

Rogue Waves

Based on the article: “Physical mechanisms of the rogue wave phenomenon”, by Kharif and Pelinovsky (2003)

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Introduction

Storm wave height can reach up to 8-10 m in the deep sea under extreme wind conditions (Lin, 2008). Nevertheless, observations were indeed reported for suddenly emerged huge waves on an otherwise quiet and calm background wave field in deep water. Such waves are called rogue waves. These waves can reach a wave height over 20 m without any warning, representing a huge danger for ships.

Naval architects have always worked on the assumption that their vessels are extremely unlikely to encounter a rogue (most ships are built to resist just 15 m waves). Between 1969 and 1994 twenty-two supercarriers (largest type of aircraft carrier) were lost or severely damaged due to the occurrence of sudden rogue waves; a total of 542 lives were lost as a result (Lawton, 2001).



Figure 1. A rogue wave in the Bay of Biscay, France. Published in Fall 1993 issue of Mariner's Weather Log. Credits: NOAA Photo Library



Figure 2. NOAA Ship Discoverer gets pounded by waves in the Bering Sea. Credits: Commander Richard Behn, NOAA Corps, 1979.

Also called “freak” or “giant” waves, “rogues” are waves such height, H_{fr} , exceeds at least twice the significant wave height, H_s , i.e. $H_{fr}/H_s > 2$. The ratio H_{fr}/H_s is the so called abnormality index (AI). This is the only parameter defining whether a wave is a “rogue”.

