ENERGY FLOWS IN ELECTRICAL GRIDS

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FLOW OF POWER

Generation of Electricity

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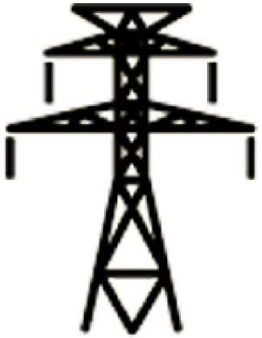
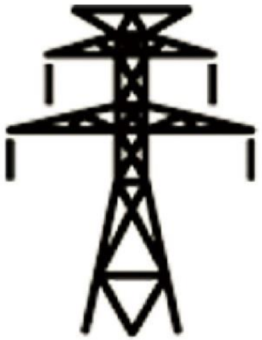


Transmission

FLOW OF POWER

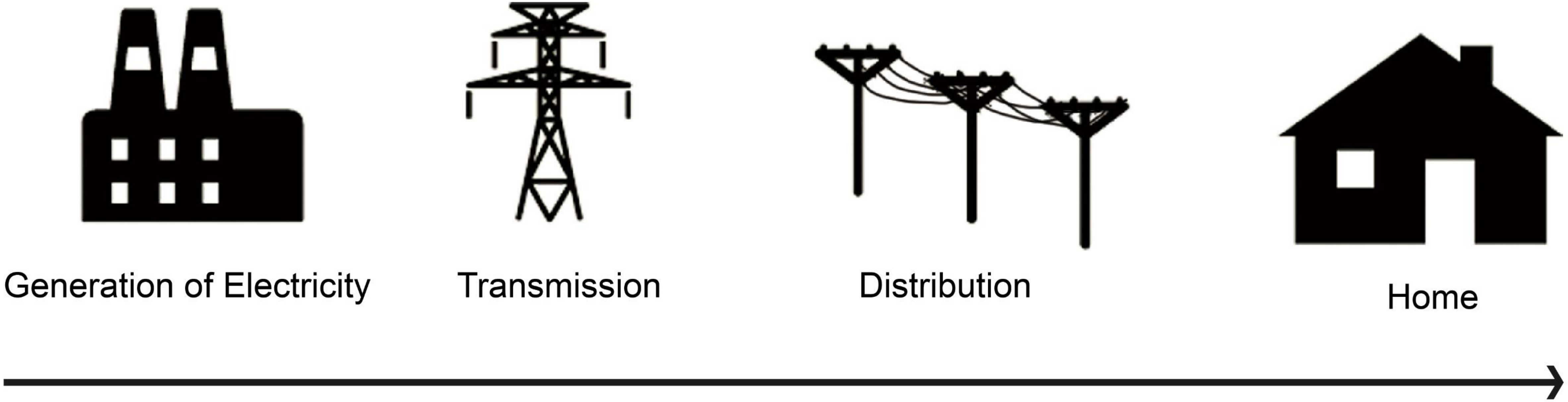
Generation of Electricity



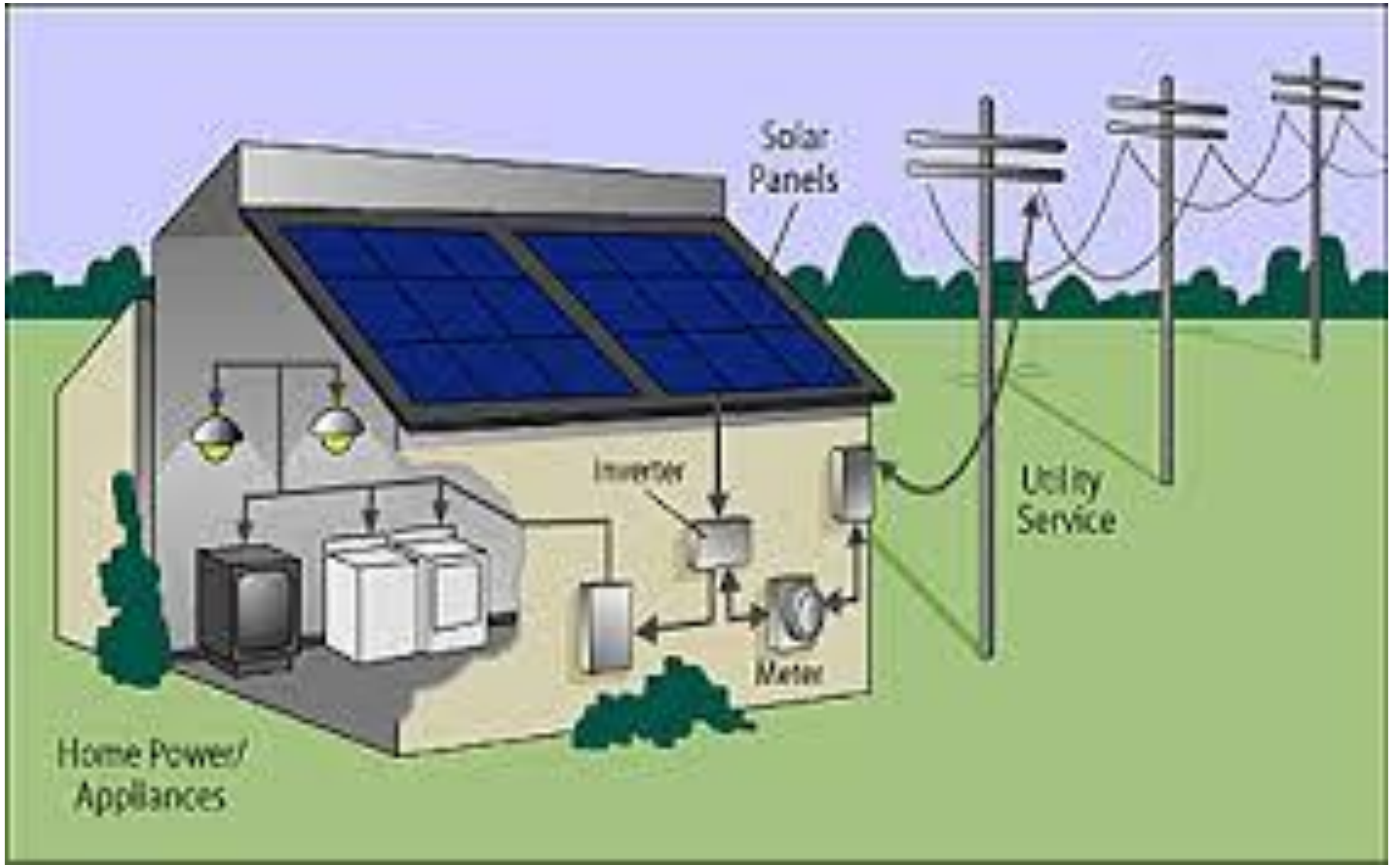
Transmission

Distribution

FLOW OF POWER



APPLICATIONS



MOTIVATION

- Voltage fluctuates along a feeder line
  - Having consumers also produce is a strain on the grid
- Better integration of renewable energy sources without causing instability
- Goal: To model the effects of variability power along the electrical line.

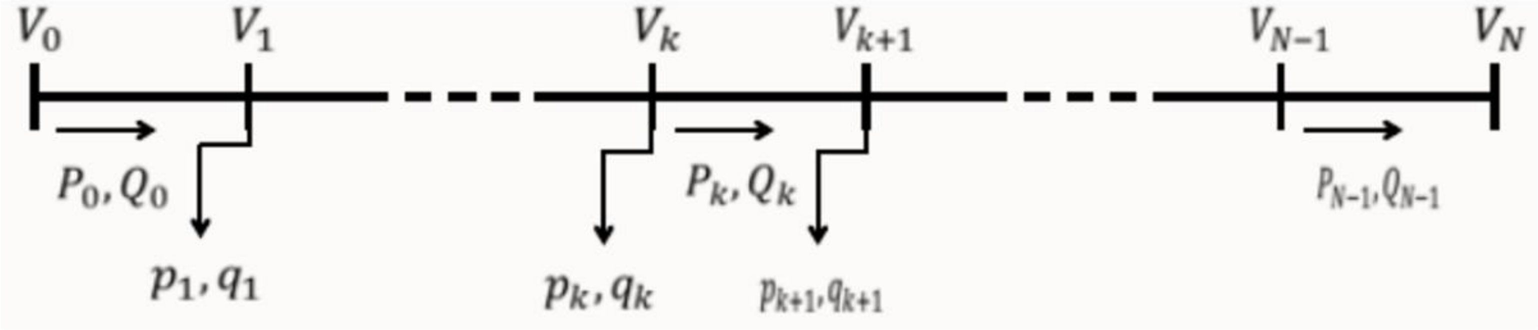
TERMS

DistFlow ODEs variables

- k = 0,...,N-1 enumerates buses of the feeder (k=consumers)
- P_k is real power flowing from bus k to bus k + 1
  - Real power is energy that can be used to do work
- Q_k is reactive power flowing from bus k to bus k + 1
  - Reactive power is needed in an alternating current transmission system to support the transfer of real power over the network
- p_k and q_k are net consumption for consumers
- r_k, x_k represent the resistance and inductance of the line element connecting bus k to bus k+1

VISUAL MODEL

General idea for how energy flows in electrical grids.



