Extreme Bungee Jumping!

A Specialized Application of the Large-Amplitude Dynamics of the Elastic Pendulum.

By:

Corey Zammit, Nirantha Balagopal,

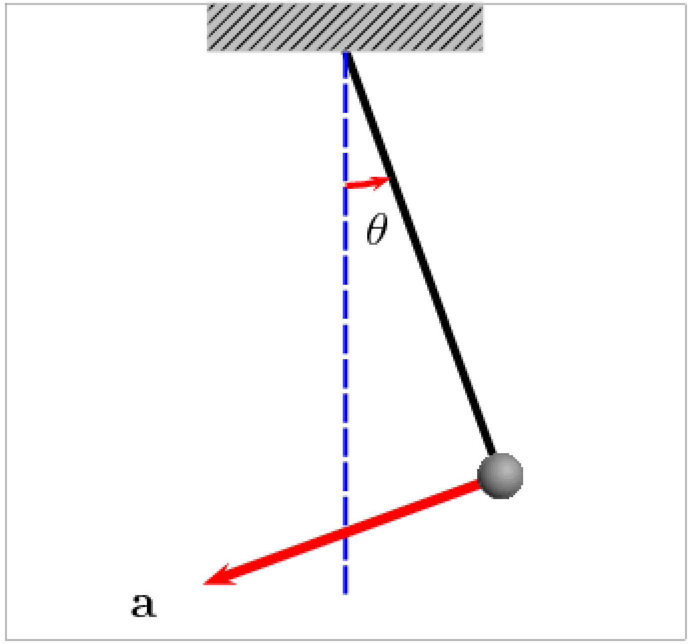
Zijun Li,

Qisong Xiao, & Shenghao Xia.

An introduction to the system

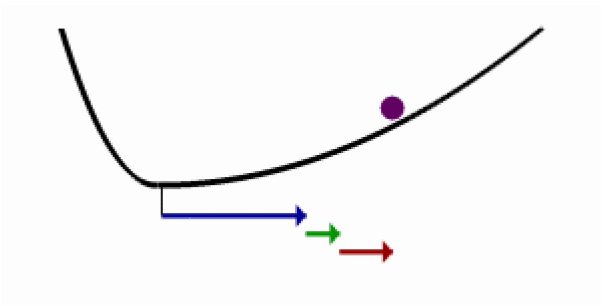
- Elastic problem

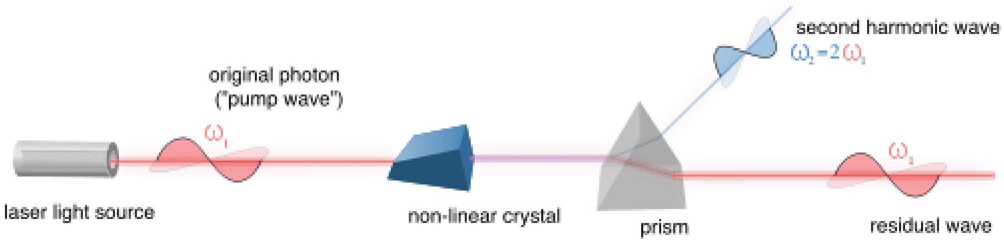
- Pendulum problem



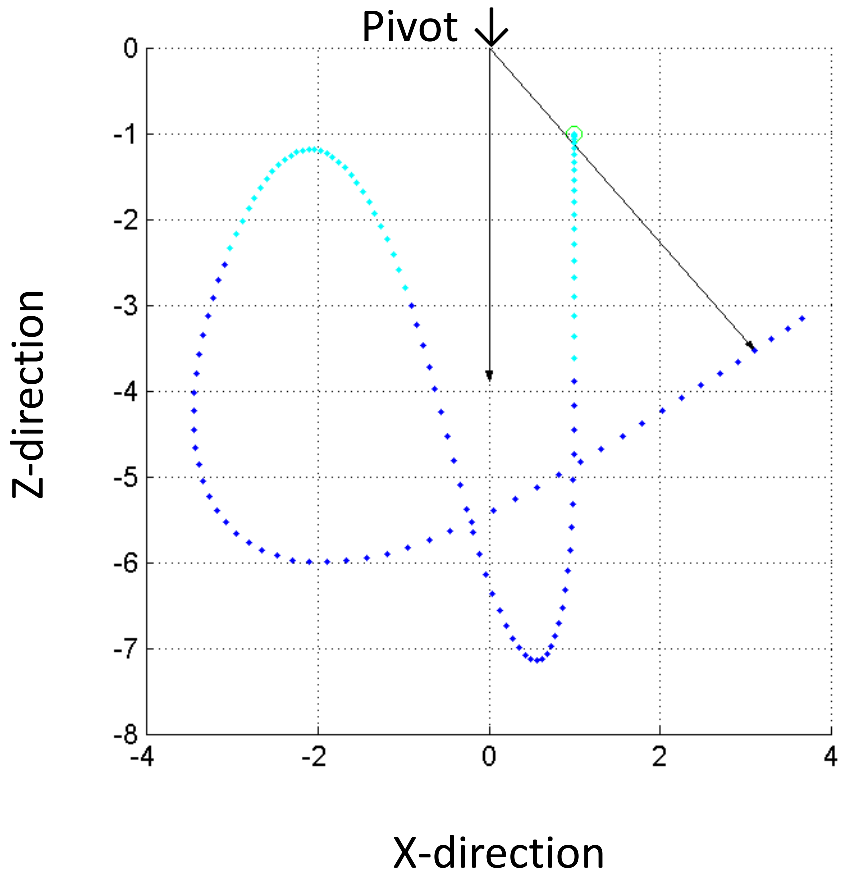
Another application of the system model:

- Under specific conditions this mathematical model can be used to model optical second harmonic generation





The numerical bungee jumping model:



The system considered is a mass freely swinging from an elastic band, acted on by the constant force of gravity, and the extension-dependent force of the elastic response. This does not include spring compression.

