Energy Flow in Electrical Grids

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Overview

http://www.altenergymag.com/articles/09.04.01/smartgrid/grid.jpg
Motivation

- Along a feed line voltage fluctuates
  - Desire: Reduce voltage drop at end of line
- Determine maximum length supporting consumption
- Goal: Create low-parametric model of power and voltage distribution along an electrical line using [1] to understand energy flow in grids
- Adjust model to include consumption variations in loads

Applications

Smart Grids

PV systems

Transporting Energy

Superbowl Power Outage
Background

Alternating Current

\[ v(t) = \sqrt{2}V \sin(\omega t + \alpha) \quad i(t) = \sqrt{2}I \sin(\omega t + \beta) \]
\[ p(t) = VI \cos(\alpha - \beta) - VI \cos(2\omega t + \alpha + \beta) \]

\[ v = \text{Voltage} \]
\[ i = \text{Current} \]
\[ p = \text{Power} \]
Background

Other Basic Equations

\[ S = VI = P + jQ \]
\[ z = r + jx \]

S = Apparent Power
z = impedance
x = inductance
r = resistance
Set-up

Assumptions: One directional flow, uniform consumption at loads, static

\[ S_1 = S_0 - S_l - S_L \]

\[ V_1 = V_0 - z_1 I_0 \]
Developing the Model
Problem Formulation

Discrete form

\[ P_{k+1} - P_k = p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2} \]

\[ Q_{k+1} - Q_k = q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2} \]

\[ v_{k+1}^2 - v_k^2 = -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{v_k^2} \]

where

- \( k = 0, \ldots, N \) enumerates buses of the feeder
- \( P_k, Q_k \) real and reactive power flowing from bus \( k \) to bus \( k + 1 \)
- \( p_k, q_k \) overall consumption of real and reactive power at bus \( k \)
- \( r_k, x_k \) line resistance and reactance connecting bus \( k \) to bus \( k + 1 \)

with Boundary Conditions

\[ v_0 = 1, P_N = Q_N = 0 \]
Continuous and Homogenous Form

Transform the discrete finite element to a continuous form:

- assume large number of consumers \( N \gg 1 \)
- continuous form with limit \( N \to \infty \)
- \( \frac{r_k}{x_k} \) is set constant so \( r_k = r \frac{l_k}{L} \) and \( x_k = x \frac{l_k}{L} \)
- \( r \) and \( x \) are constant values
- \( L \) total length of the feeder line and \( l_k \) length of line from bus \( k \) to bus \( k+1 \)
- \( F_k = F(z) + \bar{F}(L_k)/N \)
  - \( F(z) \) which is the change from node \( k \) to \( k+1 \)
  - \( \bar{F}(L_k)/N \) which is the averaging term
- \( z = L_k = \sum_{i=0}^{k-1} l_k \)
- \( F_{k+1} - F_k \approx F'(z)l_k/L \)
Boundary Value Problem

\[
\frac{d}{dz} \begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r \frac{P^2 + Q^2}{v} \\ q - x \frac{P^2 + Q^2}{v^2} \\ -rP + xQ \end{pmatrix}
\]

with Boundary Conditions

\[v_0 = 1, \quad P(L) = Q(L) = 0\]
Re-scaled Form

- Assuming \( p = \text{constant} \)
- New Variable \( s = \frac{\sqrt{|p| r}}{v(L)} (L - z) \)

Dimensionless Variables for \( P, Q \) and \( v \)

\[
\varrho(s) = \sqrt{\frac{r}{|p|}} \frac{P(z)}{v(L)}, \tau(s) = \sqrt{\frac{r}{|p|}} \frac{Q(z)}{v(L)}, v(s) = \frac{v(z)}{v(L)}
\]
Initial Value Problem

\[
\frac{d}{ds} \begin{pmatrix} \varphi \\ \tau \\ v \end{pmatrix} = \begin{pmatrix} \text{sign}(p) - \frac{\varphi^2 + \tau^2}{v^2} \\ A - B\frac{\varphi^2 + \tau^2}{v^2} \\ -\frac{\varphi + B\tau}{v} \end{pmatrix}
\]

