WAVE STRUCTURES ON A JET ENTERING THE BULK LIQUID

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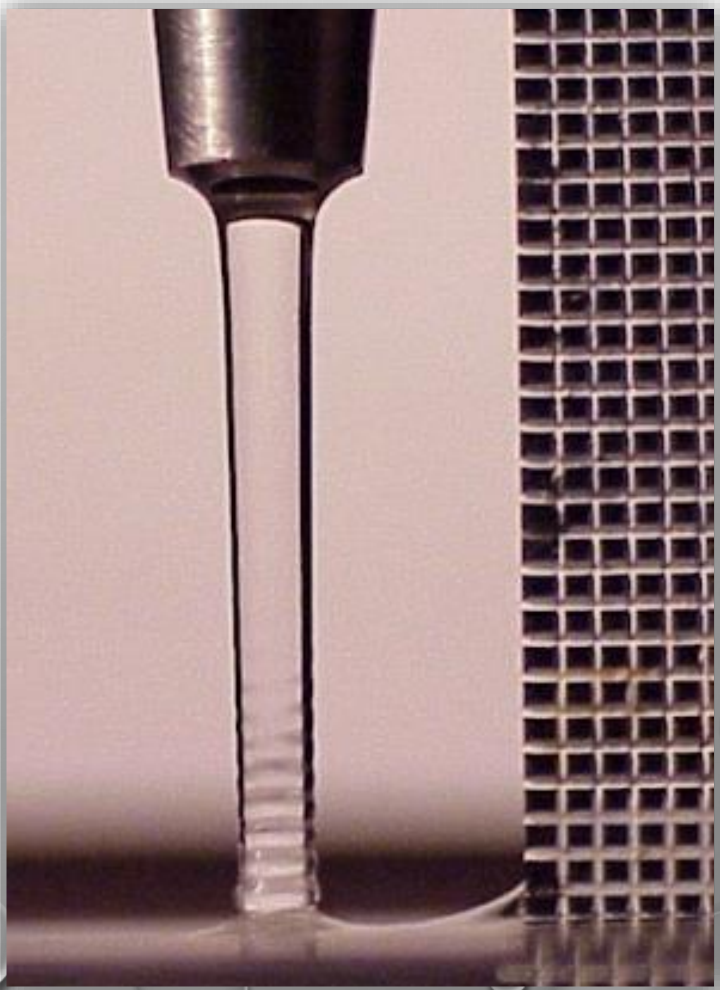
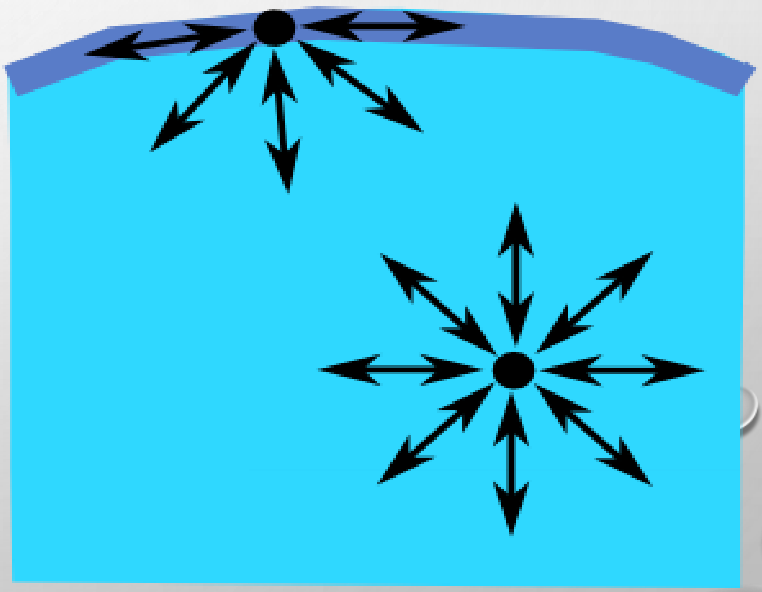
INTRODUCTION

- When a jet impacts the liquid surface, there will be wave on the jet. Such wave is called **Capillary Wave**.
- It is generated from the vibration under the effect of **Surface Tension**.



