Project Description

- We are interested in the dynamics of interacting languages and the possibility of two languages coexisting simultaneously.
- The authors of [1] make the following assumptions in the derivation of their model:
  - Two languages, X and Y, are fixed and compete with each other for speakers.
  - Speakers are monolingual and highly connected to other speakers of the same language.
  - The attractiveness of a language increases with the number of speakers and perceived status.
- The Abrams-Strogatz model is:
  \[ \frac{dx}{dt} = yP_{Y;X}(x,s) - xP_{X;Y}(x,s) \]
  where 
  \[ P_{Y;X}(x,s) = cx^a \]
  and \( x + y = 1 \).
- The authors of [1] determined that coexistence is not possible.

Scientific Challenges

- Verifying the model by comparing its results to past and present language interactions would be informative, but it is made challenging due to the difficulty of collecting data.

Potential Applications

- Understanding the dynamics of language competition can aid efforts to prevent the death of languages in the future.
- This understanding can also help preserve cultural heritage as well as facilitate communication between different generations.

Methodology

1. Reproduced and confirmed the findings of [1] and [2].
2. Extended the previous work by adding the existence of a bilingual population.
3. Analyzed the new system of equations by determining stable fixed points and conditions guaranteeing that they are physically sensible.
4. Used the software PPLANE to determine the stability of fixed points.

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The Model

The Bilingual Model was derived by incorporating a bilingual population:

\[ \frac{dx}{dt} = c((1 - x)(1 - k)x_Y (1 - y) - x) \]
\[ \frac{dy}{dt} = c((1 - y)(1 - k)y_Y (1 - x) - y) \]

where \( x + y + b = 1 \).

This model is presented in [4].

Results

To ease analysis, \( a \) was set to 1. As [2] indicates, this slight change exhibits qualitatively similar behavior to the \( a = 1.3 \) case. We determined that coexistence can only occur when both

\[ k > \frac{1}{s_x + 1} \]
\[ k > \frac{2s_y - 1}{s_y} \]

are satisfied. This area represents approximately 23% of the possible combinations of \( k \) and \( s_x \).

Fixed points are \((1,0)\), \((0,1)\), \((1,1)\), and \((x_y, y_Y)\), where

\[ (x_y, y_Y) = \left( \frac{(1 - k)(1 - x_Y)x_Y}{1 - s_x}, - \frac{(1 - k)(1 - y_Y)y_Y}{s_y(1 - (1 - k)^2)} \right) \]

The fixed points \((1,0)\), \((0,1)\), and \((1,1)\) are all unstable while \((x_y, y_Y)\) is always stable as long as the above conditions are satisfied. Therefore, when these conditions are observed, there is stable coexistence of both languages and a bilingual population.

Glossary of Technical Terms

- \( x \): proportion of population speaking language \( X \)
- \( y \): proportion of population speaking language \( Y \)
- \( P_{X;Y}(x,s) \): probability an \( X \) speaker converts to \( Y \)
- \( P_{Y;X}(x,s) \): probability a \( Y \) speaker converts to \( X \)
- \( s \): status; attractiveness of language \( X \)
- \( k \): the similarity between \( X \) and \( Y \)
- \( c \): conversion rate
- \( a \): constant derived from measured data, usually valued at 1.3

References


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