THE DISCRETE MODEL

DistFlow Equations

$$P_{k+1} - P_k = p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2}$$

$$Q_{k+1} - Q_k = q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2}$$

$$v_{k+1}^2 - v_k^2 = -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{v_k^2}$$

with boundary condition $P_{N+1} = Q_{N+1} = 0, \quad v_0 = 1$

HOMOGENIZATION

- Large number of consumers ($N >> 1$)
- Continuous form with limit $N \to \infty$

DistFlow ODEs (B.V.P.)

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\dfrac{P^2 - Q^2}{v^2} \\ q - x\dfrac{P^2 - Q^2}{v^2} \\ -\dfrac{rP + xQ}{v} \end{pmatrix}$$

boundary conditions

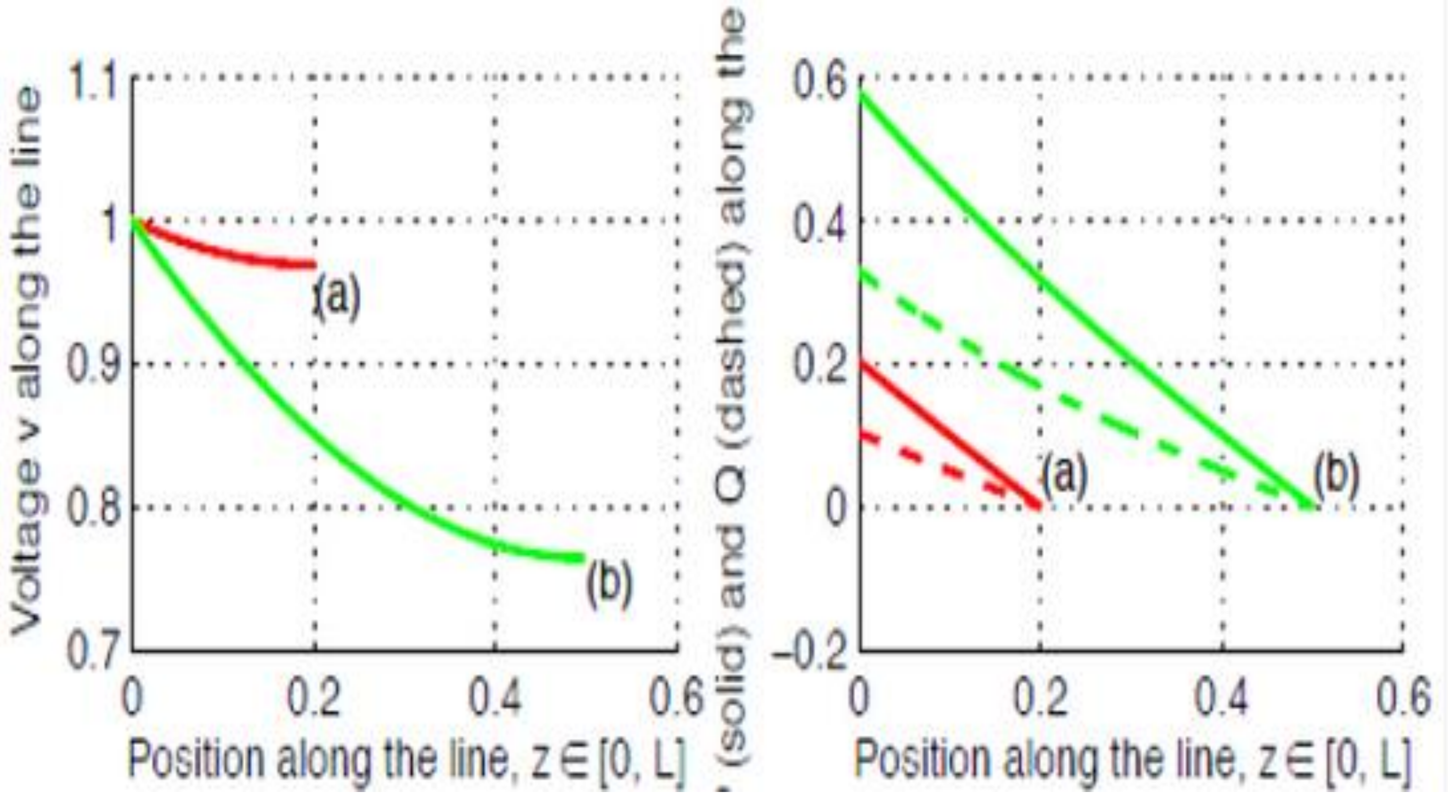
$$V_0 = 1, P(L) = Q(L) = 0$$

- By Solving the ODEs for known length of feeder line, one can evaluate real power, reactive power, and voltage along the line.
- Power vs Position
- Voltage vs Position