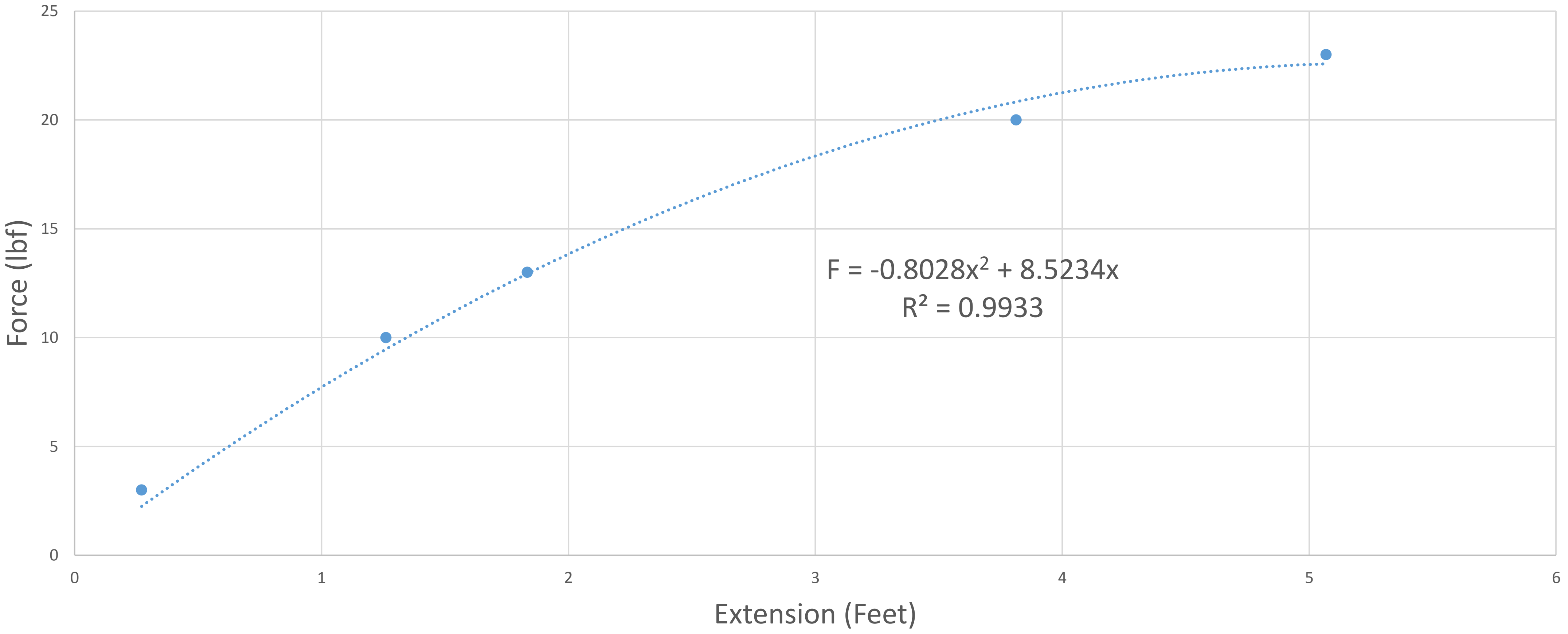
In the arbitrary example shown in the image, cyan shows free-fall.

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{a}{4}(r - l_0)^4 - \frac{b}{2}(r - l_0)^2 - mgz$$

- This system shows the

$$\begin{cases} \ddot{x} = -\frac{a}{m} \cdot \frac{(r - l_0)^3}{r} x - \frac{b}{m} \cdot \frac{r - l_0}{r} x \\ \ddot{y} = -\frac{a}{m} \cdot \frac{(r - l_0)^3}{r} y - \frac{b}{m} \cdot \frac{r - l_0}{r} y \\ \ddot{z} = -\frac{a}{m} \cdot \frac{(r - l_0)^3}{r} z - \frac{b}{m} \cdot \frac{r - l_0}{r} z - g \end{cases}$$

