Competing Species

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Project Description:

In this project we seek to:

- model coexistence and chaos in ecosystems,
- show the effects of adaptation and mutation on population dynamics,
- reproduce the results of Sprott, et al [1], who employed the Lotka-Volterra Equations to model population dynamics,
- and to observe the effect that changes in adaptation have on the model.

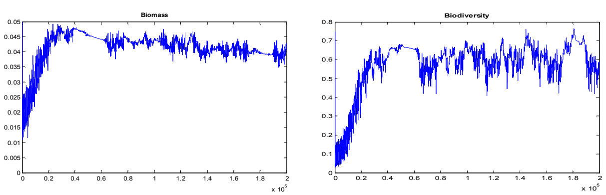
Through the use of a computer simulation we hope to accurately model large numbers of species in an ecosystem.

Scientific Challenges

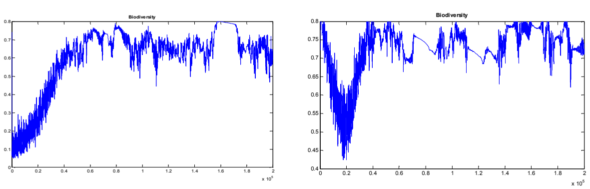
- To accurately capture the chaos amongst populations through a numerical simulation

Potential Applications

- Survival and extinction in the wild
- Financial markets
- Traffic flow



Biomass and Biodiversity graphs with initial conditions of clamp size = 10^{-6} and mutation at every 20 time steps with a time step size of 0.0125



Biodiversity graphs with mutation at every 20 time steps with a time step size of 0.025 and clamp sizes of 10^{-4} and 10^{-2}, respectively

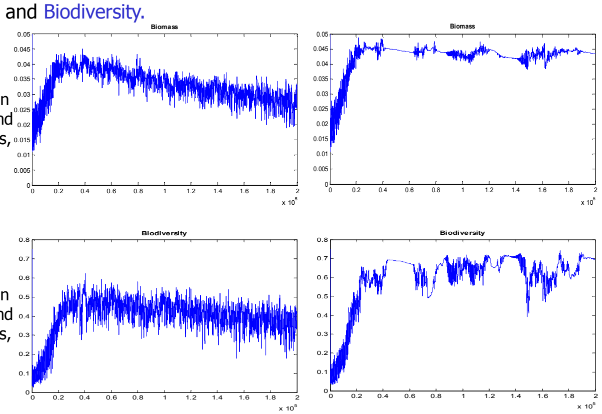
Methodology

To achieve our goals, we:

1. simulated population dynamics using the generalized Lotka-Volterra equation, which includes a competition matrix that dictates how the various species interact,
2. employed a classical Runge Kutta method in MATLAB,
3. simulated adaptation by altering the interaction matrix every twenty time steps. We increased the off-diagonal elements of the matrix by a small value to model increasing environmental stress, as well as enhanced competition,
4. prevented extinction by clamping a failing species at a small value,
5. investigated the role of adaptation by changing how often we altered the interaction matrix and changing the value at which we clamped a dying species.
6. In order to obtain easily understandable results, we generated the measures Biomass and Biodiversity.

Biomass with mutation at every 5 and 40 time steps, respectively

Biodiversity with mutation at every 5 and 40 time steps, respectively



Results

1. We were able to approximately replicate the results of Sprott [1] in terms of biomass and biodiversity.
2. For the modifications of mutation, noticeable visual differences were seen between the different mutation time steps. Higher mutation rates produced a lower biomass and a lower biodiversity. Fluctuations decrease as adaptation is simulated less frequently (as seen above). This mutation is due to environmental stress [1], and therefore with higher mutation rates, each species is less likely to flourish.
3. With our initial clamp size of 10^{-6} there was a sharp decline in biodiversity over the initial few time steps. When the clamp size was changed by a factor of 100 in either direction, there was negligible change; this was most likely due to the fact that these values are still extremely small in comparison to the population levels of the various species.

Glossary of Technical Terms

Generalized Lotka-Volterra Equation:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \sum_{j=1}^{N} a_{ij} x_j\right)$$

Biomass (M): the total mass of living organisms in a certain ecosystem divided by the total number of species, or the average population size

$$M = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Biodiversity (D): the diversity of plant and animal life in a specific habitat

$$D = 1 - \frac{1}{2(N-1)}\sum_{i=1}^{N}\left|\frac{x_i}{M} - 1\right|$$

x_i = size of population of species i
r_i = growth rate of population i
a_{ij} = competition between species i & j
x_j = other species interacting with species i
N = total number of species

References

1. Sprott, J.C., J.C. Vano, J.C. Wildenberg, M.B. Anderson, and J.K. Noel. "Coexistence and chaos in complex ecologies." *Physics Letters A* 335. (2005): 207-12. Web. 23 Feb 2010.
2. Fox, J. W., and D. A. Vasseur. 2008. Character convergence under competition for nutritionally-essential resources. American Naturalist 172:667-680

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