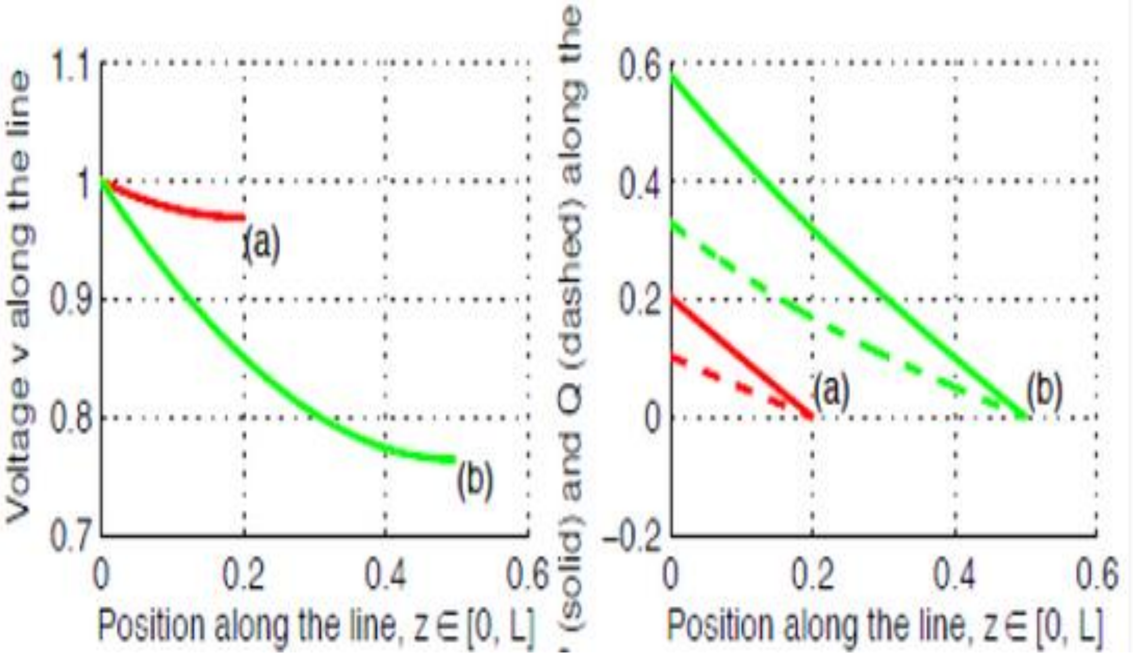
REPRODUCTION OF RESULTS

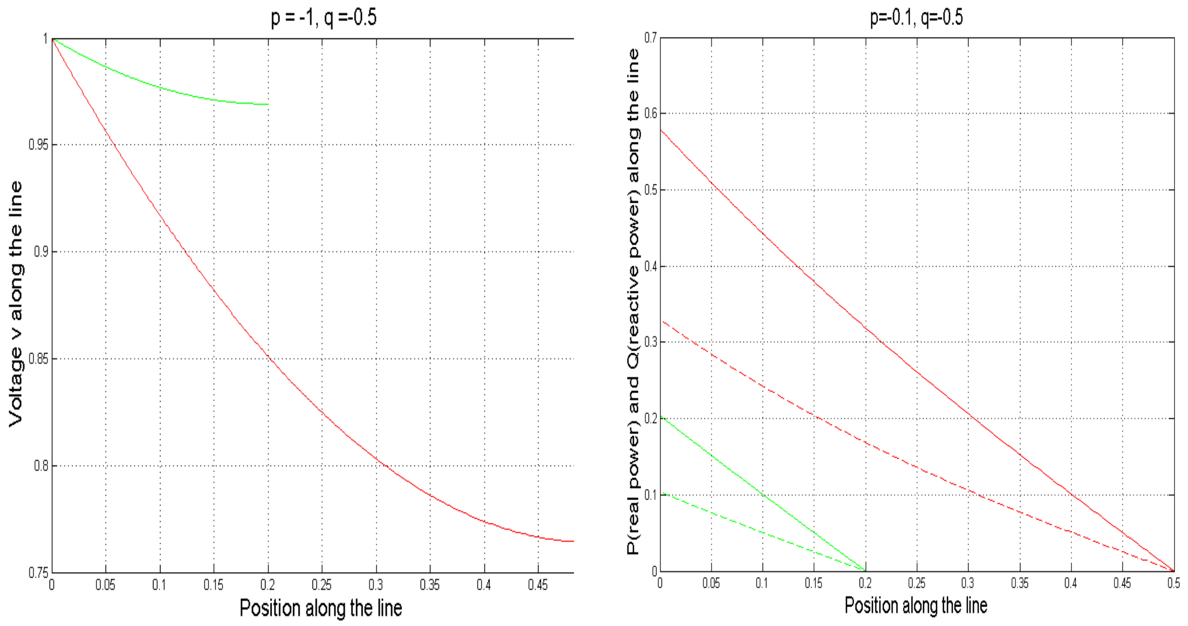
PREVIOUS WORK

- Set $r = x = 1, q = \frac{p}{2}$
- p=-1, q=-0.5
- p and q are negative along the feeder -- consuming



PREVIOUS RESULTS vs. OUR RESULTS





OUR WORK

ADDING STOCHASTICITY

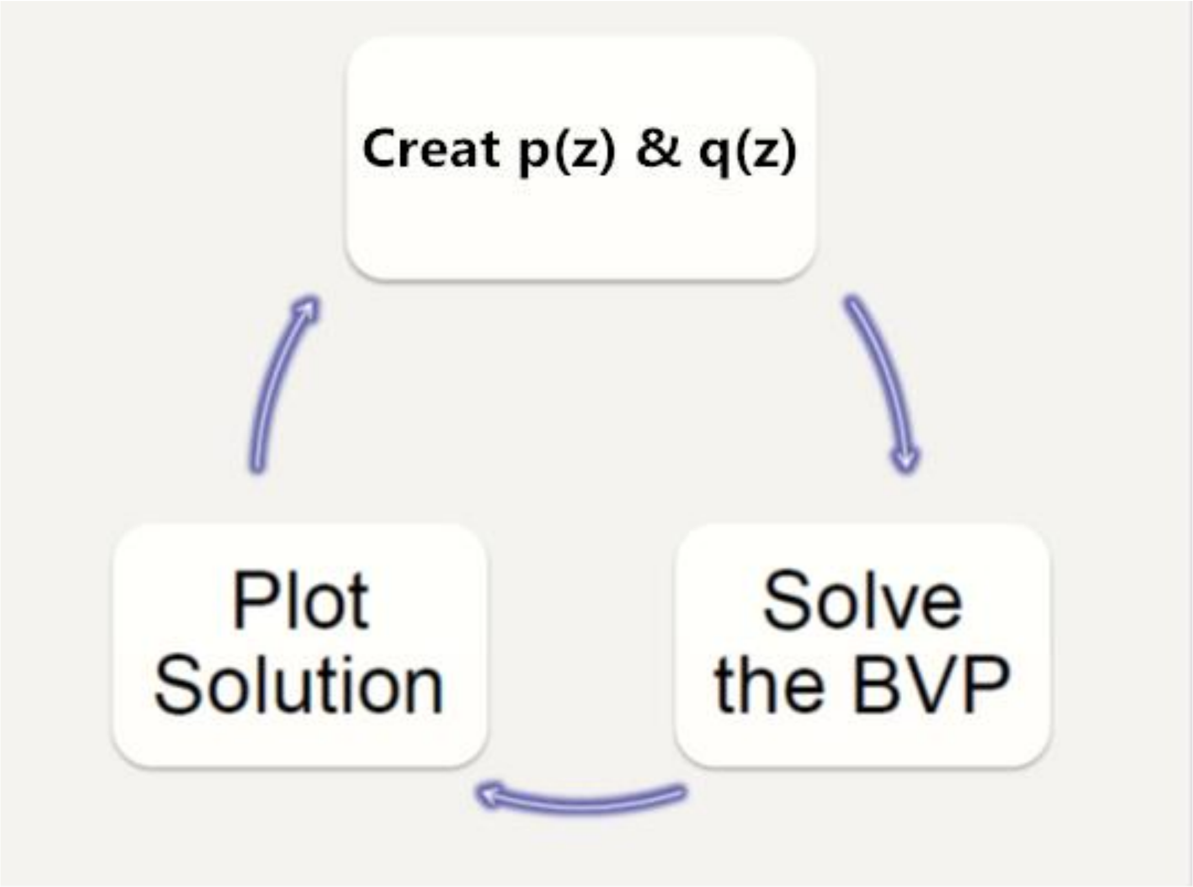
ADDING STOCHASTICITY

- A stochastic process, or a random process, is a collection of random values. This is often used to represent the evolution of some random variable, or system, over time

- ***Independent and identically distributed*** method was used
  - Independent and identically distributed of each random variable has the same probability distribution as the others and all are mutually independent

METHODOLOGY

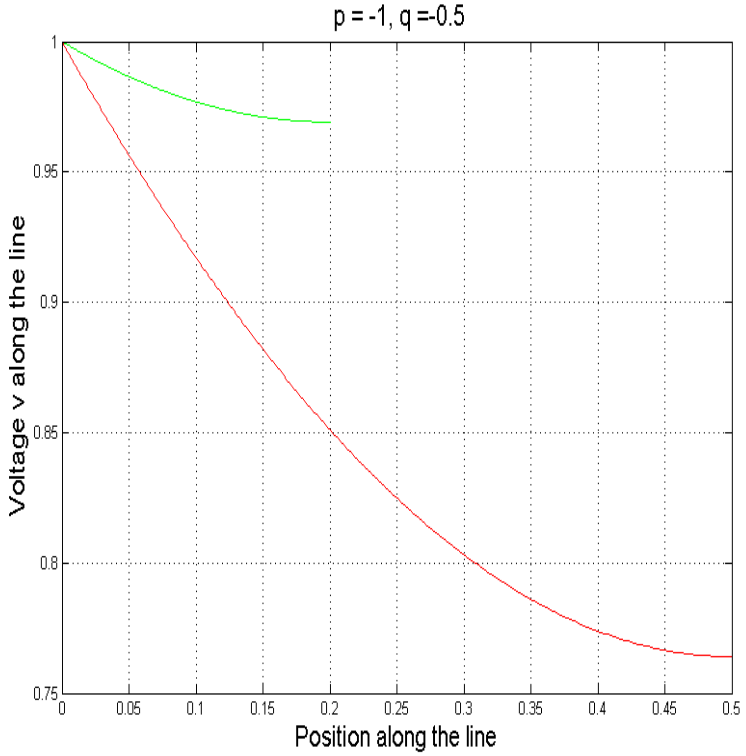
$$p(z) = -1 + 0.3 \times n$$
$$q(z) = -0.5 + 0.15 \times n$$