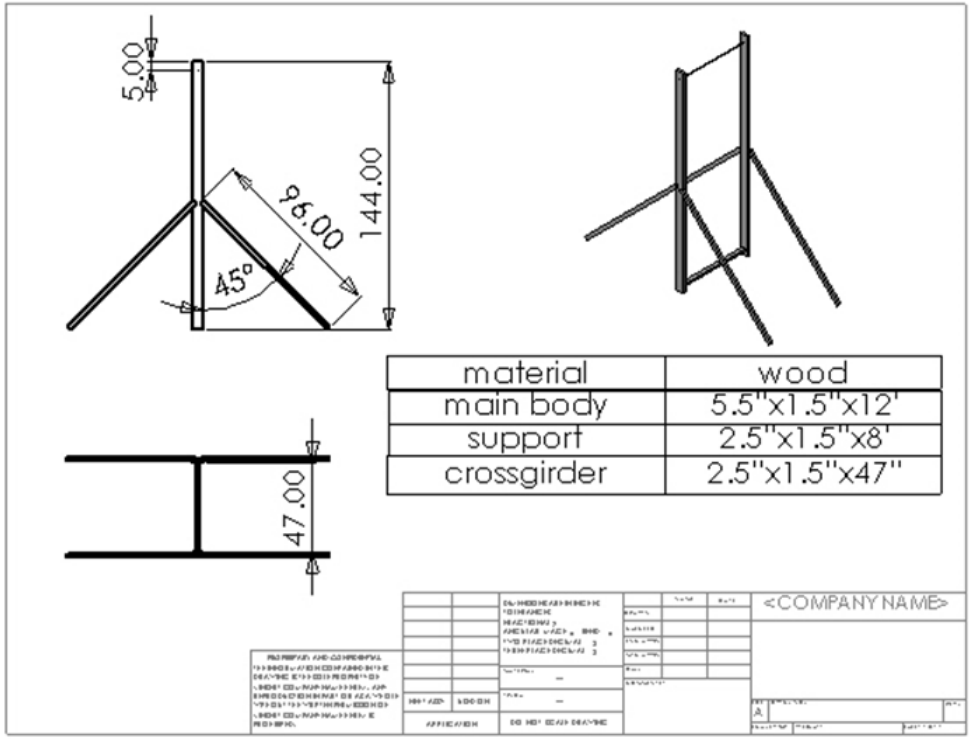
Elastic Response as a Function of Extension



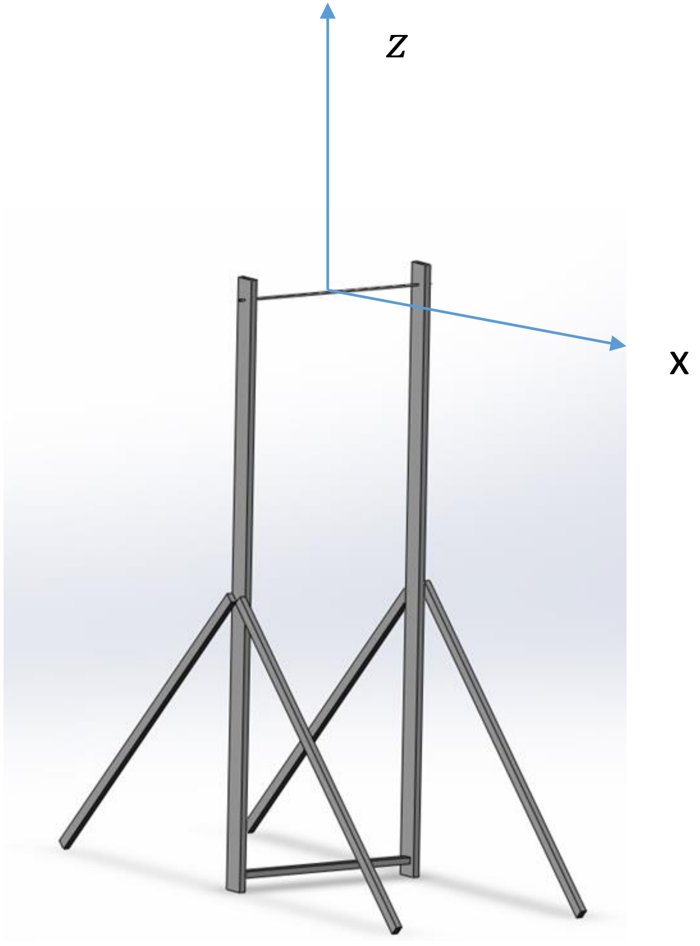
Equations of motion customized for our experiment:

$$\begin{cases} \ddot{x} = -\dfrac{a}{m} \cdot \dfrac{(r - l_0)^2}{r} x - \dfrac{b}{m} \cdot \dfrac{r - l_0}{r} x \\ \ddot{y} = -\dfrac{a}{m} \cdot \dfrac{(r - l_0)^2}{r} y - \dfrac{b}{m} \cdot \dfrac{r - l_0}{r} y \\ \ddot{z} = -\dfrac{a}{m} \cdot \dfrac{(r - l_0)^2}{r} z - \dfrac{b}{m} \cdot \dfrac{r - l_0}{r} z - g \end{cases}$$

Bungee project design



material	wood
main body	5.5"x1.5"x12'
support	2.5"x1.5"x8'
crossgirder	2.5"x1.5"x47"



Bungee project design

The mass is released from rest positioned horizontally at the characteristic length of the bungee. $x_0 = 3.875$ feet