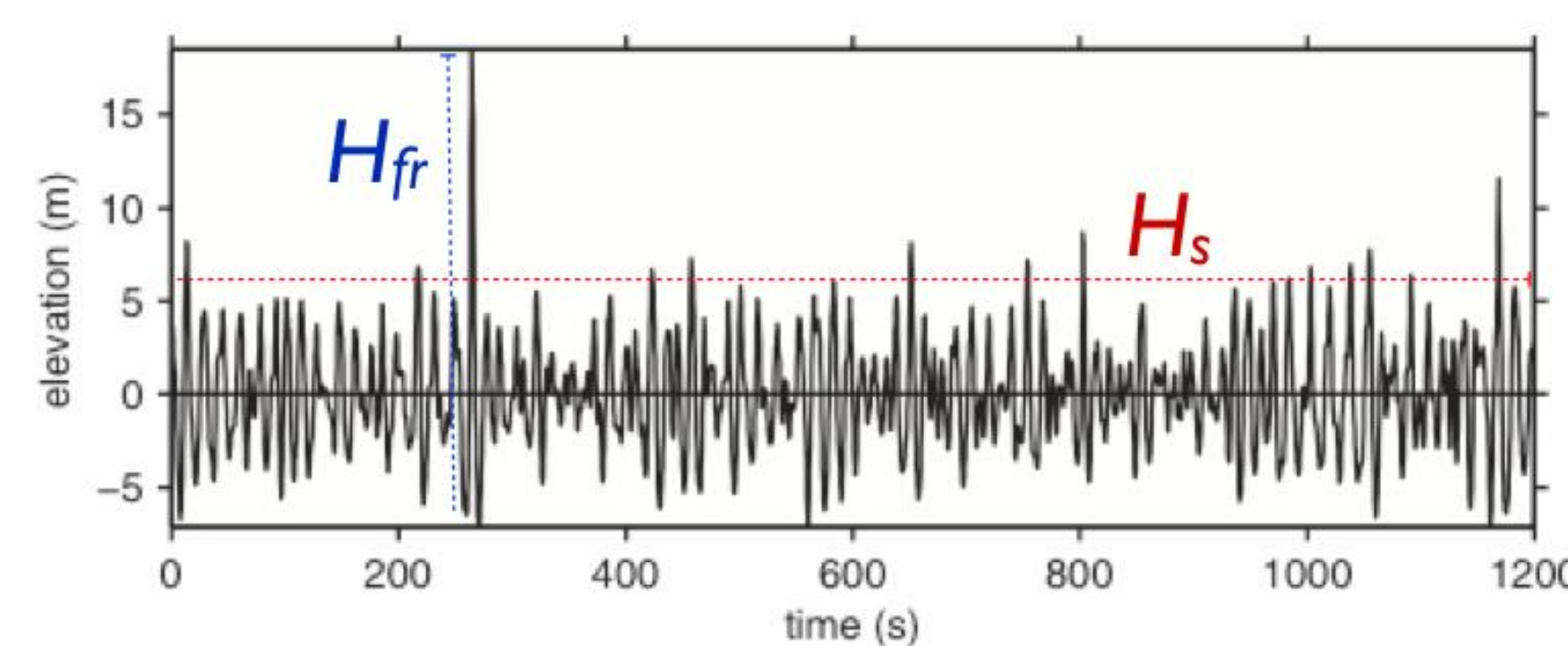


Figure 3. The “New Year Wave” recorded in a gas platform in the North Sea on New Year's Day 1995, with a wave height of 26 m and an abnormality index $AI=2.24$

Modeling Rogue Waves

In the coastal region, viscous forces can be neglected. Under this conditions, the fluid is considered irrotational.

Irrotationality implies that the flow must satisfy the Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \nabla^2 \phi = \Delta \phi = 0$$

where ϕ =velocity potential of the flow, Δ =Laplacian operator.

To solve the Laplace equation, conditions on boundaries are needed: (i) a kinematic boundary condition (nonlinear), (ii) a dynamic boundary condition (nonlinear) and (iii) a sea bottom boundary condition.

Linear mechanisms causing rogue waves

For small-amplitude waves, all the nonlinear terms can be neglected. As a result of this assumption, it can be shown that water waves with different wavelengths will propagate at different speeds.

In linear theory, the wind wave field can be sought as the sum of a very large number of small-amplitude waves with different frequencies and directions of propagations (Kharif and Pelinovsky, 2003).

The focusing due to dispersion is one of the three main methods suggested in linear theory as the origin of rogue waves.

Focusing due to dispersion

If short waves (with small group velocities), then, long waves will overtake short waves, and a large amplitude wave can appear at some fixed time owing to the superposition of all the waves located at the same place.