with Initial Conditions

\[v(0) = 1, \varphi(0) = \tau(0) = 0\]
Evaluating for End Points

Solving IVB for some value of $s_*$
$s : 0 \rightarrow s_*$ we obtain $\varrho(s_*)$, $\tau(s_*)$ and $v(s_*)$
Then we can compute the value of $L$ and the end values

$$L = \frac{s_*}{v(s_*) \sqrt{|p|/r}}$$
$$v(L) = \frac{1}{v(s_*)}$$
$$P(0) = \frac{\varrho(s_*) \sqrt{|p|/r}}{v(s_*)}$$
$$Q(0) = \frac{\tau(s_*) \sqrt{|p|/r}}{v(s_*)}$$
Reproduction of Results
Initial and Boundary Value Problems

- IVP
  \[-\frac{d}{ds} \begin{pmatrix} \rho \\ \tau \\ v \end{pmatrix} = \begin{pmatrix} \text{sign}(p) - \frac{p^2 + \tau^2}{A - B\frac{\rho^2 + \tau^2}{v}} \\ \frac{\rho^2 + \tau^2}{v} \end{pmatrix} \]

- BVP
  \[\frac{d}{dz} \begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{p^2 + Q^2}{v^2} \\ q - \frac{p^2 + Q^2}{v} \\ -rP + xQ \end{pmatrix} \]

Graphs generated from each problem:
- End Voltage vs. Length
- Power Utilization vs. Length
- Voltage vs. Position
- Power vs. Position
End Voltage vs. Length

$p = -1, q = -0.5$
End Voltage vs. Length

$p = -1, q = 0$
Two End Voltages for One Length

\[ v(L) = \frac{1}{v(s_*)} \]

\[ L = \frac{s_*}{v(s_*) \sqrt{|p|r}} \]
Power Utilization vs. Length

Power utilization = \frac{\text{Initial Injected Power}}{\text{Power Consumed}} = \frac{P(0)}{p*L}
Power Utilization vs. Length

$p = -1, q = -.5$
Voltage Along the Line

The graph shows the voltage along the line for different values of $L$. The red line represents $L = 0.2$ and the green line represents $L = 0.5$. The voltage decreases as the position along the line increases, indicating a typical exponential decay pattern.
Voltage Along the Line

Voltage along the line: $p = -1, q = 0$

- Red line: $L = 0.4$
- Green line: $L = 0.7$

Position along the line, $z$

Voltage along the line, $v(z)$
Power Along the Line

Voltage along the line: \( p = -1, q = -0.5 \)

- Green line: \( P: L = 0.5 \)
- Dotted green line: \( Q: L = 0.5 \)
- Red line: \( P: L = 0.2 \)
- Dotted red line: \( Q: L = 0.2 \)

Position along the line, \( z \)

Power along the line, \( P(z) \) and \( Q(z) \)
Our graphs vs. Article
Stochastic Addition
Non-uniform Consumption

- Major assumption in the article:
  - Uniform consumption of loads $p$
- In reality
  - Slight variations across a line
Adding Stochasticity

- Let $p(l) = p_0 + W(l)$, where $W(l)$ is a Wiener Process.
- Substitute for $p$ in DistFlow ODEs
- Solve for boundary value problem
Methodology

- Monte Carlo Method

Create $p(l)$

Plot Solution

Solve the BVP
Results
Results
Discussion

• Small perturbations of power consumption have relatively little effect on voltage at the end of the line.

• Assuming a constant power consumption is statistically valid.

• Suggests that power consumption alone does not serve as an explanation to sudden voltage drops.
Future Work

Dynamic Model
- Implementation energy variations with respect to time
- Better understanding of ‘jumps’ in stability
- Introducing producers along the line; effect of renewable energy sources

Branching
- Take into consideration the grid layout of energy distribution systems
- Use discrete form of the model to examine energy flow dynamics of three and four bus systems
References

• The papers used for this presentation were:


Questions?