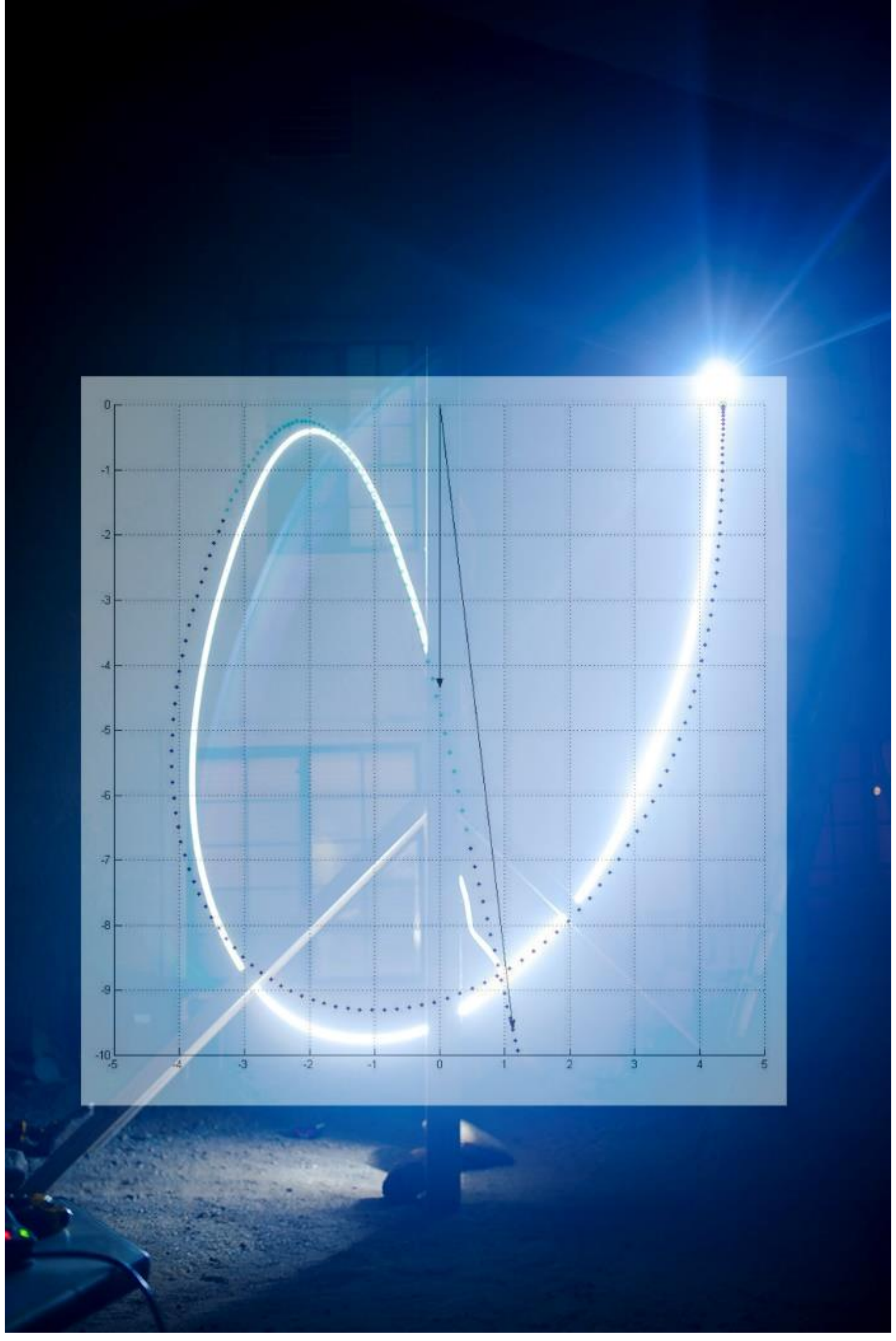
Time interval is ¼ second; A three pound dumbbell swings from a band with an elastic response curve fit to

$$F_{elastic} = -0.3807(r - l_0)^2 + 3.589(r - l_0)$$

A ten-pound medicine ball…

$$F_{elastic} = -0.8028(r - l_0)^2 + 8.5234(\mathrm{r} - \mathrm{l_0})$$

Long exposures were illuminating…



High speed video…

Raw Data

- Origin is fixed at pivot point
- 2-D Position data points are measured on each 30 frame (0.25sec) and scale up with a ratio
- Velocity is calculated as average velocity for each 30 frame (0.25sec) of x and z directions

Data Evaluation

- Magnitude of velocity,

$$v = \sqrt{{v_x}^2 + {v_z}^2}$$

- Total energy,

$$E_t = K + U$$

$$E_t = \frac{1}{2}v^2 + \frac{1}{m}\int_{l_0}^{r} F_r dr + zg$$

$$E_t = \frac{1}{2}v^2 + \frac{1}{m}[\frac{a}{3}(r - l_0)^3 + \frac{b}{2}(r - l_0)^2] + zg$$

