP_{yx}(x,s) - xP_{xy}(x,s) \qquad (1)$$

where y is the proportion of y language speakers and x is the proportion of x language speakers such that x and y together represent the total population ($x+y=1$). The status term, s, is taken to be in the interval [0,1] such that if $s > 0.5$ language x is the most attractive, and if $s < 0.5$ y is the only attractive language. The other terms found in this model relate to the probability that speakers will switch languages. Therefore, $yP_{yx}(x,s)$ is the proportion of y speakers that are converting to language x, and $xP_{xy}(x,s)$ is the proportion of X speakers that have switched over to language Y. It is obvious that this equation is simply showing the dynamical changes in the proportion of X speakers over time, and because this model uses proportions to model languages, if x is known, y is also known, taking this from a coupled system of differential equations down to a one dimensional problem.

Through the use of real world data, Abrams and Strogatz were able to define the probability terms as $P_{yx}(x,s)=cx^{a}s$ and $P_{xy}(x,s)=c(1-x)^{a}(1-s)$, where c and a are both constants derived from comparison with the recorded data and s is the status term. In this paper, it is important to note that c and a are both positive coefficients such that a is always presumed to be greater than one (more generally 1.31±0.25 as suggested by the data). The absolute status c represents the maximum conversion possible (i.e. s = 1), and the actual conversion present in the transition probabilities will decrease as s decreases towards 0.

After analyzing this one-dimensional differential equation, three fixed points are observed. These exist at $x=0$, $x=1$ (both being stable fixed points), and where the location of the third fixed point varies with the initial conditions. Although this third fixed point may appear promising in terms of language survival, after observing its trace and determinant we concluded that it is always unstable. Thus, in this model we observe no language coexistence in the long run. Depending on the status, the populations will either eventually convert to a society composed of a single language, either X or Y.

Through the fixed-point analysis of the Abrams and Strogatz model, we observe that competing languages cannot coexist simultaneously in the long run. Although mathematically sound, this result does not account for the actual existence of bilingual societies observed in the world today. As such, it appears that the Abrams and Strogatz model, although a solid foundation for the analysis of language dynamics, needs some revision to account for these anomalies still existing today.

Stauffer et al, 2007

The Stauffer et al. group began with the same model that Abrams and Strogatz provided but made different assumptions and presented a deeper analysis of the model's dynamics [3]. Many of these changed assumptions are made to more easily evaluate the equation while keeping the same qualitative behavior provided by the [2]. The differential equation Stauffer et al. uses for analysis is:

$$\frac{dx}{dt} = (1-x)sx^{a} - x(1-s)(1-x)^{a} \qquad (2)$$

The major simplification of equation (2) over equation (1) is that a has also been set to 1. The case $a = 1$ is an appropriate case for analysis because it provides the same qualitative behavior of Equation (2) as does the case $a = 1.3$, which is the value that Abrams and Strogatz [2] empirically found to apply most frequently to real competing language situations. However, Stauffer et al. pointed out that although $a = 1$ provides a much simpler linear case to analyze the dynamics, it is still important to recognize the other possibilities of values of a.

For values $a > 1$ we confirm the behavior as described for the Abrams-Strogatz model. For values $a < 1$ the stabilities flip; meaning, that $x = 1$ and $x = 0$ become unstable and the third equilibrium point $0 < x < 1$ becomes stable. This would imply the opportunity for coexistence of both languages, however, the authors did not elaborate on this case. We feel that this was done because the case $a < 1$ is not reflective of a situation likely to occur in actual interactions.

For the case $a = 1$ we observe two equilibrium points, at $x = 0$ and $x = 1$. $x = 1$ is an unstable fixed point and $x = 0$ is found to be stable. Figure 1 depicts a direction field plot from which it is easily seen that these two values are unstable.

