Search for a car parked on a forest road

Math 485 Final Presentation

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Reference and link

Yu Baryshnikov and V Zharnitsky,
Search on the brink of chaos, Nonlinearity, 25 (2012), 3023–3047, doi:10.1088/0951-7715/25/11/3023

Abstract

Overall Goal:

Analysis of a linear search problem for a hidden object from a given probability density function.

Search for a Parked Car on a Forest Road

Project Goal

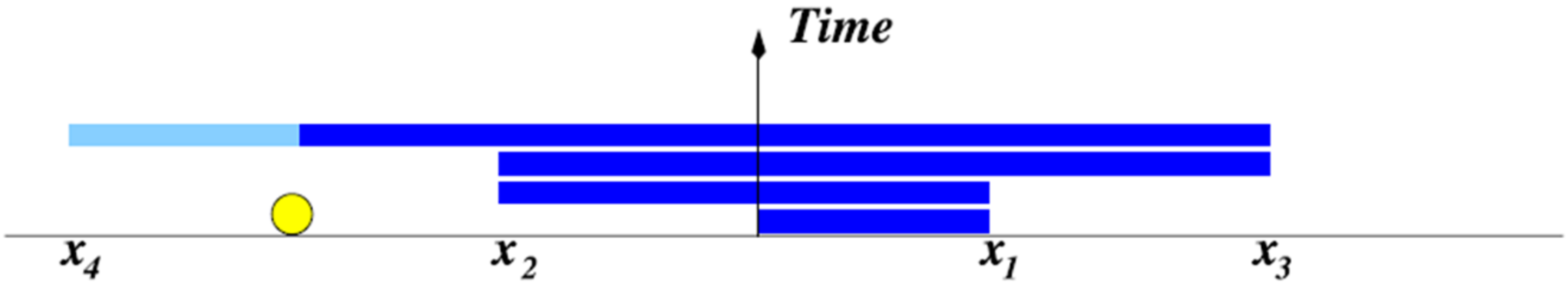
- Develop a numerical method to simulate a linear search problem.
- Predict the position of a hidden object by finding a model.
- Minimize searching time to reduce the cost
- Find the best distant to the object by determining the optimal path.
- Provide insight of the situation into real world.

Introduction

The linear search problem:

- An object is placed on a line using a given probability density function

- Starting with an initial step length, the searcher goes back and forth using step lengths given by a specific formula

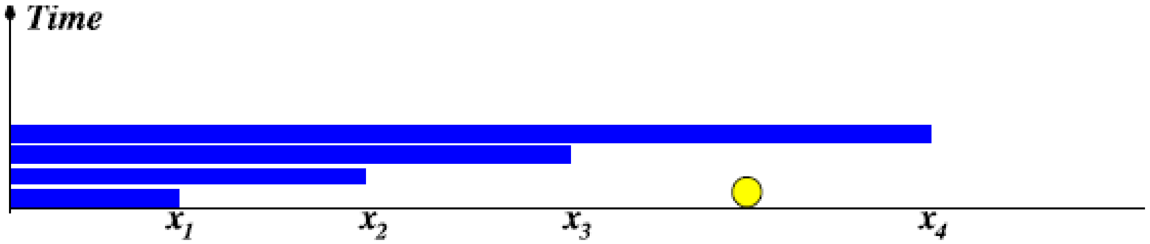


Mathematical Model

Variables

EL = expectation length of search, given by:

$$EL = 2\left[\sum_{n=1}^{\infty} x_n - \sum_{m=2}^{\infty} x_m f(x_{m-1})\right]$$