x	z (height)	r	x	z (height	r	vx	vz
3.2	0	3.2	-16.32	0	-16.32	0	0
3.1	-1	3.35	-15.81	5.1	-17.085	2.04	20.4
2.5	-3.6	4.4	-12.75	18.36	-22.44	12.24	53.04
0.4	-5.6	5.7	-2.04	28.56	-29.07	42.84	40.8
-1.8	-5.1	5.4	9.18	26.01	-27.54	44.88	-10.2
-2.7	-2.5	3.6	13.77	12.75	-18.36	18.36	-53.04
-2.5	-0.5	2.8	12.75	2.55	-14.28	-4.08	-40.8
-2.3	-0.7	2.4	11.73	3.57	-12.24	-4.08	4.08
-1.8	-2.8	3.3	9.18	14.28	-16.83	-10.2	42.84
-0.9	-5.4	5.5	4.59	27.54	-28.05	-18.36	53.04
0.85	-6	6.1	-4.335	30.6	-31.11	-35.7	12.24
1.9	-3.45	4	-9.69	17.595	-20.4	-21.42	-52.02
2.3	-1.5	2.6	-11.73	7.65	-13.26	-8.16	-39.78
2.7	-1.3	2.9	-13.77	6.63	-14.79	-8.16	-4.08
2.9	-2.6	3.7	-14.79	13.26	-18.87	-4.08	26.52
1.7	-4.5	4.8	-8.67	22.95	-24.48	24.48	38.76
-0.6	-5	5.1	3.06	25.5	-26.01	46.92	10.2
-2.5	-3.4	4.3	12.75	17.34	-21.93	38.76	-32.64
-2.8	-1.45	3.2	14.28	7.395	-16.32	6.12	-39.78
-2.4	-1.1	2.8	12.24	5.61	-14.28	-8.16	-7.14
-1.9	-2.6	3.3	9.69	13.26	-16.83	-10.2	30.6

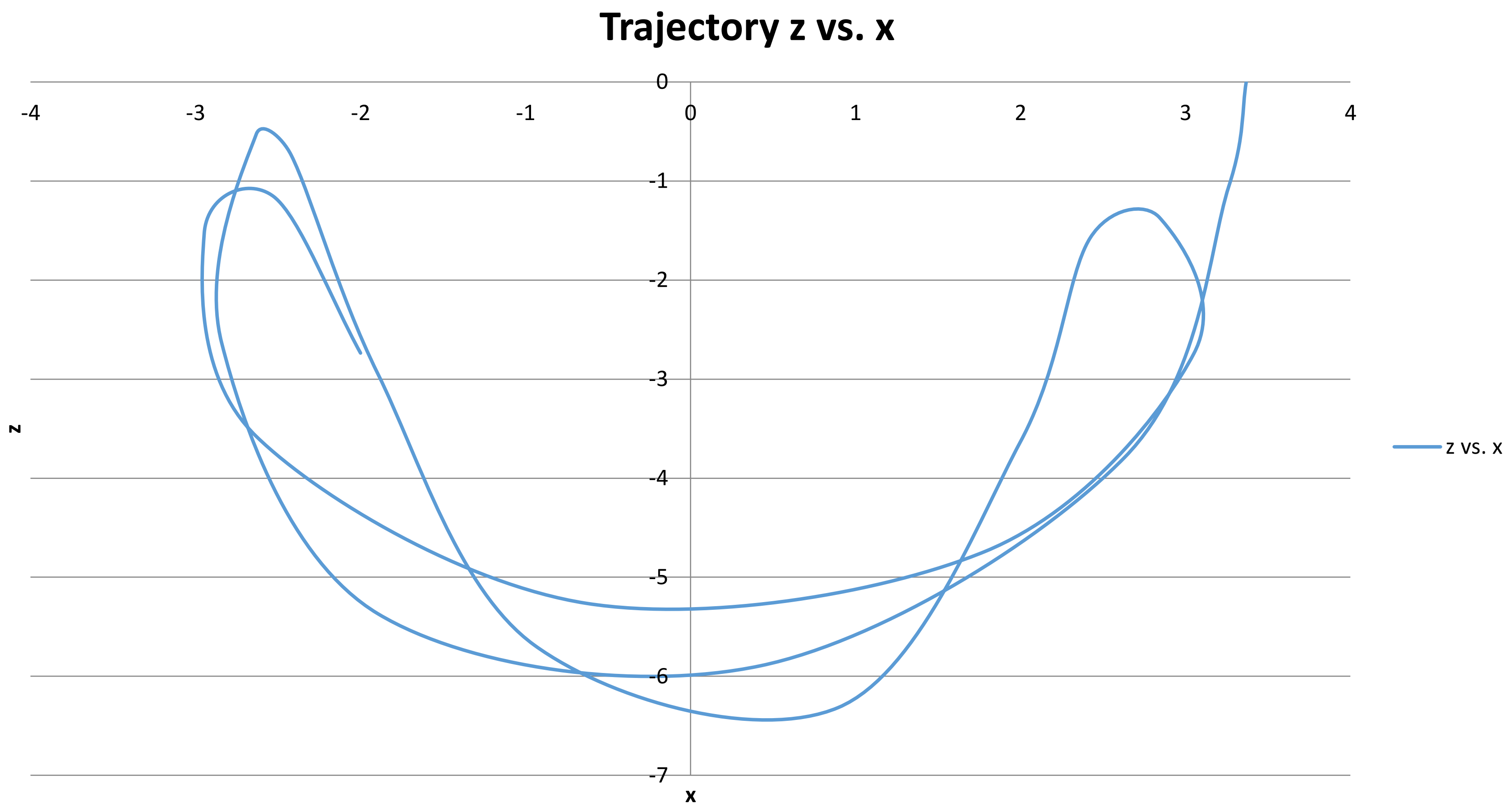
By measuring all the data from high speed camera. Time interval is ¼ second.