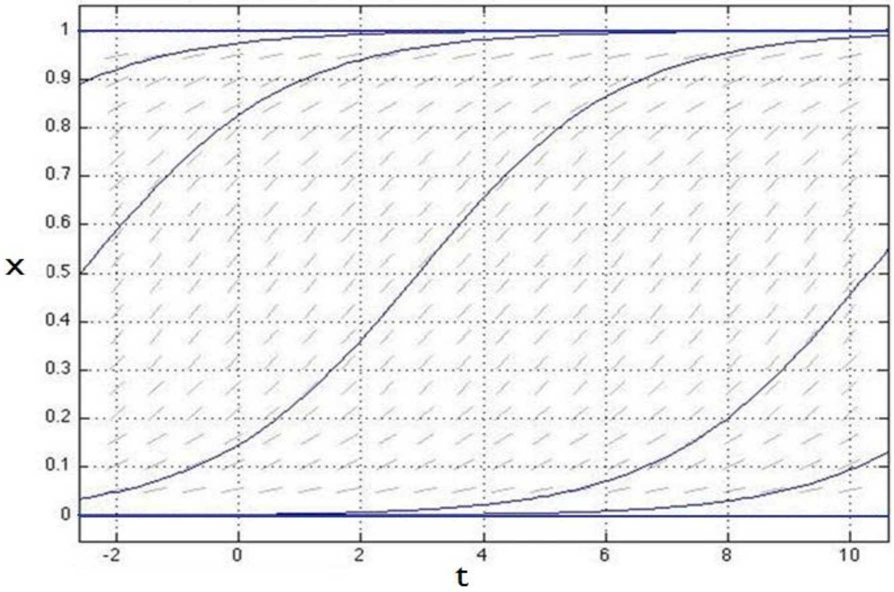

Figure 1: Direction field plot of Equation (2) with $a = 1$ and $s = 0.8$.

Figure 1 shows trajectories along lines $x = 0$ and $x = 1$ to be straight which implies that they are fixed points. All trajectories which begin between 0 and 1 trend towards the value x = 1, which is more popular with $s = 0.8$, hence x = 1 is unstable. Figure 2 shows a case where coexistence is possible according to this model. In the case where both languages have equal

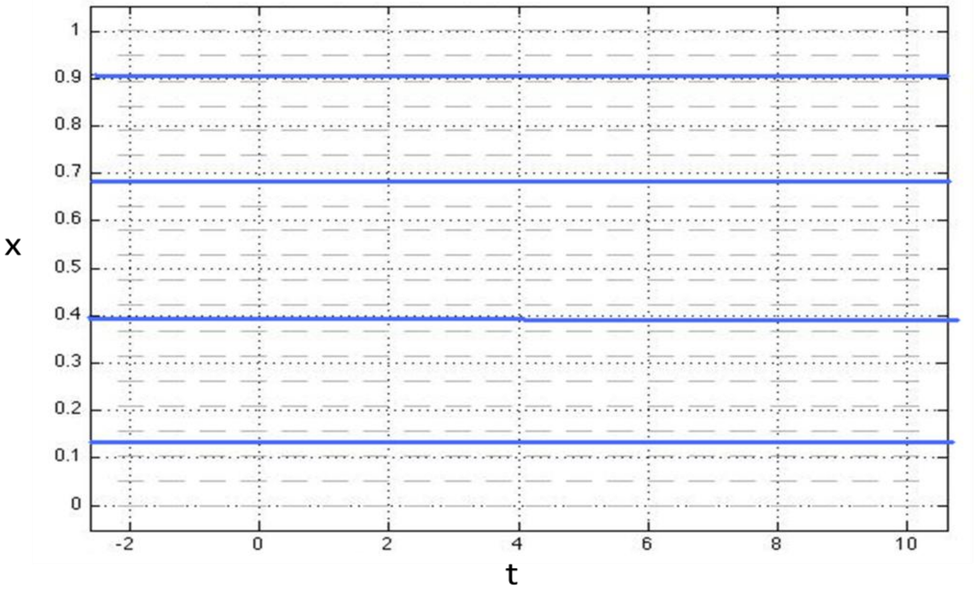

Figure 2: Direction field plot with $a = 1$ and $s = 0.5$.

status and $a = 1$, any initial value of x is a fixed point.

The Stauffer et al. paper provides us with conclusions that coexistence is only possible if languages have equal prestige or if $a < 1$. If neither of these conditions is met, then given enough time, the less prestigious language will eventually die out.