SURFACE TENSION

- THE SURFACE TENSION IS DUE TO THE IMBALANCE OF THE MOLECULAR ATTRACTION ON SURFACE LAYER.
- <u>Inside the liquid</u>, each molecule receives the attraction (incl. repulsive force) from the adjacent molecule in each direction. Therefore, the resultant force that the internal molecule receives is zero.
- However, <u>on the boundary</u> between the liquid and gas, the attraction receives in each direction is unbalanced. Then the molecules are **pulled** into the interior part of the liquid.
- Therefore, the surface tension makes the liquid surface try to obtain the smallest smooth area.



CAPILLARY WAVE

- When a water jet impacts the water surface, the particles flow away from the equilibrium position due to the **interference**. Then, they move along the horizontal surface vertically.
- Under the effect of the **surface tension as the restoring force**, the water particles will return to the equilibrium position.
- Afterwards, the particles continue to move toward the other side under the inertia effect, which forms vibration. **As the vibration spreads**, the capillary wave comes into being.

CAPILLARY WAVE

- The left figure shows the pure water, while the right figure shows the water with surfactant, with larger surface tension which is regarded as restoring force.
- Therefore, in right figure, the liquid particles can restore equilibrium rapidly, and we could see the wave length is shorter.

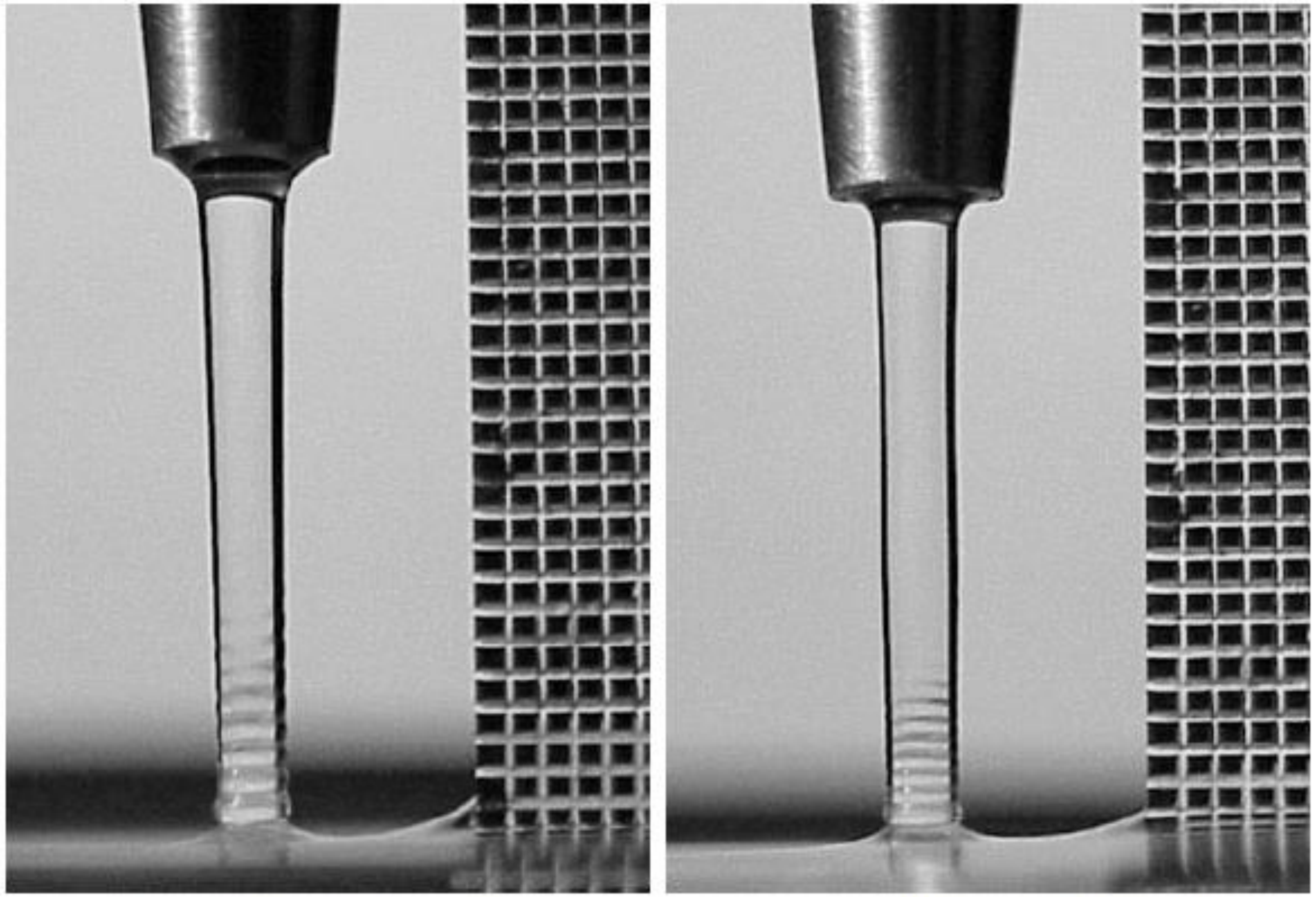


FIGURE 1. Water jets impinging on a pure water reservoir when (*a*) $Q = 3.2\,cm^3\,s^{-1}$, (*b*) $Q = 4.6\,cm^3\,s^{-1}$. The grid on the right is millimetric.

CAPILLARY WAVE

- The water velocity increases gradually as it falls down. When the water velocity is equal to the wave velocity, we can view the capillary wave with naked eyes.
- In the following experiment, we measure the wave velocity based on this principle.

PURPOSE

In this project, our purpose is to explore the dispersion relation between the length and velocity of capillary wave.