x = step length, given by the geometric series: $x = \Delta\alpha^n$

Δ = geometric constant

α = geometric ratio

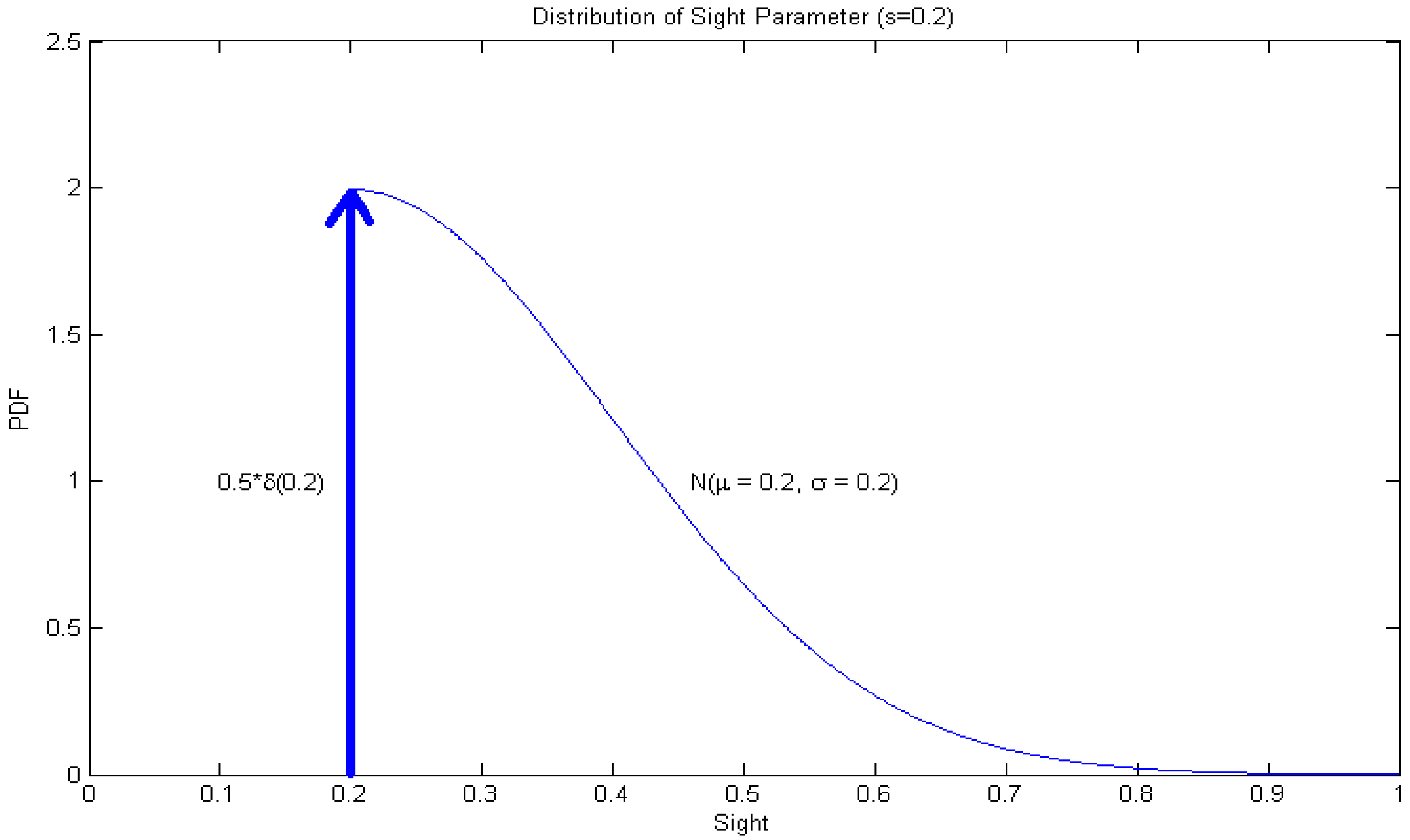
F(x) = probability density function

f(x) = cumulative distribution function for F(x)

Mathematical Model

Variables

S = sight parameter, determined by the following distribution:



Explanation of Code – Sight Parameter

```
aa= 0.2;
bb = normrnd(0,aa);
if aa > bb
        rr = aa;
        end
if bb >= aa
      rr = bb;
       end
```

- Sight parameter at each step is chosen based on a normal distribution
- In the case at the left, the mean value used in the distribution is 0.2, which is the minimum distance for sight
- The sight parameter used in calculations is the maximum distance: either 0.2 or the value chosen for that instance according to the normal distribution

Theory

Introducing the sight density function

$$S(x_n,\mu) = \begin{cases} \frac{1}{2}\delta(x_n-\mu), & x=\mu \\ \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x_n-\mu)^2}{2\sigma^2}}, & x>\mu; x<\mu \end{cases}$$