Coexistence

The third paper considered, entitled “Coexistence of Languages is Possible” by J.P. Pinasco and L. Romanelli [4], takes into account the total population of two languages rather than the proportions of population as seen in [2]. This paper assumes that language *X* is the only attractive language which allows only for language conversion from language *Y* to language *X*. The rate of this conversion from *Y* to X is given by *c* in the model. Finally, the parameters α_x ,α_y include both the birth and death rates of each population. The growth rates of the relative populations are given by

$$\alpha_x X\left(1 - \frac{X}{S_x}\right) \quad \text{and} \quad \alpha_y Y\left(1 - \frac{Y}{S_y}\right) \tag{3}$$

where S_x and S_y are the carrying capacities of the respective languages in the absence of competition, or when $c = 0$. These growth rates become a penalty term in order to keep the populations at carrying capacity.

In order to model the language competition, the following differential equations are used:

$$\frac{dX}{dt} = cXY + \alpha_x X\left(1 - \frac{X}{S_x}\right) \quad \text{and} \tag{4}$$

$$\frac{dY}{dt} = -cXY + \alpha_y Y\left(1 - \frac{Y}{S_y}\right) \tag{5}$$

Equation (4) shows that the change in population *x* over time is given by the positive probability of conversion from *y* to *x* plus the environmental term. The second equation is similar, however, the probability of conversion is negative because *y* is not an attractive language and speakers are only leaving the population of *y* to join that of *x*.

The system of equations, (3) and (4), yield three fixed points found at (0,0) in which both populations are at zero, (0, S_y) in which population *x* is zero and population *y* has reached its carrying capacity, and finally (S_x,0) in which population *y* is zero and population *x* has reached its carrying capacity. However, a fourth equilibrium point is found when the threshold condition

$$S_x < \frac{\alpha_y}{c}. \qquad (6)$$

is satisfied. The threshold condition (6) restricts the model to the first quadrant since we are only interested in positive populations. This threshold condition includes three conditions for language survival:

- Carrying capacity S_x is small and easily reached
- The growth rate of population *y* is high
- There is a low shift from language *y* to *x*

This fourth equilibrium point is found to be stable since the trace of the linearized system is negative and the determinant is positive.

Figure 3 illustrates the language dynamics in the absence of conversion (when $c = 0$). Thus, the threshold condition is satisfied trivially. With no conversion and equal growth rates, both populations reach their carrying capacity. Therefore, the fourth equilibrium point is found at the point (500, 600).

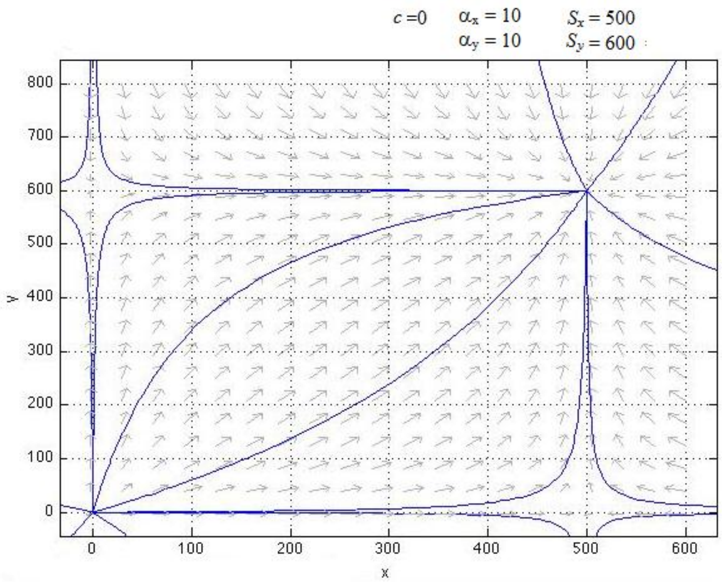

Figure 3: Phase Portrait of (3) and (4) with no competition

Now, when the threshold condition is satisfied with a conversion rate of $c = 0.005$, Figure 4 again shows the existence of a positive fourth equilibrium point thus showing that coexistence of tow languages is possible.