DERIVATION OF FORMULAS

Laplace's equation

$$\nabla^2 \phi = 0$$

$$\mathbf{V} = \nabla \phi$$

$$Z = h(x, y, t)$$

$$\frac{\partial h}{\partial t} + \mathbf{V}_x \frac{\partial h}{\partial x} + \mathbf{V}_y \frac{\partial h}{\partial y} = \mathbf{V}_z$$

$$So \ \frac{\partial h}{\partial t} = \mathbf{V}_z \quad \text{Since } \mathbf{V}_x \ and \ \mathbf{V}_y = 0$$

$$\mathrm{h}(x,y,t)=e^{ikx-i\omega t}$$

$$\phi=-i\omega h(x,y,t)f(z)$$

Plug in $\frac{\partial h}{\partial t}=\mathbf{V}_z$

So $-i\omega h=-i\omega hf^{'}(z=h)$

Hence, $f^{'}(z=h)=1$

Then solve for the Laplace's Equation,

$$f=\frac{1}{k}e^{k(z-h)}=\frac{1}{k}\quad\cdots\cdots\text{ Equation 1}$$

Bernoulli Equation

$$P_1 = P - \rho(\frac{\partial \phi}{\partial t} + \frac{1}{2}v^2 + gh)$$

We will ignore gravity, so $gh = 0$

So $\frac{\partial \phi}{\partial t} = \frac{P - P_1}{\rho}$

Young's Equation

$$P - P_1 = \sigma(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 y}{\partial y^2})$$

Then $\frac{\partial \phi}{\partial t} = \frac{\sigma}{\rho}(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 y}{\partial y^2})$

Hence, $(-i\omega)(-i\omega h) = \frac{\sigma}{\rho}(-k^2 h)$

So $-\omega^2 h = \frac{\sigma}{\rho}(-k^2 h)$

$$f(z = h) = \frac{\frac{\sigma}{\rho}k^2}{\omega^2} \quad \cdots\cdots \text{Equation 2}$$

According equation 1 and 2:

$$\frac{1}{k} = \frac{\frac{\sigma}{\rho} k^2}{\omega^2}$$

$$\omega^2 = \frac{\sigma}{\rho} k^3$$

- ω is angular frequency
- σ is surface tension number
- ρ is density of the water
- k is wavenumber and it is equal $\frac{2\pi}{\lambda}$

V is phase velocity, also said linear velocity, is $\frac{\omega}{k}$

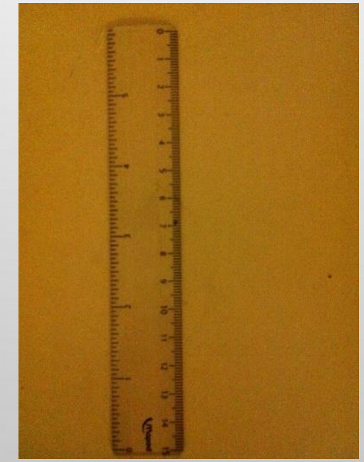
$$v^2 = \frac{\omega^2}{k^2} = \frac{\sigma}{\rho}\left|k\right| = \frac{\sigma}{\rho} \cdot \frac{2\pi}{\lambda}$$

$$V = \sqrt{\frac{\sigma}{\rho} \cdot \frac{2\pi}{\lambda}} \text{ and } \sigma = \frac{v^2 \rho \lambda}{2\pi}$$