VOLTAGE ALONG THE FEEDER LINE

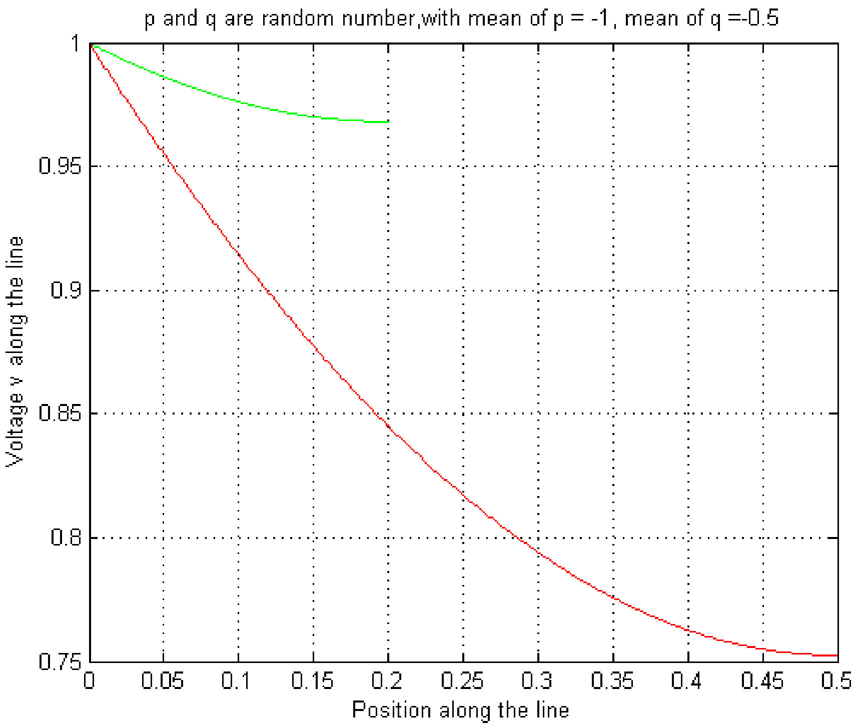
Set $r = x = 1$

- Real power, p, and reactive power, q, are constant



Set $r = x = 1$

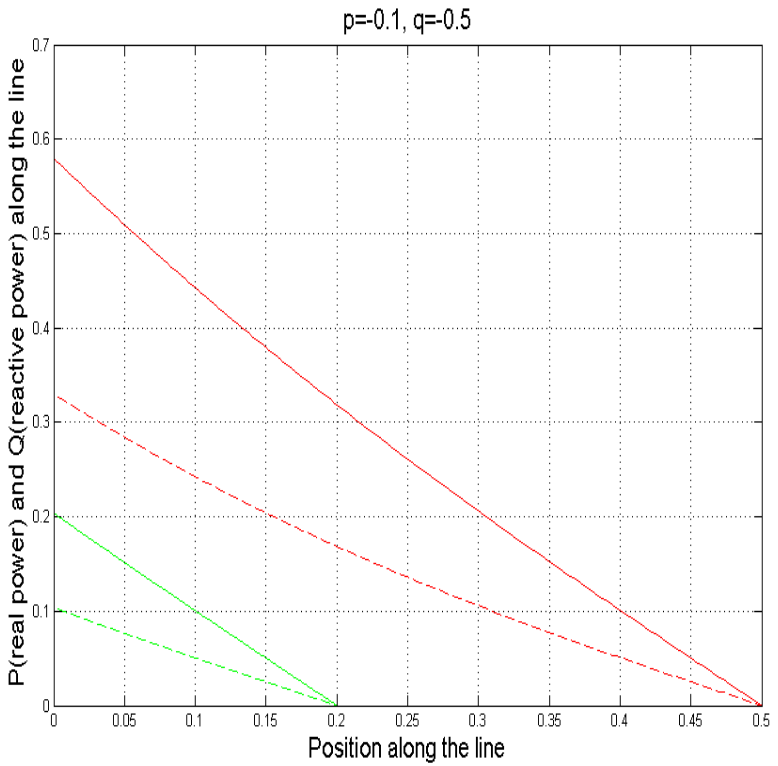
- Real power, p, and reactive power, q, are random numbers



POWER ALONG THE FEEDER LINE

Set $r = x = 1$

- Real power, p, and reactive power, q, are constant



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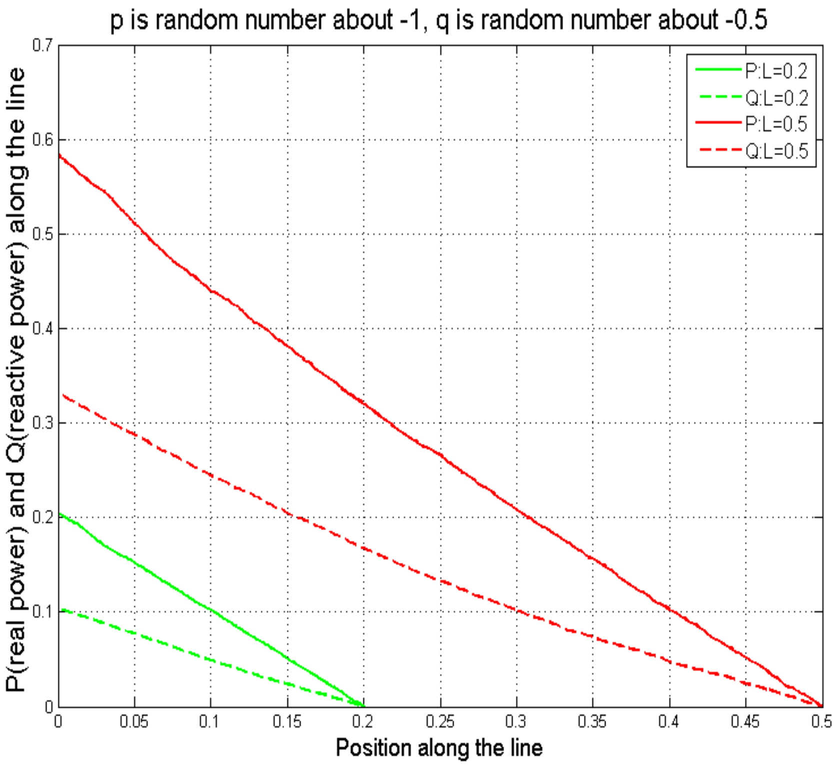
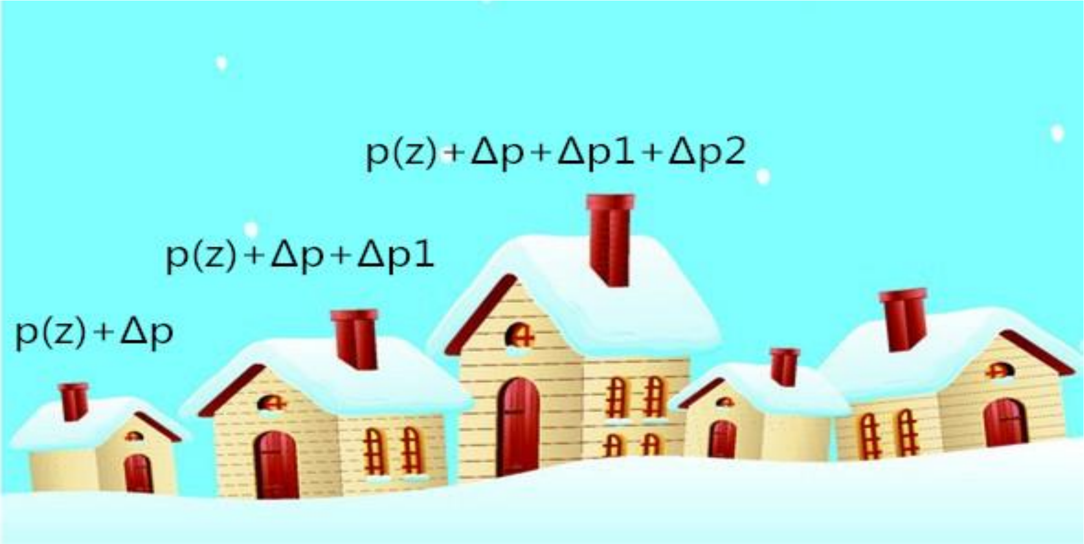