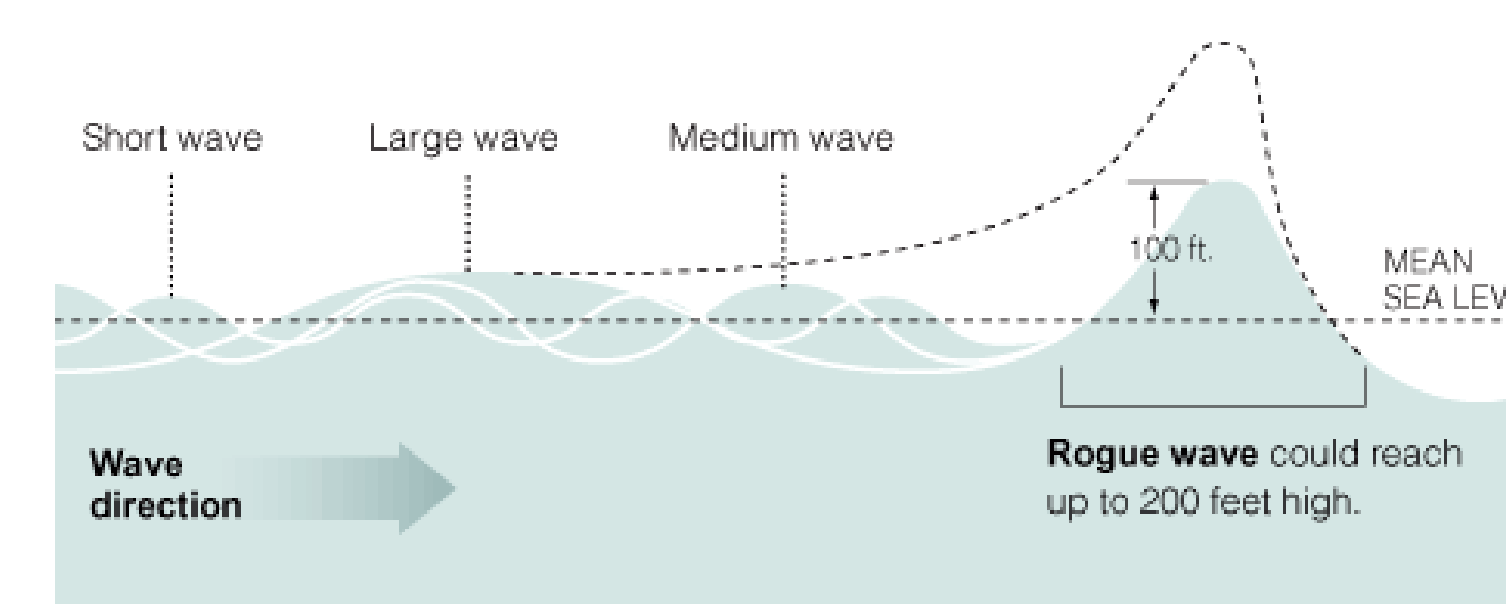


Figure 4. Trains of waves traveling in the same direction but at different speeds pass through one another. When they synchronize, they combine to form large waves. Credits: The New York Times, 2006.

Rogue waves in shallow water due to dispersion can be modeled by using the following equation

$$\eta(x, t) = \frac{A_0}{k^3 \sqrt{\frac{h^2 ct}{2}}} \exp\left(\frac{1}{2h^2 ct k^2} \left(x - ct + \frac{6}{77h^2 ct k^4}\right)\right) \text{Ai}\left(\frac{x - ct + \frac{9}{77h^2 ct k^4}}{\sqrt{\frac{h^2 ct}{2}}}\right)$$

where: A_0 =Initial wave train amplitude, k =wavenumber, h =water depth, c =phase velocity, t =time, Ai =Airy function.

Nonlinear mechanisms causing rogue waves

When wave amplitude increases, the linear theory may become inadequate. By solving the Laplace equation without neglecting nonlinear terms, the so-called Stokes waves are obtained. Stokes' waves can become unstable and grow exponentially in time due to the Benjamin-Feir instability (BFI). In 1968, Zakharov discovered the existence of BFI in Stokes' waves. Furthermore, in the context of modulated water waves, he obtained the famous Nonlinear Schrödinger equation (NLS), (Kharif and Pelinovsky, 2009).

The simplest NLS has many exact solutions. One of them has been particularly popular as a candidate to explain freak waves in deep water. It is called a “breather” and starts out as a periodic wave train where the amplitude is weakly modulated. After some time it develops a particularly strong focusing of wave energy by which a small part of the wave train “breathes” itself up at the expense of the neighborhood (Dysthe, 2007).

One of the breather solutions (a singular breather on an infinite domain) corresponds to the so-called algebraic breather:

$$A(x, t) = A_0 \cdot \exp(i\omega t) \left[1 - \frac{4(1 + 2i\omega t)}{1 + 16k^2 x^2 + 4\omega^2 t^2} \right]$$

where: A_0 =Initial wave train amplitude, k =wavenumber, ω =frequency, t =time.

Results

To illustrate the generation of freak waves through the nonlinear theory, the Airy function associated with the focusing due to dispersion theory was plotted in Matlab. The graphs are compared below with those obtained from the base article.