Procedure of this Experiment

EQUIPMENT FOR THIS EXPERIMENT

1. Cup with volume
2. Stopwatch to measure time
3. Ruler to measure the length of waves
4. Camera to get pictures.



PROCEDURES OF THE EXPERIMENT:

- 1.We need to measure out the volume of the cup, so we can find out the capacity of the water.
- 2. Use the stop-watch to measurement the time.
- 3. After we record the time about the flux, we need to move the cup until we can see the waves clearly.
- 4. Put the ruler on the side, and the use camera to take pictures.
- 5. Repeat those steps more times and take pictures for different length of waves.

PROCEDURE TO MEASURE WAVE LENGTH AND CROSS SECTIONAL AREA.

1. Use ruler to find out the waves length in pictures.
2. Use ruler to get the proportion about the ruler in pictures.
3. Use proportion to find out the real length about the waves.
4. Repeat those steps and find out the radius about the cross sectional in different high.
5. After all those steps, we will use the measurements to do following test.

RESULT OF EXPERIMENT

1. Time (second)
2. cup volume: $4.93\times10^{6} \quad (mm^{3})$
3. $V=(r^{2})\pi h$ (h is the height of cup)
4. Wave length
5. a: water wave cross section area

GET THE VALUE OF FLOW

Time(s)	Flow(mm^3)
105.4	4677.4
110.4	4465.6
99.5	4954.8
109.3	4510.5
105.5	4672.9
110.4	4465.6
110.9	4445.4
107.3	4594.8
108	4564.8
107.3	4594.6
109.8	4489.9

flow = volume of cup / time

GET THE VALUE OF VELOCITY

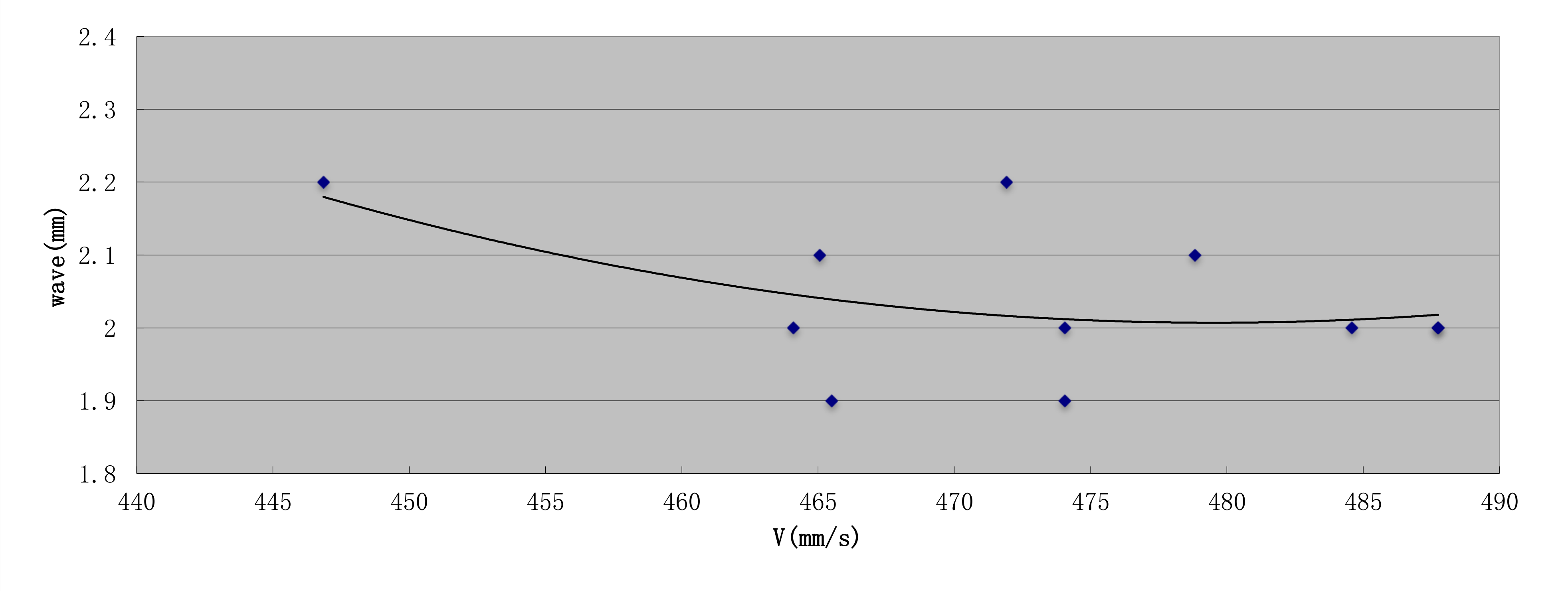
r (mm)	v (mm/2)
1.6	465.5
1.5	474.1
1.7	464.1
1.5	478.8
1.6	465.1
1.5	474.1
1.5	471.9
1.5	487.8
1.5	484.6
1.5	487.7
1.6	446.9