$$\begin{cases} \ddot{x} = -\frac{a}{m} \cdot \frac{(r - l_0)^2}{r} x - \frac{b}{m} \cdot \frac{r - l_0}{r} x \\ \ddot{y} = -\frac{a}{m} \cdot \frac{(r - l_0)^2}{r} y - \frac{b}{m} \cdot \frac{r - l_0}{r} y \\ \ddot{z} = -\frac{a}{m} \cdot \frac{(r - l_0)^2}{r} z - \frac{b}{m} \cdot \frac{r - l_0}{r} z - g \end{cases}$$

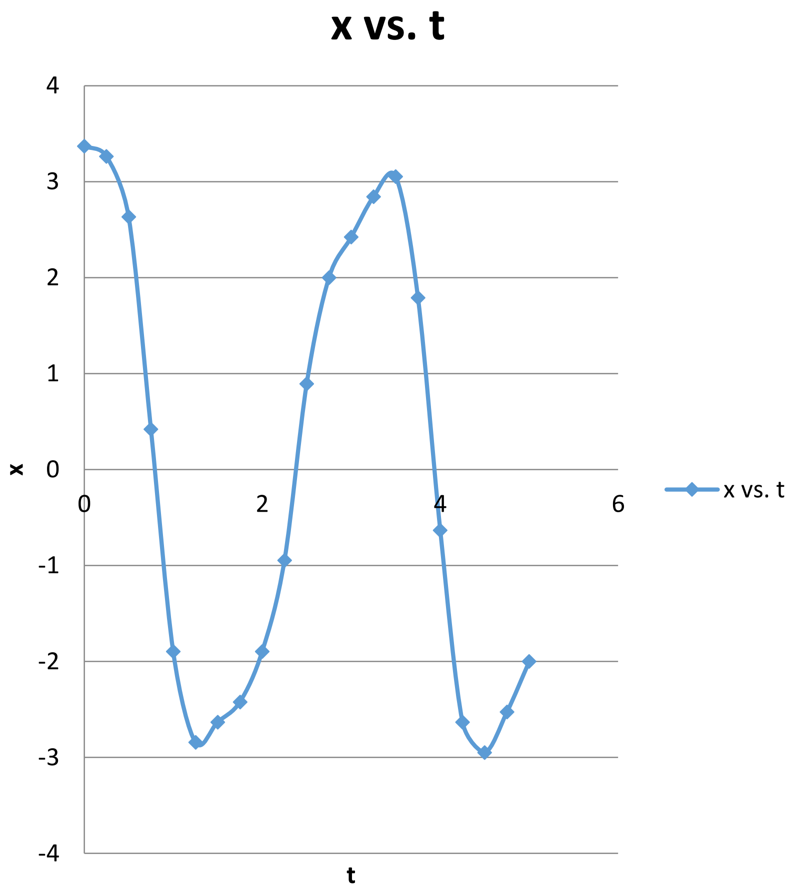
Bring all the data to the Lagrangian Equation:

