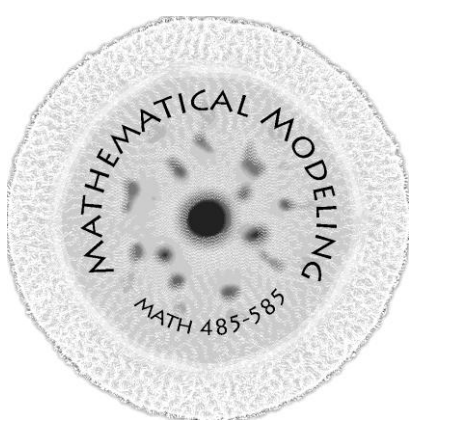




Modeling Language Competition



Project Description

- We are interested in the dynamics of interacting languages and the possibility of two languages coexisting simultaneously.
- The authors of [1] make the following assumptions in the derivation of their model:
 - Two languages, X and Y, are fixed and compete with each other for speakers.
 - Speakers are monolingual and highly connected to other speakers of the same language.
 - The attractiveness of a language increases with the number of speakers and perceived status.
- The Abrams-Strogatz model is:

$$\frac{dx}{dt} = yP_{yx}(x,s) - xP_{xy}(x,s) \quad \text{where} \quad \begin{aligned} P_{yx}(x,s) &= cx^a s \\ P_{xy}(x,s) &= c(1-x)^a (1-s) \end{aligned}$$
 and $x + y = 1$.
- The authors of [1] determined that coexistence is not possible.

Scientific Challenges

- Verifying the model by comparing its results to past and present language interactions would be informative, but it is made challenging due to the difficulty of collecting data.

Potential Applications

- Understanding the dynamics of language competition can aid efforts to prevent the death of languages in the future.
- This understanding can also help preserve cultural heritage as well as facilitate communication between different generations.

Methodology

- Reproduced and confirmed the findings of [1] and [2].
- Extended the previous work by adding the existence of a bilingual population.
- Analyzed the new system of equations by determining stable fixed points and conditions guaranteeing that they are physically sensible.
- Used the software PPLANE to determine the stability of fixed points.

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The Model

The Bilingual Model was derived by incorporating a bilingual population:

$$\begin{aligned} \frac{dx}{dt} &= c[(1-x)(1-k)s_x(1-y)^a - x(1-s_x)(1-x)^a] \\ \frac{dy}{dt} &= c[(1-y)(1-k)(1-s_x)(1-x)^a - ys_x(1-y)^a] \end{aligned}$$

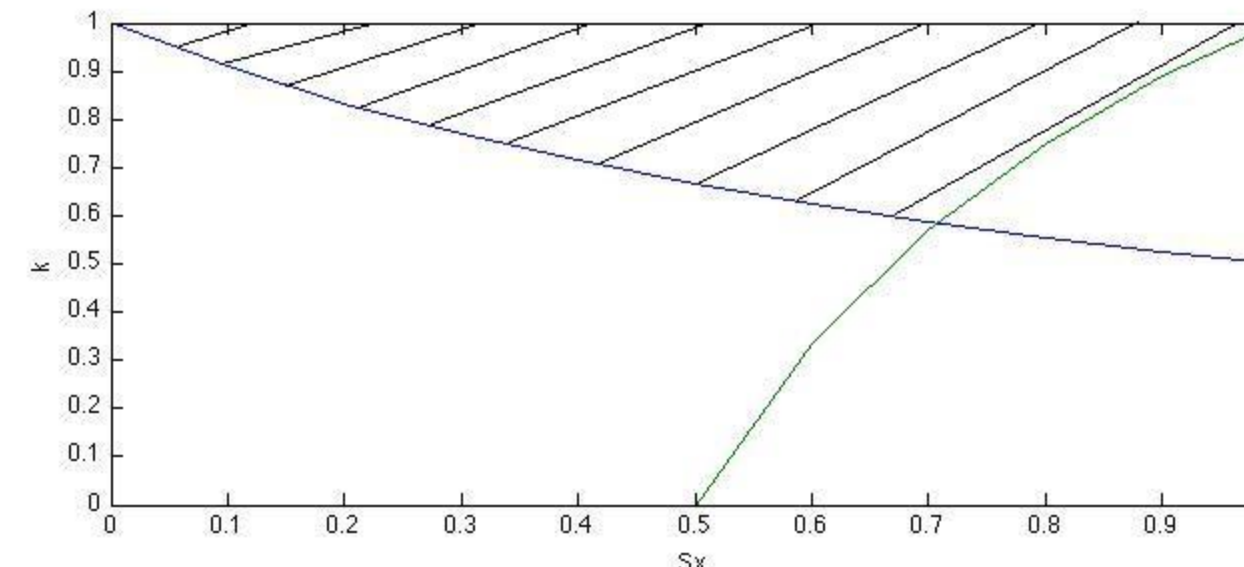
where $x + y + b = 1$.

This model is presented in [4].

Results

To ease analysis, a was set to 1. As [2] indicates, this slight change exhibits qualitatively similar behavior to the $a = 1.3$ case. We determined that coexistence can only occur when both

$$\begin{aligned} k &> \frac{1}{s_x + 1} \\ \text{and} \\ k &> \frac{2s_x - 1}{s_x} \end{aligned}$$



are satisfied. This area represents approximately 23% of the possible combinations of k and s_x .