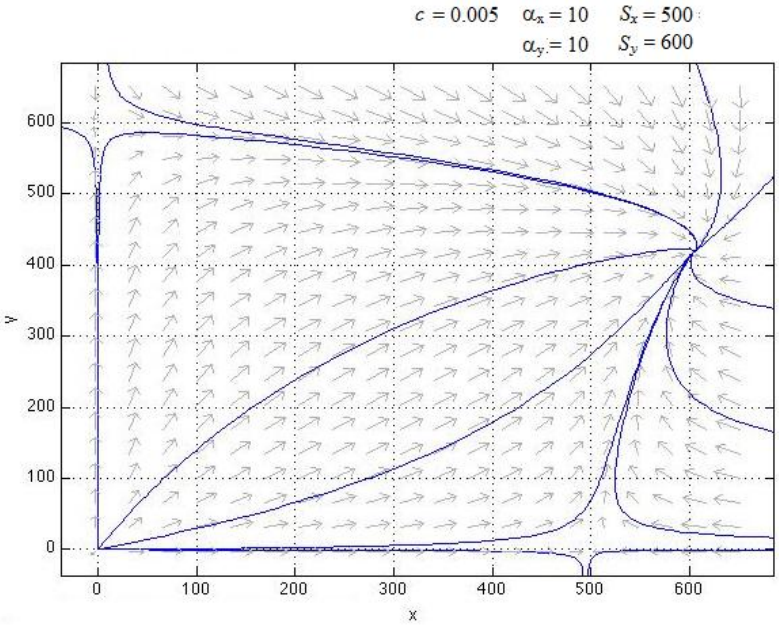

Figure 4: Phase Portrait with c = 0.005

Finally, when the threshold condition is not satisfied, the fourth equilibrium point disappears. Without the satisfaction of the threshold condition, language *y* dies out while language *x* reaches its carrying capacity, see Figure 5.

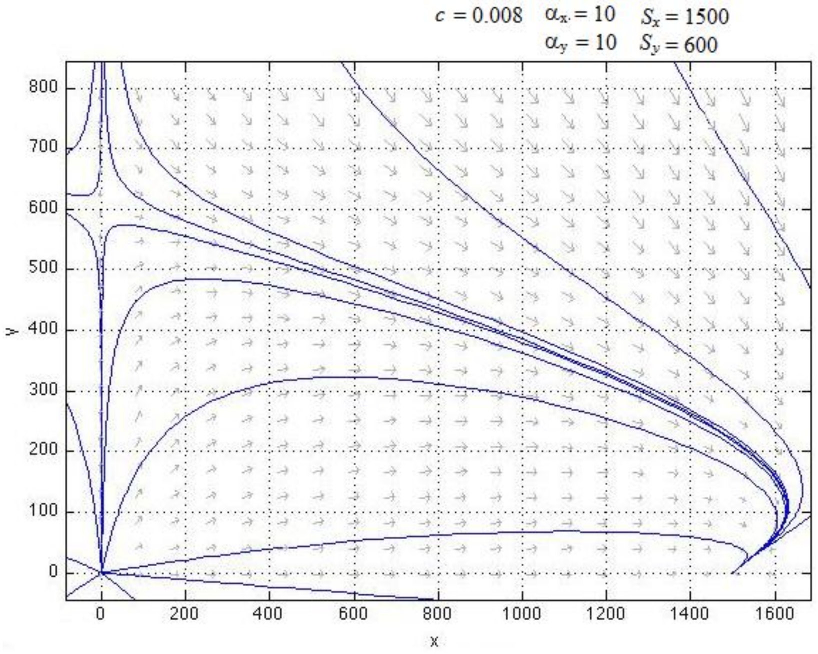

Figure 5: Phase portrait showing non-compliance with threshold condition.

Therefore, coexistence of two languages is shown to be impossible when the threshold condition is not satisfied.

Conclusion and Future Studies

The Abrams and Strogatz model uses proportions to model language transfer, resulting in a one dimensional model while the model used in the [4] uses actual population, creating a two dimensional representation. [2] allows speakers to switch from language X to language Y and vice versa. In contrast, the second model assumes that X as the only attractive language. This is more realistic as speakers will switch to the language with the higher status and not the other way around. [4] also incorporates the environment in the model, which is more realistic, but also adds more complexity. Coexistence seems possible, though the Abrams and Strogatz model predicts it under very particular circumstances while Pinasco and Romanelli allow a broader range of conditions. Although both models capture certain aspects of language competition, it is at the cost of many real life considerations such as dialects, language mutation over time, and the interaction of more than two languages. Bilingual populations are ignored and the incorporation of these seems to be the most promising aspect of language coexistence to explore further.

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