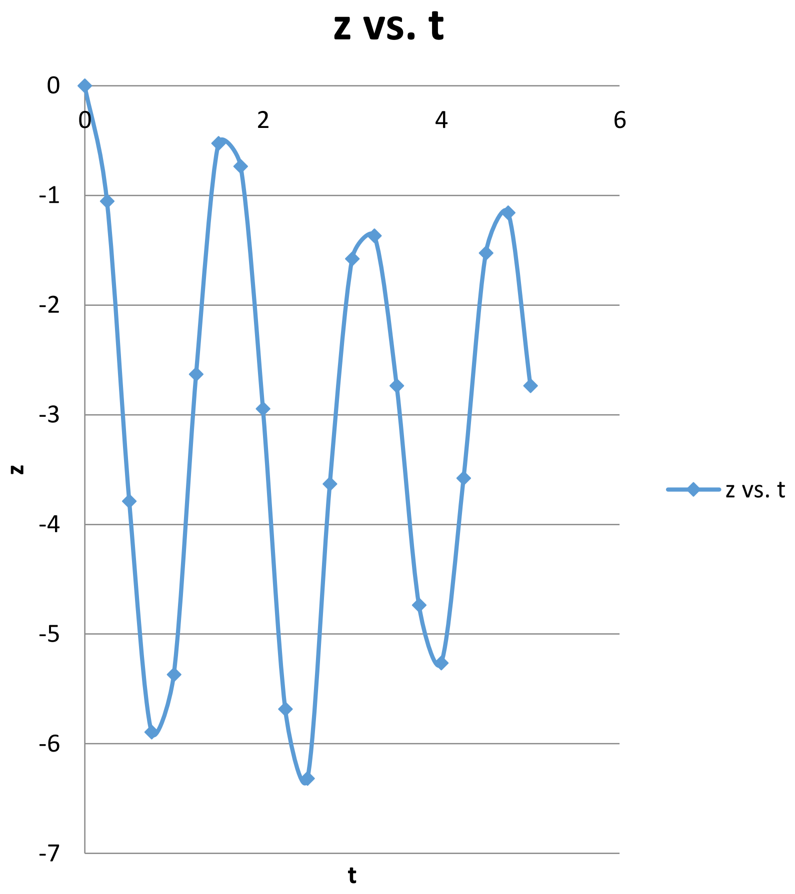
a = -0.3397   b = 4.2619

Results and Plots

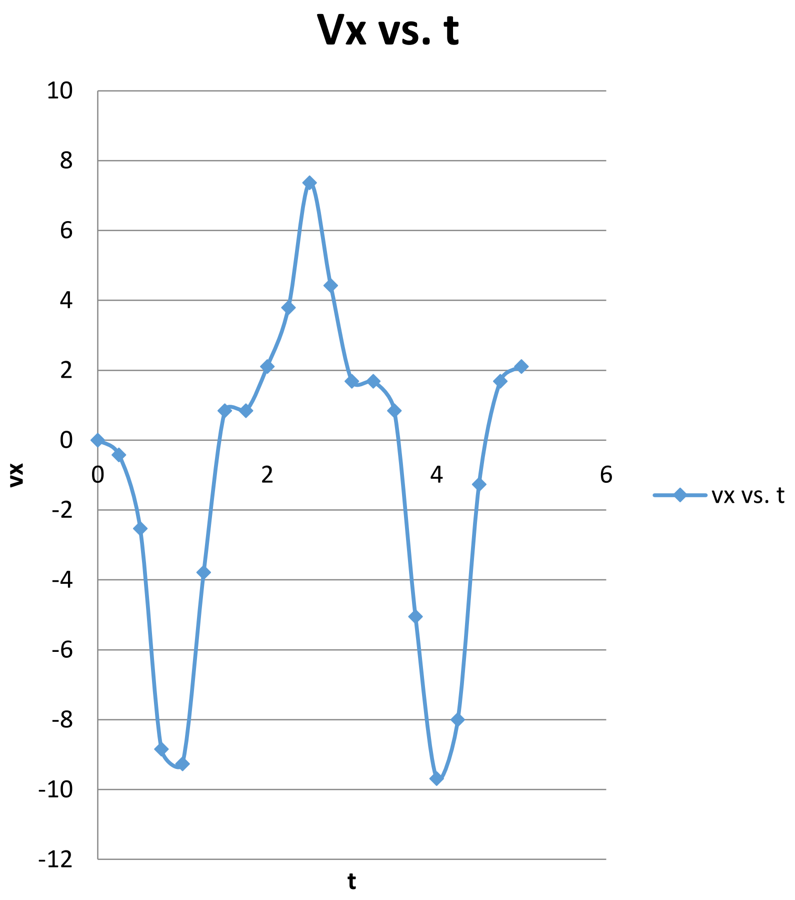


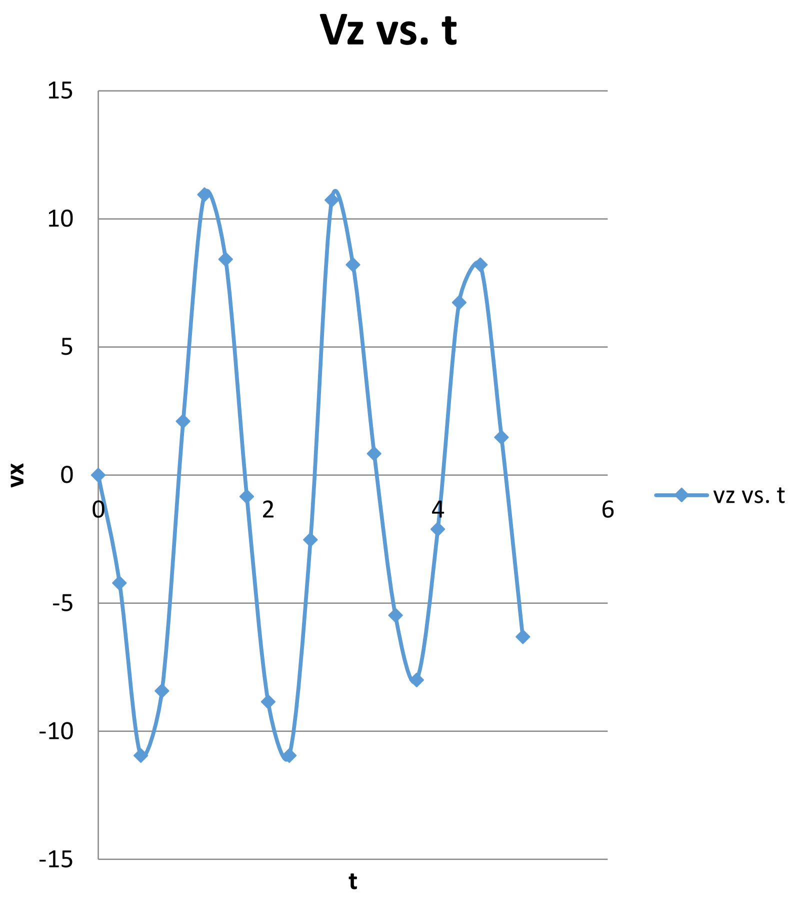
Position vs. Time



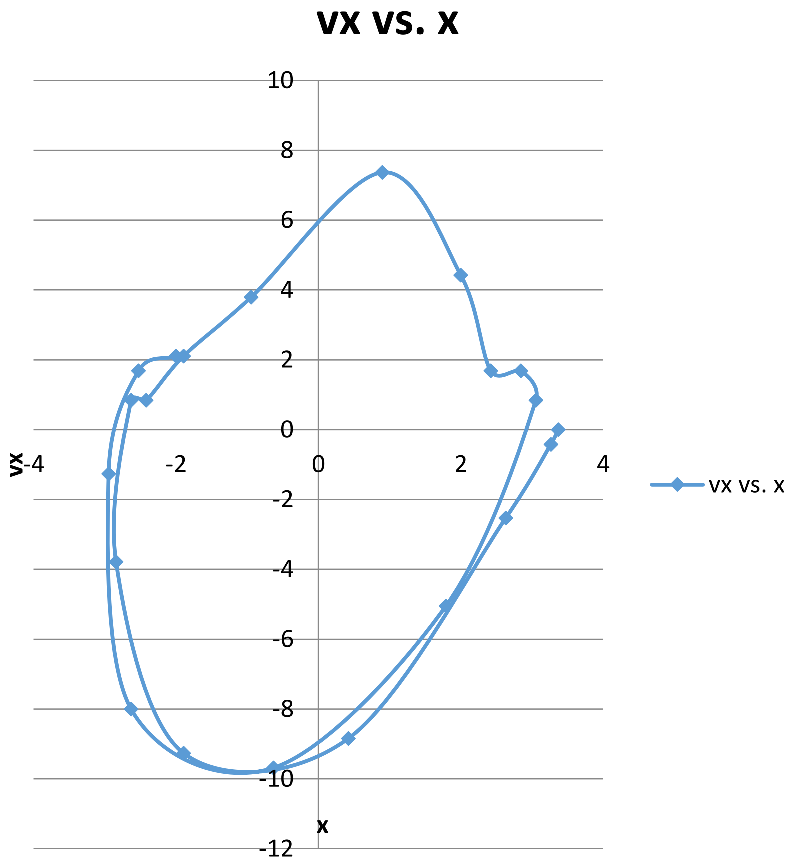


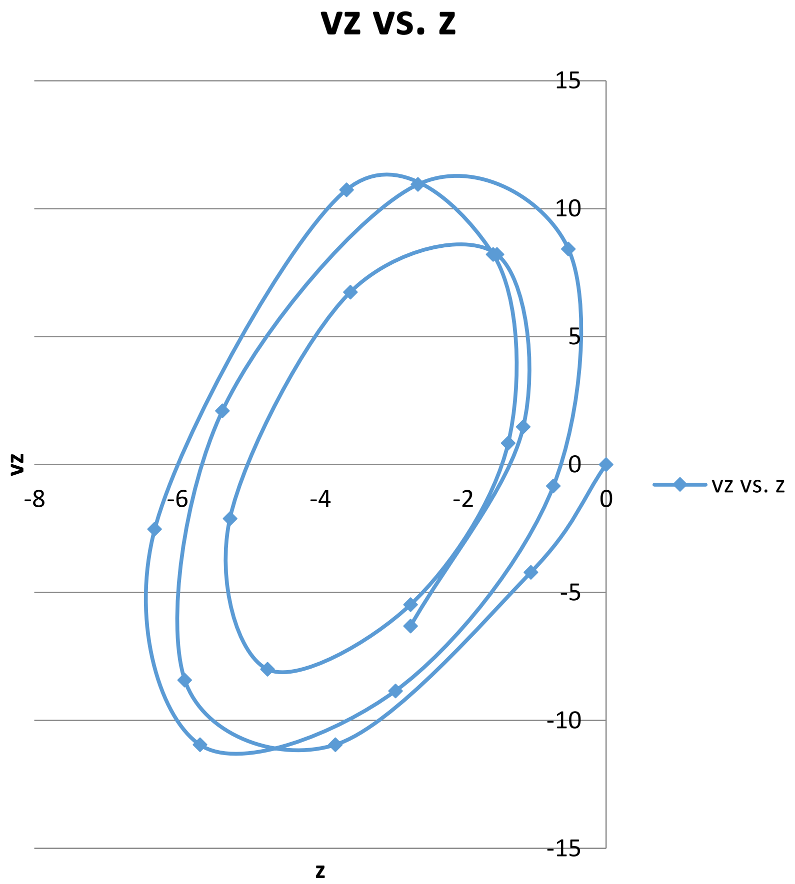
Velocity vs. Time





Phase Portrait Diagram





Conclusion

- We were able to:
  - design and test the most cost-effective physical model
  - improve our MATLAB model by comparing it with experimental data
  - confirm that one mode of the physical model met the numerical simulation model using (a) high speed camera and (b) long exposure photography for at least $0<t<3$ sec
- This model might be worth developing to serve as a prerequisite for the creation of large-scale extreme bungee jumping regimes.

References (1/2)

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