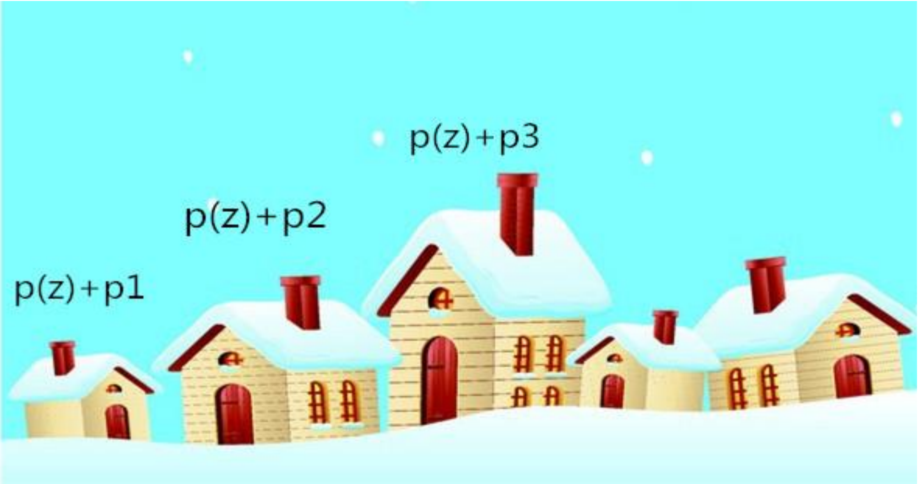
WIENER PROCESS vs. I.I.D. PROCCESS

- Let $p(I) = p_0 + W(I)$, where $W(I)$ is a Wiener Process.
- Substitute for p in DistFlow ODEs
- q=p/2, no independent noise
- Solve for boundary value problem



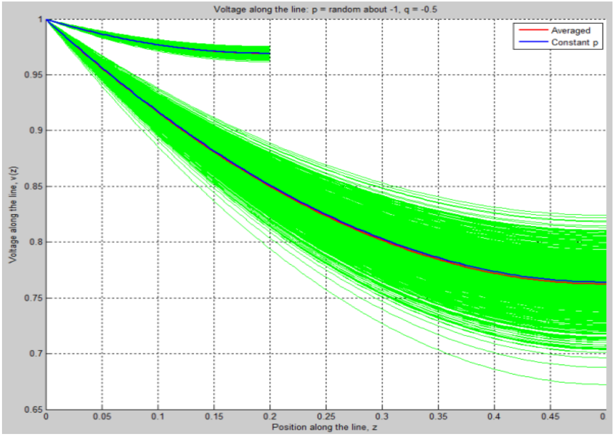
- Let $p(z) = -1 + 0.3 * n$
- Let $q(z) = -0.5 + 0.15 * n$,

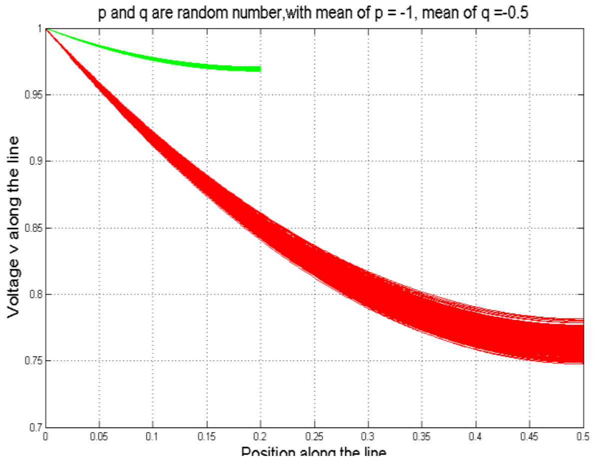
  where n ~Normal (0,1)
- Substitute for p and q in DistFlow ODEs
- Solve for boundary value problem