Overlapping
Minimum visible sight
Non-actual function

Introducing the "Stopping-time"

$$x_K + S(x_K,\mu) \geq l$$

Probability of not spotting the car /spotting the car

$$P(x_1: x_1 + S(x_1,\mu) < l) \qquad P(x_k: x_k + S(x_k,\mu) \geq l)$$

For instance, we want the individual probability that we didn't spot the car until the Kth step:

$$P_k = P(x_1: x_1 + S(x_1,\mu) < l) \cdot P(x_2: x_2 + S(x_2,\mu) < l) \cdot \ldots\ldots\ldots \cdot P(x_{k-1}: x_{k-1} + S(x_{k-1},\mu) < l) \cdot P(x_k: x_k + S(x_k,\mu) \geq l)$$

Theory

With respect to the total distance one should have travelled: $L = 2\sum_{n=0}^{K} x_n$

That does not, however tell the whole story:

$$\{P(x_1: x_1 + S(x_1,\mu) < l) \cdot P(x_2: x_2 + S(x_2,\mu) < l) \cdot \dots\dots\dots \cdot P(x_{k-1}: x_{k-1} + S(x_{k-1},\mu) < l) \cdot \int_{x_{n-1}}^{x_n} F(x) \cdot dx \cdot P(x_k: x_k + S(x_k,\mu) \geq l)\}$$

Part II: the actual sum of all the individual expectations values

$$\sum_{0}^{\infty} 2\sum_{n=0}^{K} x_n \{P(x_1: x_1 + S(x_1,\mu) < l) \cdot P(x_2: x_2 + S(x_2,\mu) < l) \cdot \dots\dots\dots \cdot P(x_{k-1}: x_{k-1} + S(x_{k-1},\mu) < l) \cdot \int_{x_{n-1}}^{x_n} F(x) \cdot dx \cdot P(x_k: x_k + S(x_k,\mu) \geq l)\}$$

S, the sight function also has a probability for the successfulness of the calculation

$$x_j + S(x_j,\mu) \geq l$$

Theory

Hot fix for this:

Introducing the actual form of Sight function, evaluated it
Evaluating the Expectation value for each “P” terms with respect to S

Logical Pattern:

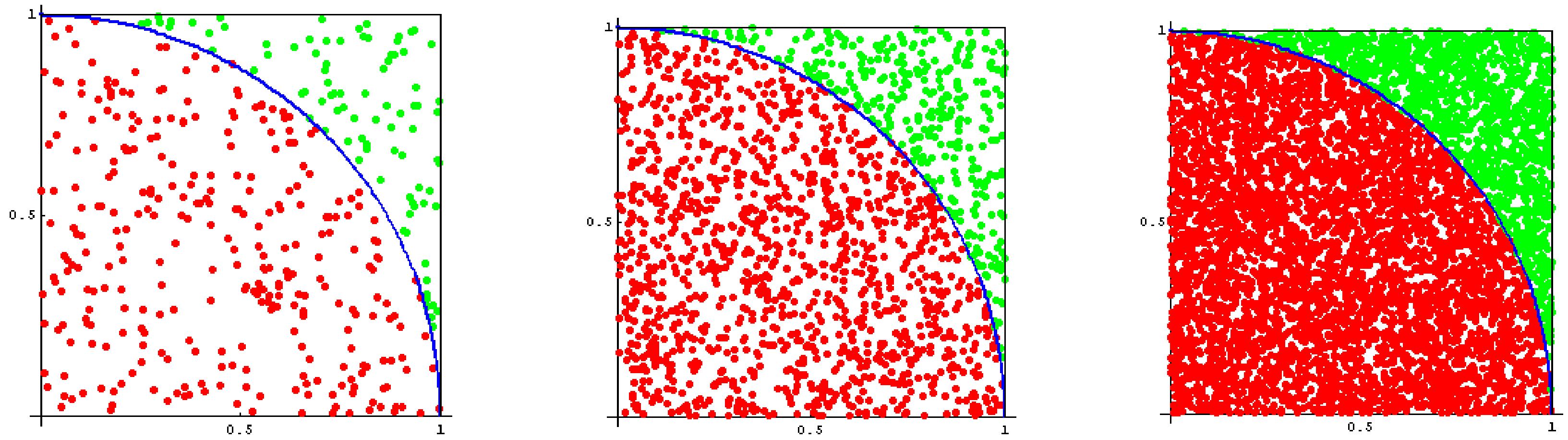
If there, execute it.
If not, continue processing