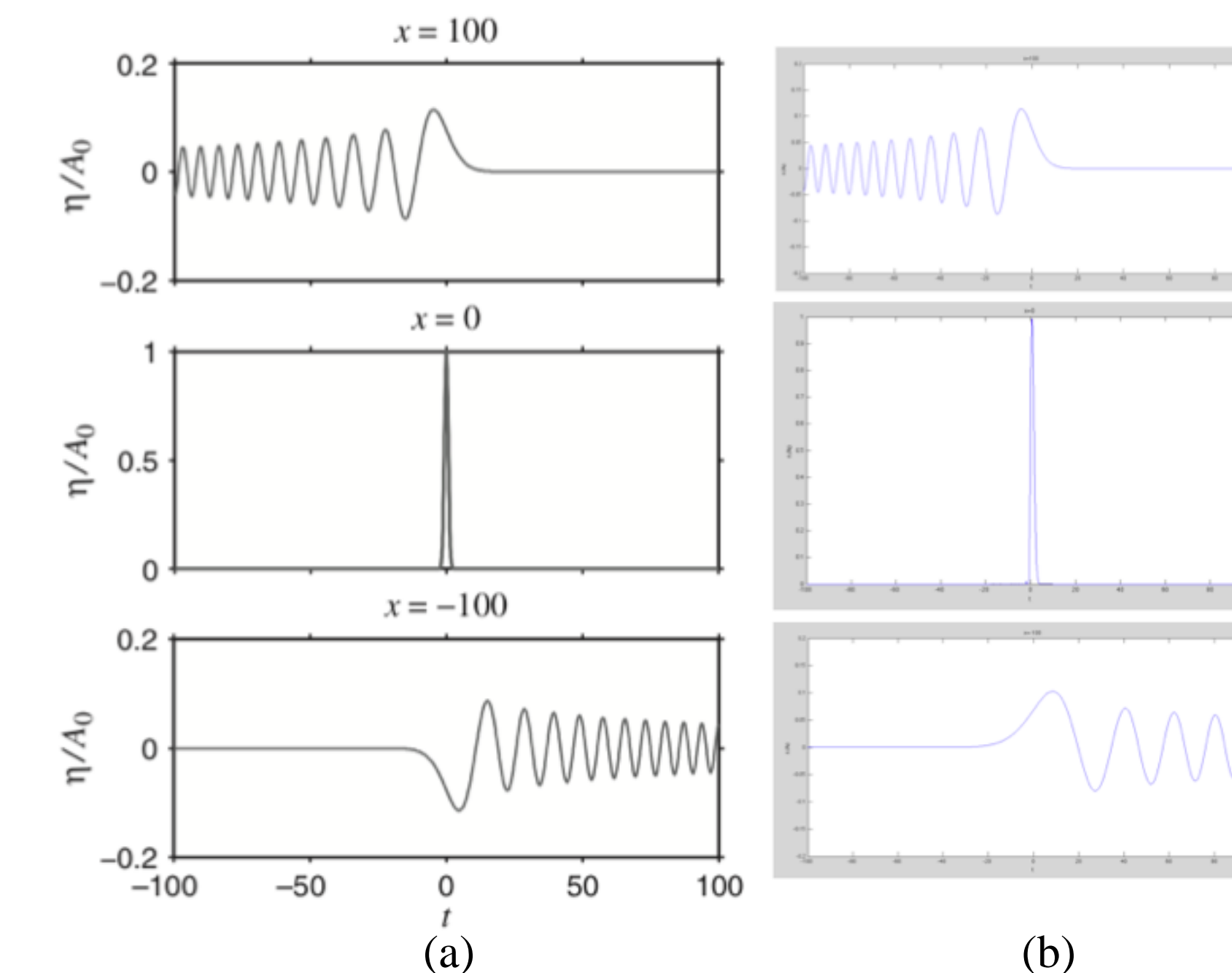


Figure 5. Formation of a freak wave due to dispersion (linear theory). (a) Graphs from the base article. (b) Own results plotted in Matlab.