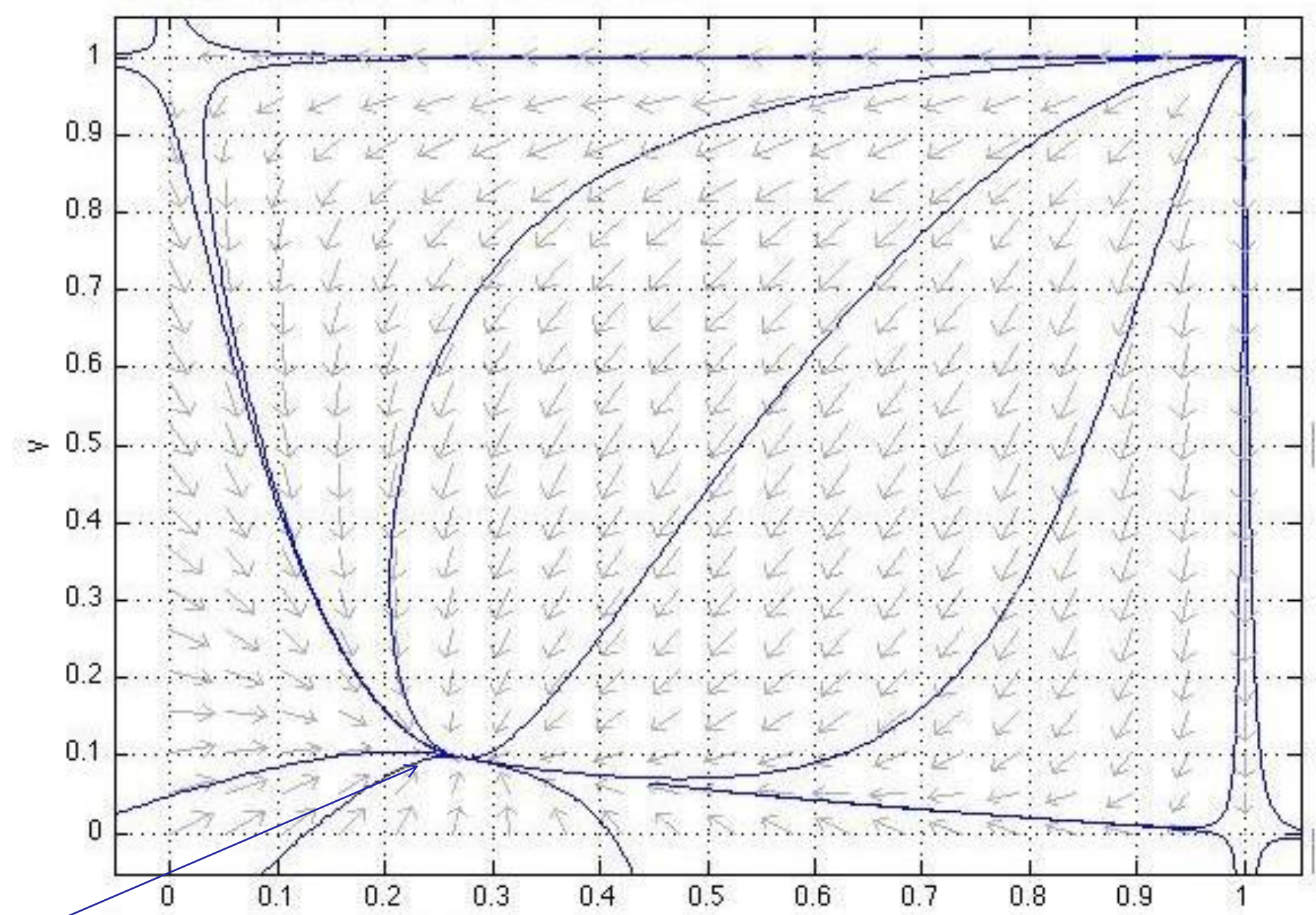
Fixed points are $(1,0)$, $(0,1)$, $(1,1)$, and (x_1, y_1) , where

$$(x_1, y_1) = \left(\frac{(1-k)(1-y_1)s_x}{1-s_x}, \frac{(1-k)((1-s_x) - (1-k)s_x)}{s_x(1-(1-k)^2)} \right)$$

The fixed points $(1,0)$, $(0,1)$, and $(1,1)$ are all unstable while (x_1, y_1) is always stable as long as the above conditions are satisfied. Therefore, when these conditions are observed, there is stable coexistence of both languages and a bilingual population.

Glossary of Technical Terms

- x : proportion of population speaking language X
- y : proportion of population speaking language Y
- $P_{yx}(x,s)$: probability an X speaker converts to Y
- $P_{xy}(x,s)$: probability a Y speaker converts to X
- s : status; attractiveness of language X
- k : the similarity between X and Y
- c : conversion rate
- a : constant derived from measured data, usually valued at 1.3



Dynamics of the model, plotted with the software PPLANE. Figure values are set at $c = 1$, $a = 1$, $k = 0.8$, and $s = 0.6$.

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