Monte Carlo Method

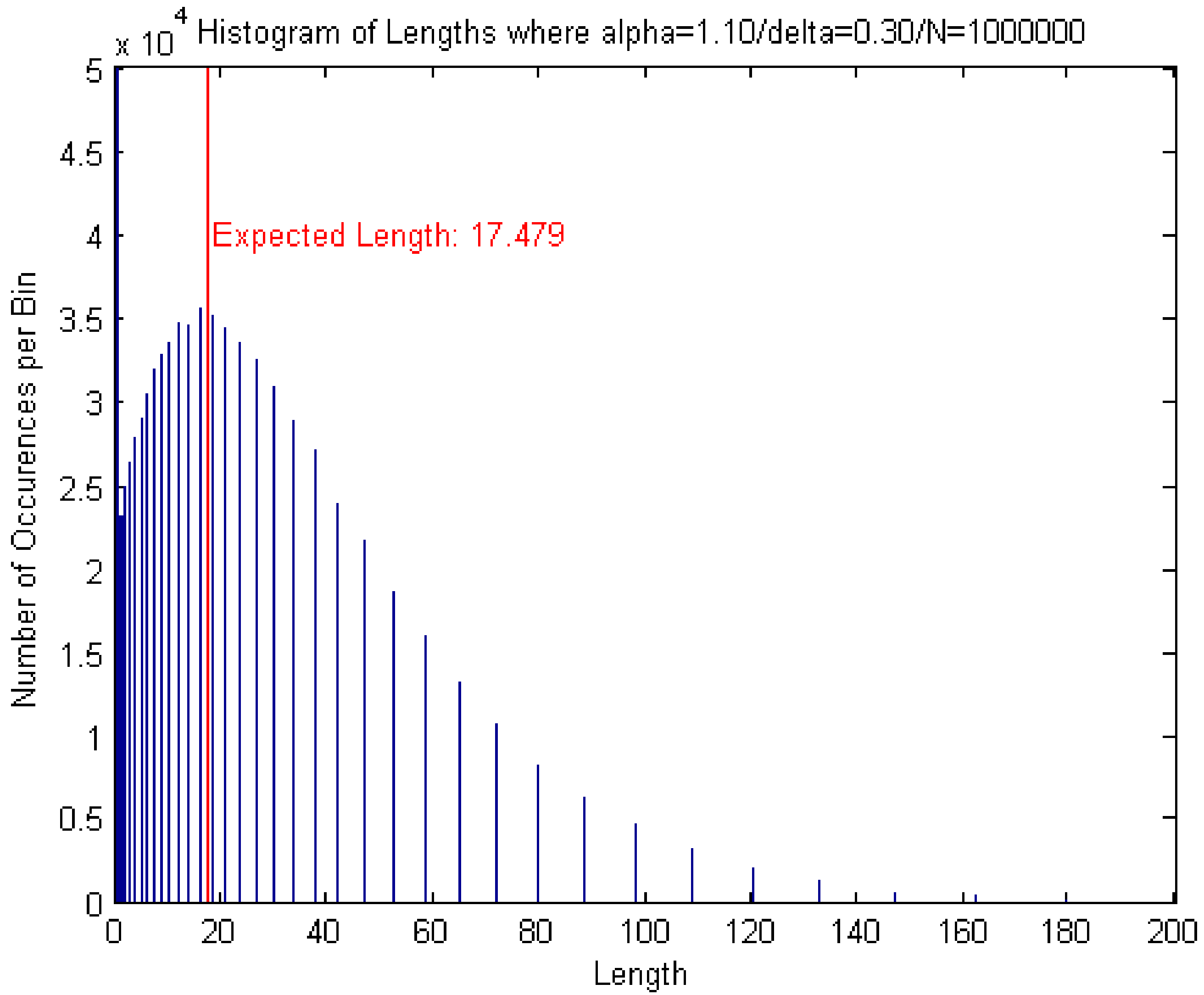
- Used when there is a random event occurring in the simulation
- Large number of random samples are taken in order to describe the behavior of the system.