Figure 5 shows how due to the effect of the dispersion, waves traveling at different speeds can focus on a specific point ($x=0$) and form a rogue wave, this wave spreads after the focusing point.

As an example of the rogue wave generation by the nonlinear theory, an algebraic breather was plotted in Matlab and is presented below.

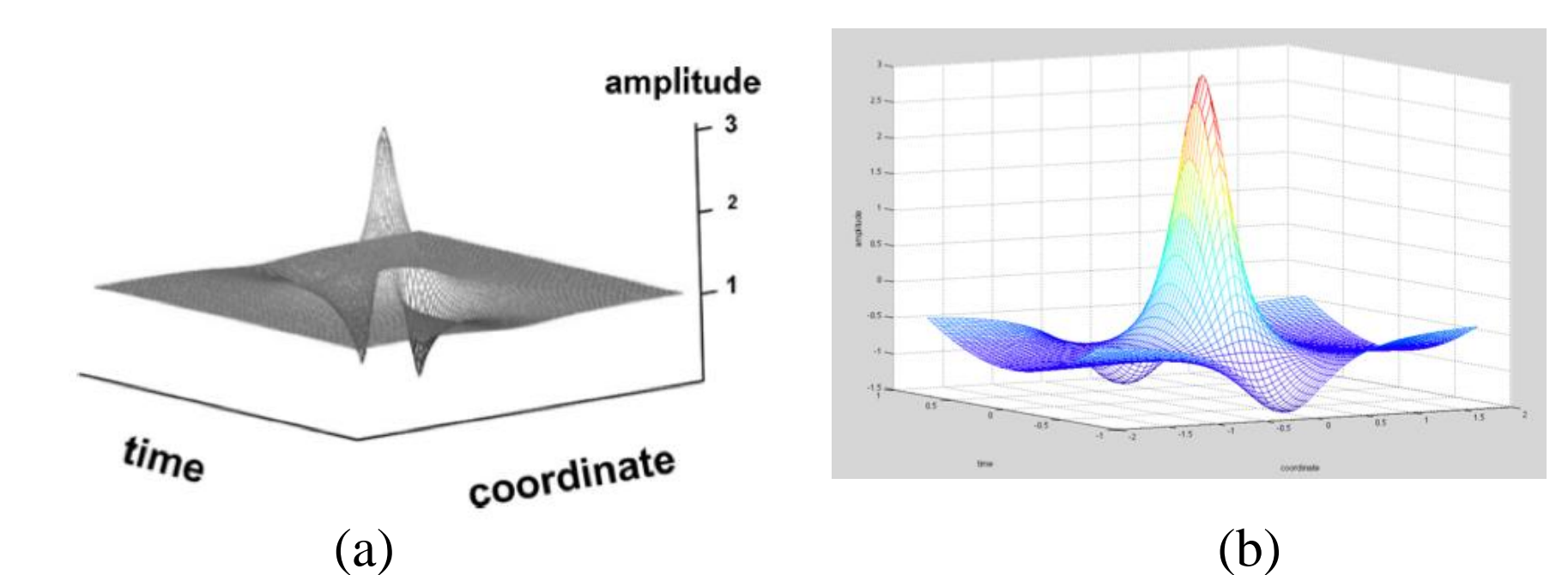


Figure 6. Formation of a freak wave due to nonlinear theory. (a) Graph from the base article. (b) Own results plotted in Matlab.

The algebraic breather in Figure 6 shows how the nonlinear instability of a weakly modulated wave train in deep water may generate rogue waves.

Conclusions

There are several theories trying to explain the rogue waves phenomenon. Unfortunately, no theory has shown to be conclusive. However, due to the importance of the rogue waves, several studies are in progress to try to understand in depth the phenomenon.

A brief revision of the linear and nonlinear theory for the analysis of rogue waves is presented. Both linear and nonlinear models were plotted in Matlab. The graphs obtained are closely similar to those presented in the base article.

Literature cited

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