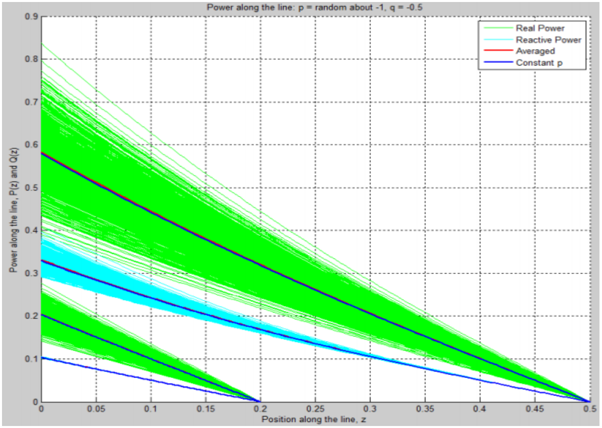


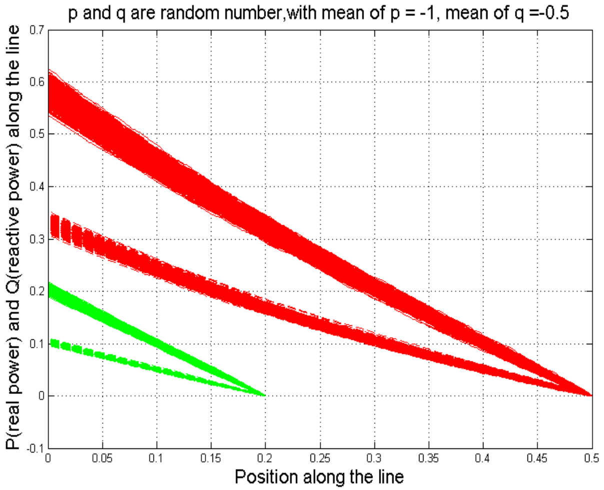
RESULTS

PREVIOUS RESULTS vs. OUR RESULTS

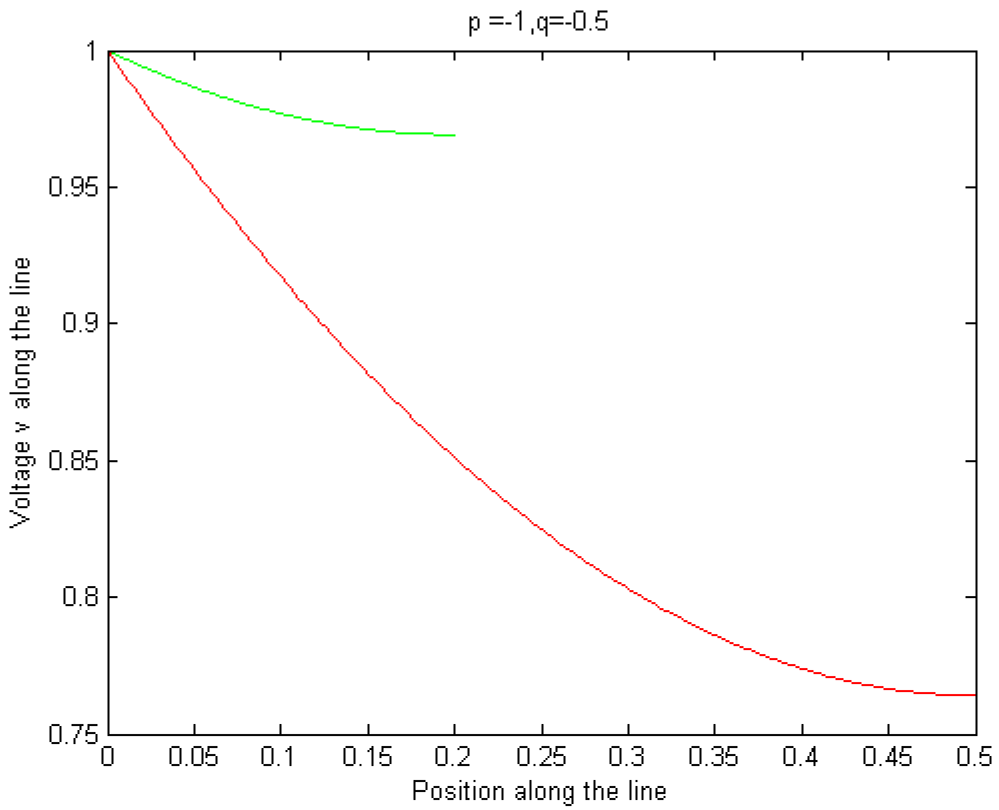


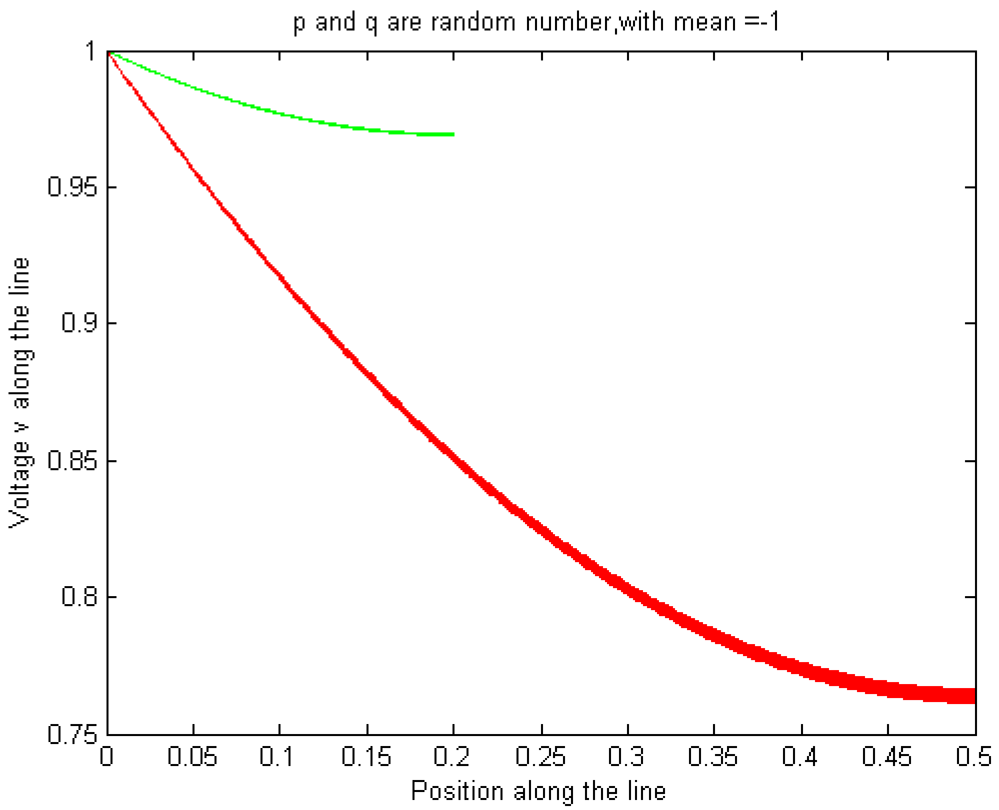






INTRODUCING NOISE INTO THE SYSTEM





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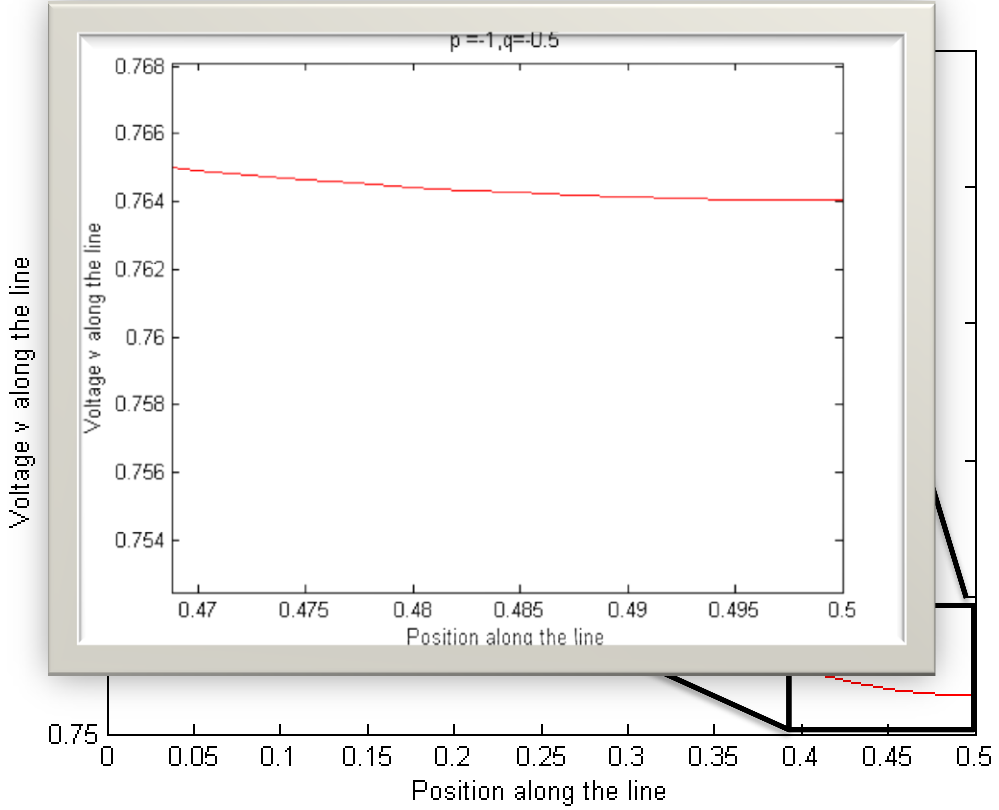

