Chaos Control in Shuttle Bus Schedules

Project Description
- Bus shuttles often exhibit chaos in their arrival times.
- This project aims to investigate the effect of shuttle speed-up on reducing chaos in arrival times.
- Nagatani [1] is an authority on the subject of traffic modeling.

Scientific Challenges
- There are a myriad of factors that can affect bus arrival times, so we combined the passenger arrival rate, the loading rate, and unloading rate in a single loading factor and performed dimensionless analysis.
- The most difficult part of the modeling project was getting the particulars of the simulation code correct.

Potential Applications
- A model for chaos arising in bus shuttles can be applied directly to improving the reliability of mass transit systems.
- Indirect applications include problems of synchronization, such as torrents and steganography, as well as chaos-reduction problems.
- Improved performance will allow commuters to reevaluate benefits of public transportation. This will result to lower green-house emissions.

Methodology
1. Designed two MATLAB codes to calculate headway of 2 buses as a function of the loading parameter and another one for arrival time. Used the following mapping, where $T_i(m)$ is arrival time of bus $i$ on trip number $m$. $T_i(m)$ is the loading factor (generalized passenger arrival rate) and $m'$ are the previous bus number and trip number respectively, and $S_i$ is the speed-up factor for bus $i$:
   \[
   T_i(m + 1) = T_i(m) + \Gamma(T_i(m) - T_i(m')) + \frac{1}{1 + S_i(T_i(m) - T_i(m'))}
   \]
2. Ran our code for various values of speedup parameter, graphing our results and analyzing the changes in each of the graphs to determine how each is being affected.
3. Determined where chaos first occurs and designed a 3-D model to determine how the various speeds of the bus affect chaos.
4. Compared our graphs with the ones presented by Nagatani [1] in his paper while also expanding on his results noting the presence of periodicity.
5. Given our results analyzed future potential applications and benefits including but not limited to mass transit, torrents, and steganography.

Results
Our simulation graphs maintained similar shape and behavior as the Nagatani [1] graphs, supporting the conclusion that speedup regulates chaos.

This behavior starts with periodicity and then progressively leads to chaos. As the gamma parameter increases, periodic behavior is implemented by changing the speedup value.

Using a speedup parameter leads to more orderly traffic flow by diminishing chaos. This means that there is some predictability to the shuttle bus system, which may be controlled for better efficiency.

Optimal efficiency is achieved when the speedup parameters are equal in comparing two buses, which is represented in our 3-D model in finding the critical value where chaotic behavior begins.

Figure 1: Graphs of headway of bus 1 vs. loading parameter. Each column and row is labeled with speedup parameters, $S_i$ indicated by bus 1 as the columns and bus 2 as the rows. The chosen speedup parameters are set to 0, 0.15, 0.30, and 0.45 respectively.

Figure 2: 3-D plot that demonstrates the relationship among speedup parameter of two buses and the starting point of chaos. The $x$- and $y$-axis represent the speedup parameters of bus 1 and bus 2, respectively. The $z$-axis is the loading parameter value of the point where chaos starts to occur.

References

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Glossary of Technical Terms
- chaos: aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions [2].
- periodicity: the recurrence of an event at regular intervals.
- headway: the difference of arrival time of two busses for each trip.
- critical loading parameter: point in the graph where the bus starts to exhibit chaotic behavior.
- speedup: the ratio of increased passenger arrival rate to the current passenger arrival rate.