Example:

- Points are randomly plotted on the diagram shown below.
- As the number of points plotted increases, the ratio of points inside the circle to total points plotted will converge to a value of $\pi/4$.



Preliminary Simulations



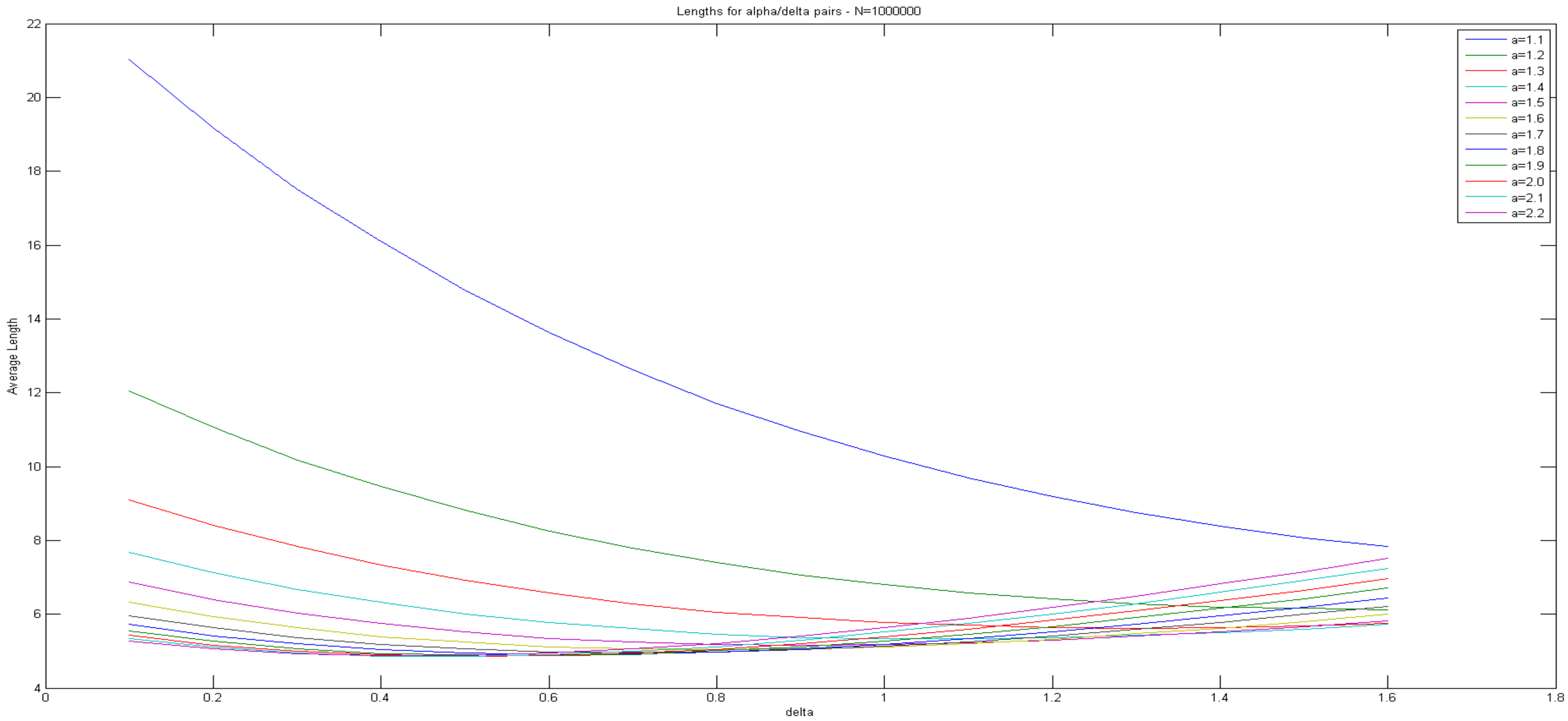