- v = flow / a
- a is the area of wave section.

THE REALTIONSHIP BETWEEN VELOCITY AND WAVE

v (mm/s)	wave (mm)
465.5	1.9
474.1	1.9
464.1	2.0
478.8	2.1
465.1	2.1
474.1	2.0
471.9	2.2
487.7	2.0
484.6	2.0
487.7	2.1
446.9	2.3

THE GRAPH OF WAVE V.S VELOCITY



- As we know the surface tension is a constant number, so there should be a line cross points.

CALCULATE THE VALUE OF A

v (mm/s)	wave (mm)	surface (N/mm)
465.5	1.9	$6.55*10^{-5}$
474.1	1.9	$6.79*10^{-5}$
464.1	2.0	$6.86*10^{-5}$
478.8	2.1	$7.64*10^{-5}$
465.1	2.1	$7.23*10^{-5}$
474.1	2.0	$7.16*10^{-5}$
471.9	2.2	$7.78*10^{-5}$
487.8	2.0	$7.56*10^{-5}$
484.6	2.0	$7.47*10^{-5}$
487.7	2.1	$7.94*10^{-5}$
446.9	2.3	$7.29*10^{-5}$

USE THE FUNCTION OF EXPERIMENT

$$V = \sqrt{\frac{\sigma}{\rho} \cdot \frac{2\pi}{\lambda}} = \frac{a}{\sqrt{\lambda}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

ERROR BAR

$$v = \frac{V}{t\pi r^2}$$

$$\ln v = \ln V + \ln t + \ln \pi + 2\ln r$$

$$\frac{\Delta v}{v} = \frac{\Delta V}{V} + \frac{\Delta t}{t} + 2\frac{\Delta r}{r} \approx 0.04$$

$$\Delta v = v \times 0.04 = 18.8$$

$$v = 470 \pm 18.8$$

$$\sigma = \frac{v^2 \rho \lambda}{2\pi}$$

$$\frac{\Delta \sigma}{\sigma} = 2\frac{\Delta v}{v} + \frac{\Delta \lambda}{\lambda} = 0.08$$

$$\Delta \sigma = 7.29 \times 0.08 \approx 0.6$$

$$\sigma = 7.3 \pm 0.6$$

SCIENTIFIC CHALLENGES

- WHILE THERE ARE THEORETICAL METHODS TO DETERMINE SURFACE TENSION, EXPERIMENTAL DATA IS DIFFICULT TO MEASURE WITH GREAT ACCURACY. SMALLER MEASUREMENTS ESPECIALLY CAN ADD LARGE ERROR TO A MODEL.

POTENTIAL APPLICATIONS

- Some insects (e.g. water skipper) can **crawl** on the water surface.
- If the surface tension is high, the water does not easily make the objects wet, and it will rebound from the surface of the objects. One of the **detergent** effects is to reduce the surface tension of the water.
- Surface tension is currently being used in zoology to communicate with dolphins. A high surface tension allows sound waves to create an imprint on the water.
- Anti-Fog agents are also developed by studying surface tension. These chemicals are designed to decrease surface tension so that individual water droplets cannot form on a solid surface.

QUESTIONS?