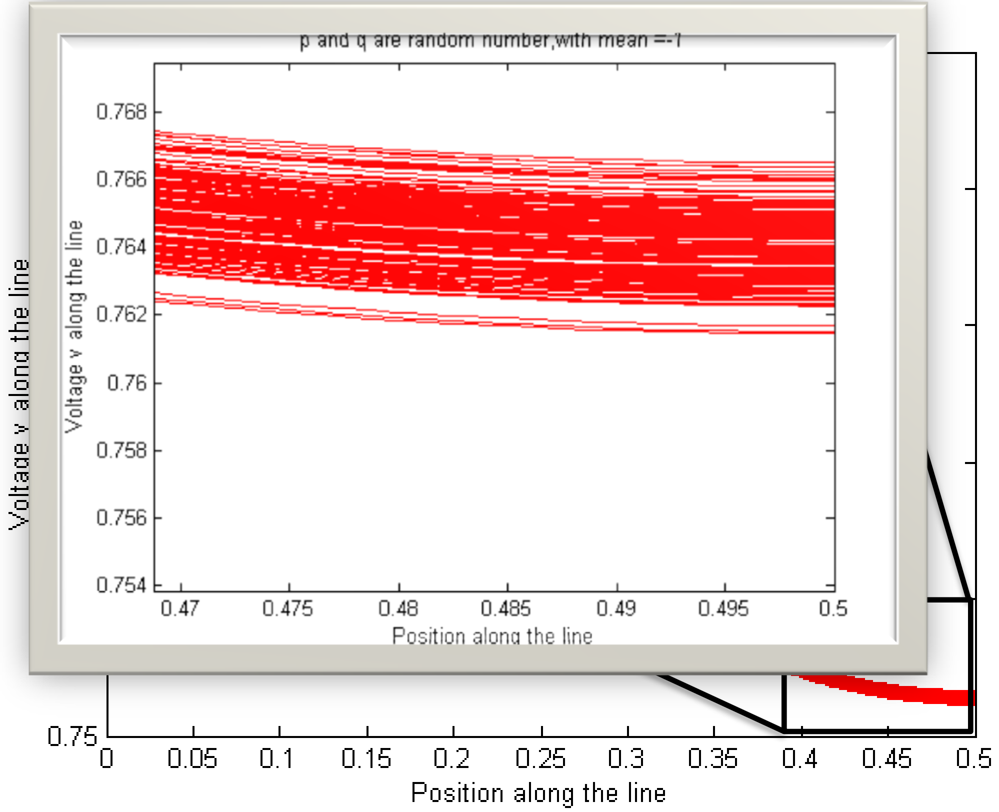

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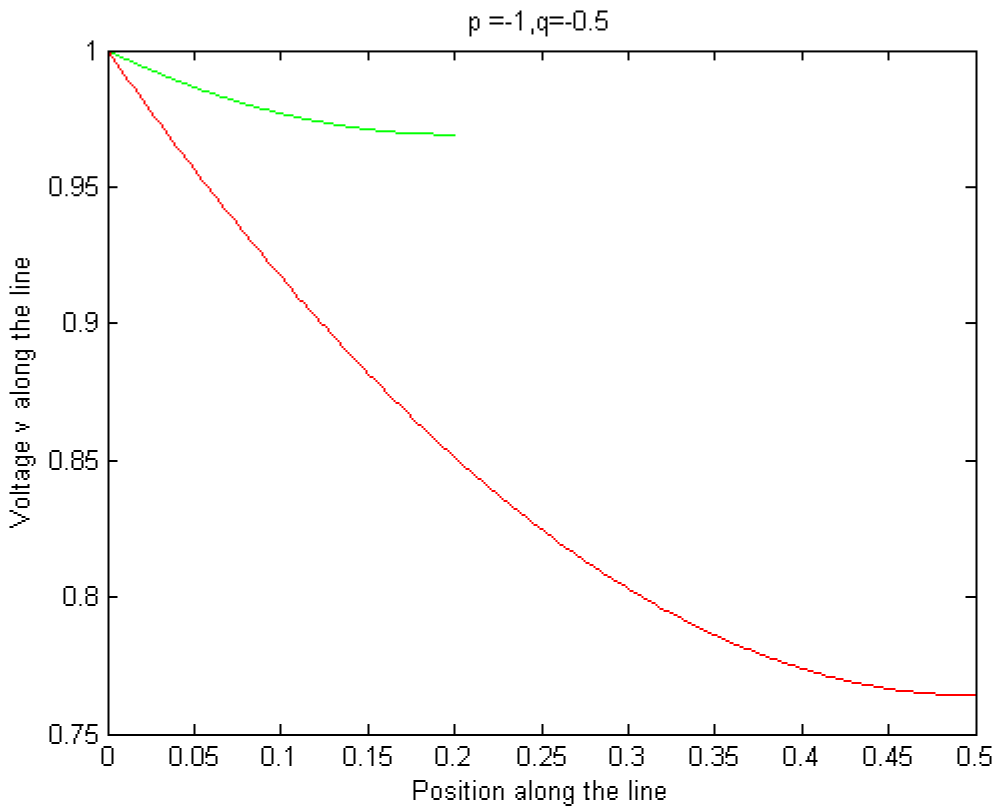
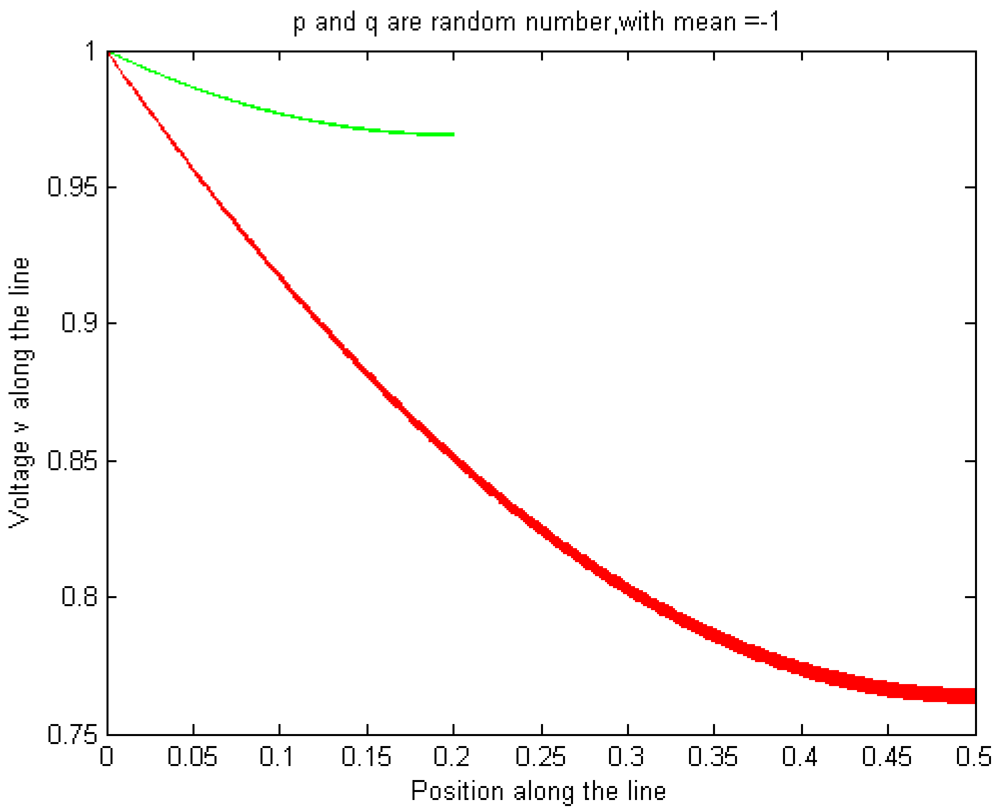
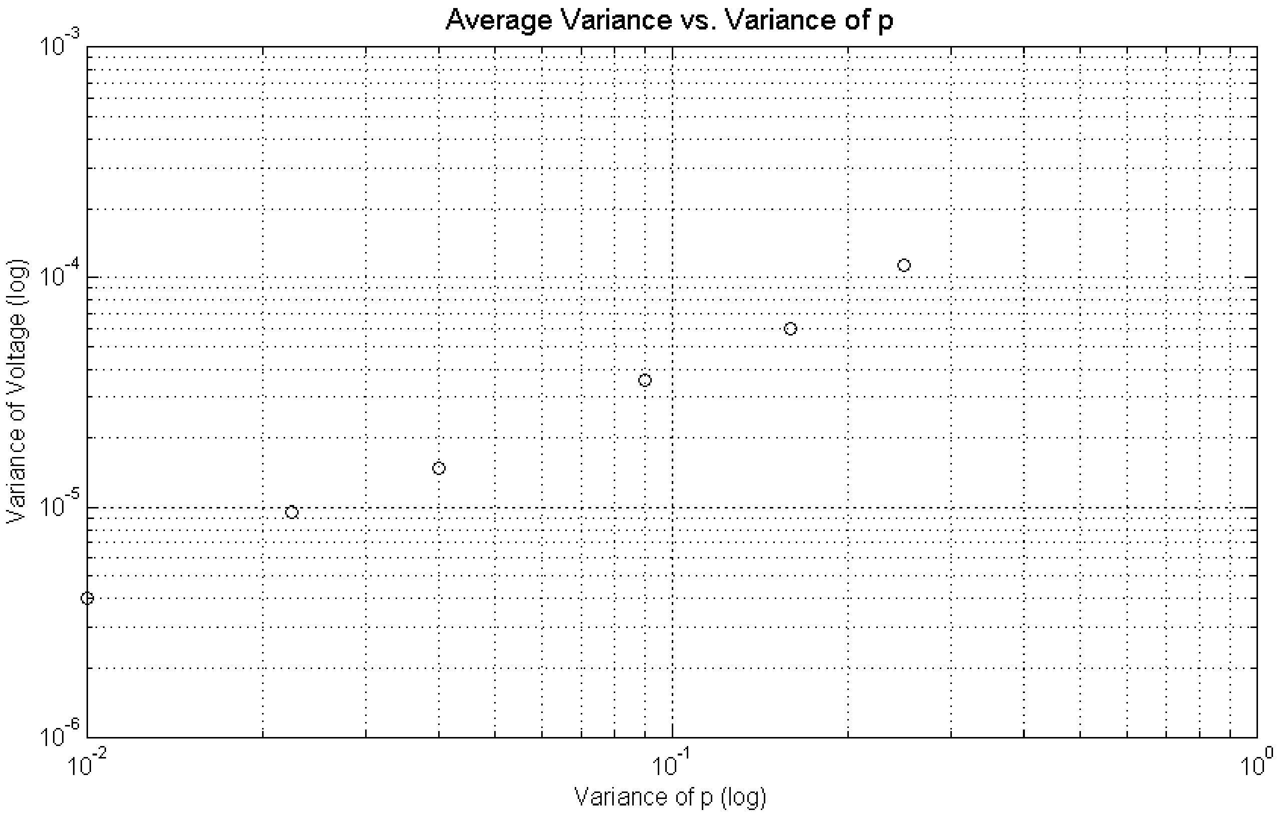


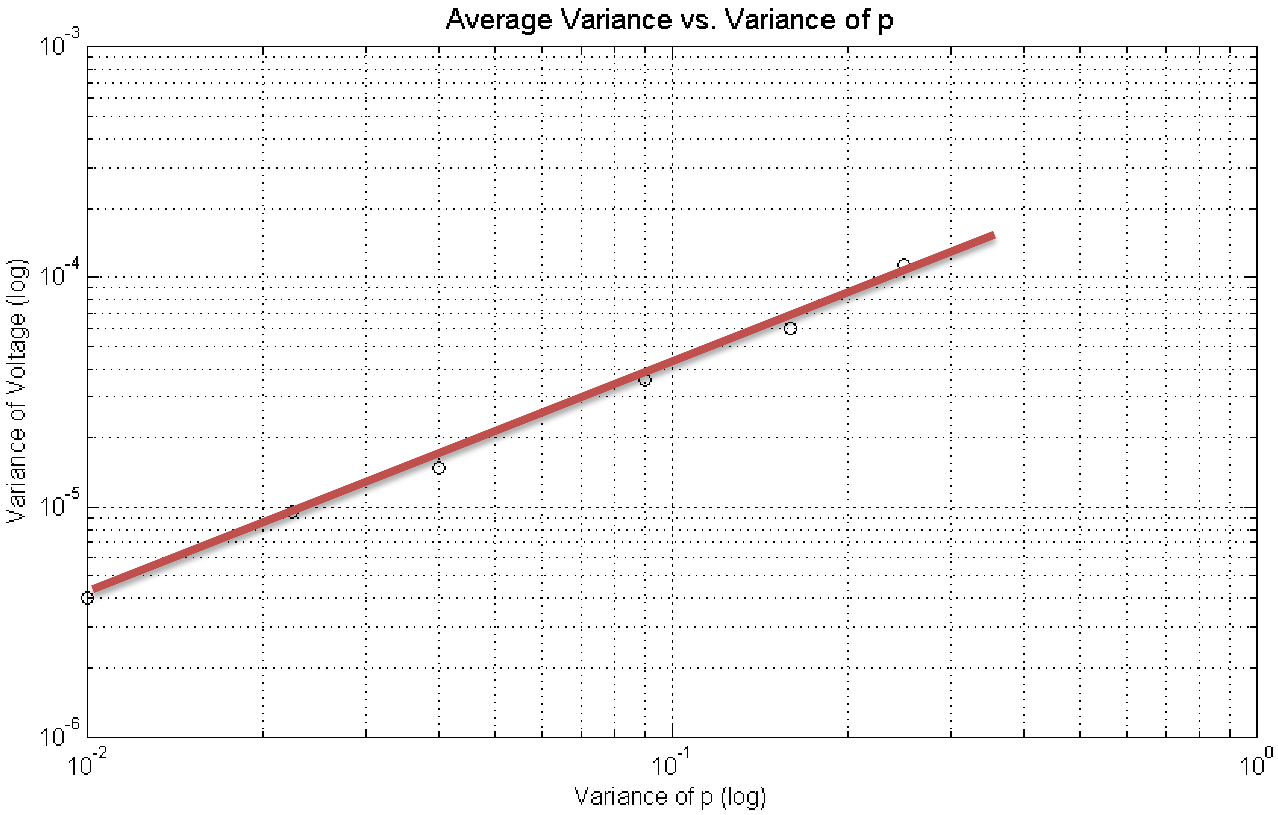


VARIANCE



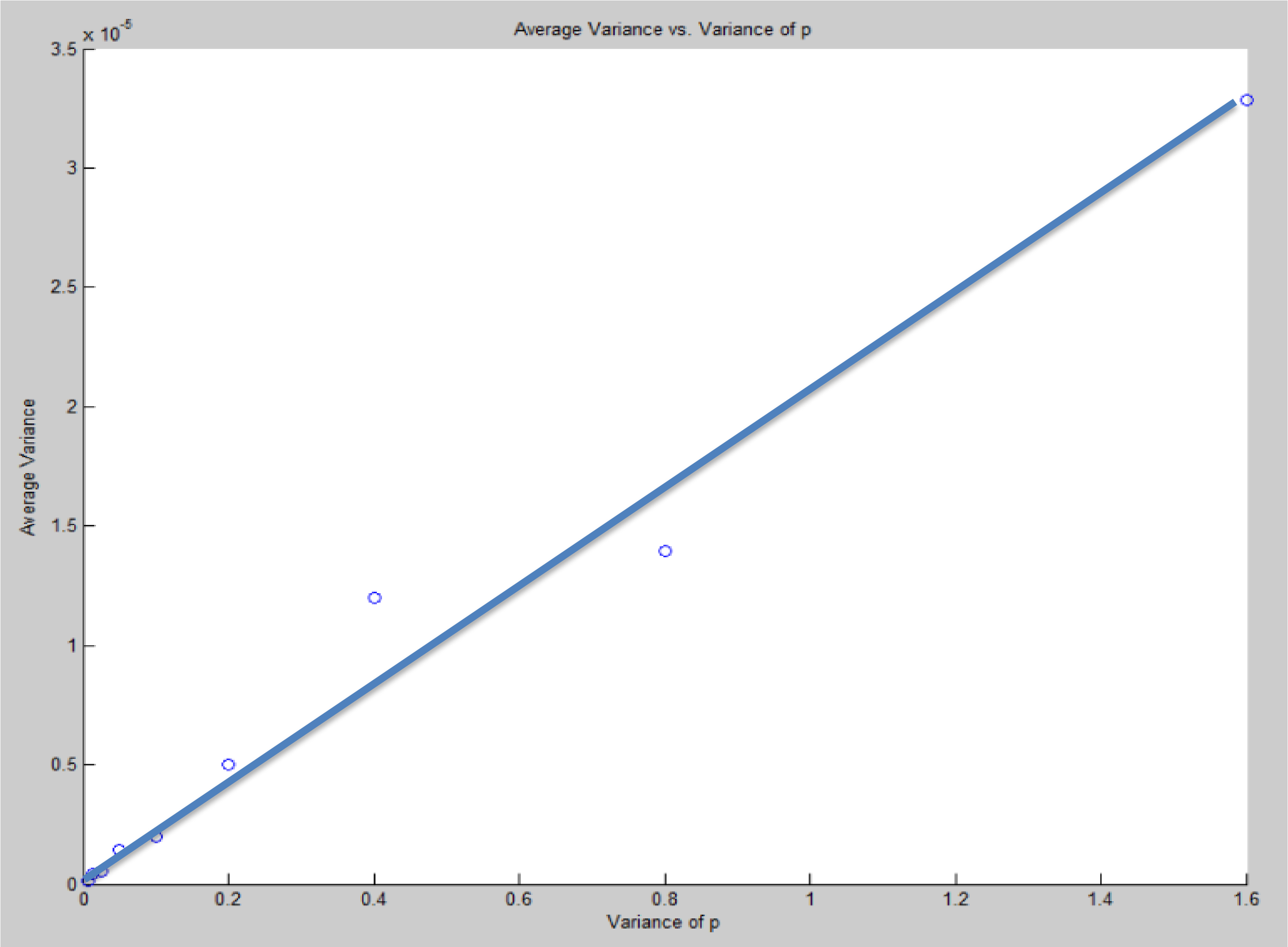
$$p(I) = -1 + \boxed{0.3} \times random\ numbers$$
$$q(I) = -0.5 + 0.15 \times random\ numbers$$

VARIANCE



- $y = a \cdot x + b$
- $y = \ln(S_v^2)$
- $x = \ln(\sigma_p^2)$
- $S_v^2 = C \cdot (\sigma_p^2)^a$
- $Value\ of\ a\ (slope) = 1$
- $S_v^2 \sim \sigma_p^2$

COMPARSION



- $Value\ of\ a\ (slope) = 2$
- $S_v^2 \sim (\sigma_p^2)^2$

DISCUSSION

DISCUSSION

- Small perturbations of power consumption have relatively little effect on voltage along the feeder line

- Assuming a constant power consumption is statistically valid

- The longer the feeder line is, the more it will be effected by the factors of line resistance, reactance, and fluctuations in power consumption

- The relation between the variance of p and the variance of voltage value is dependent on the model we use and the type of stochastic process that introduced into the system

References

1. **D Wang, K Turitsyn and M Chertkov, "DistFlow ODE: Modeling, Analyzing and Controlling Long Distribution Feeder", Proceedings of, the 51st IEEE Conference on Decision and Control (2012)**
2. **Kaplan, S. M. (2009). "Smart Grid Electrical Power Transmission: Background and Policy Issues". The Capital.Net, Government Series.Pp. 1-42**


Thank You.


Questions?