No Sight Parameter

Steps- Geometric Series:
$x_n = \Delta\alpha^n$

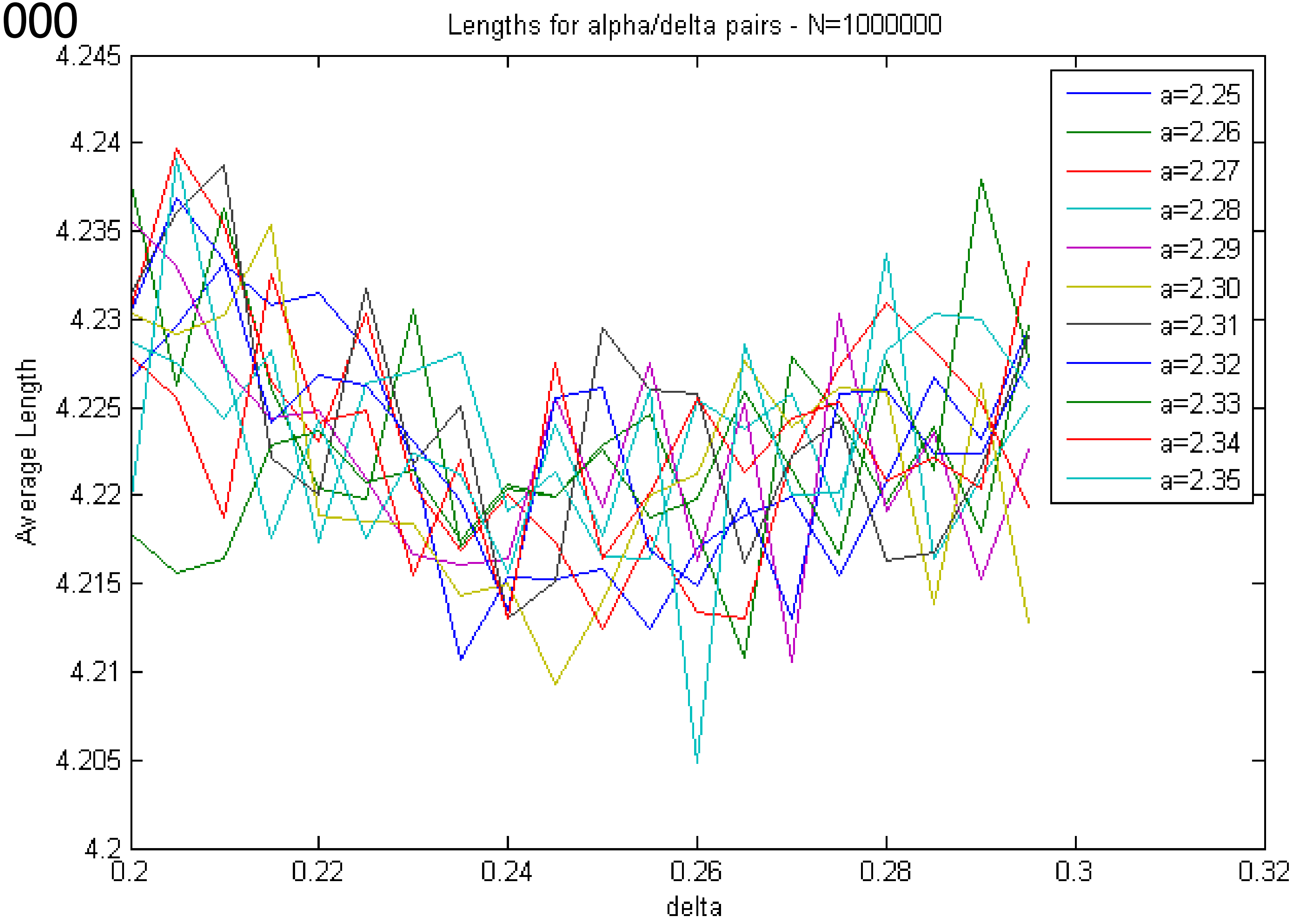
Probability Density:
$F(x) = e^{-x}$

Preliminary Simulations

No Sight Parameter: Optimal -> $\alpha = 2.1$; $\Delta = 0.5$; $L = 4.7932$: $N = 1{,}000{,}000$

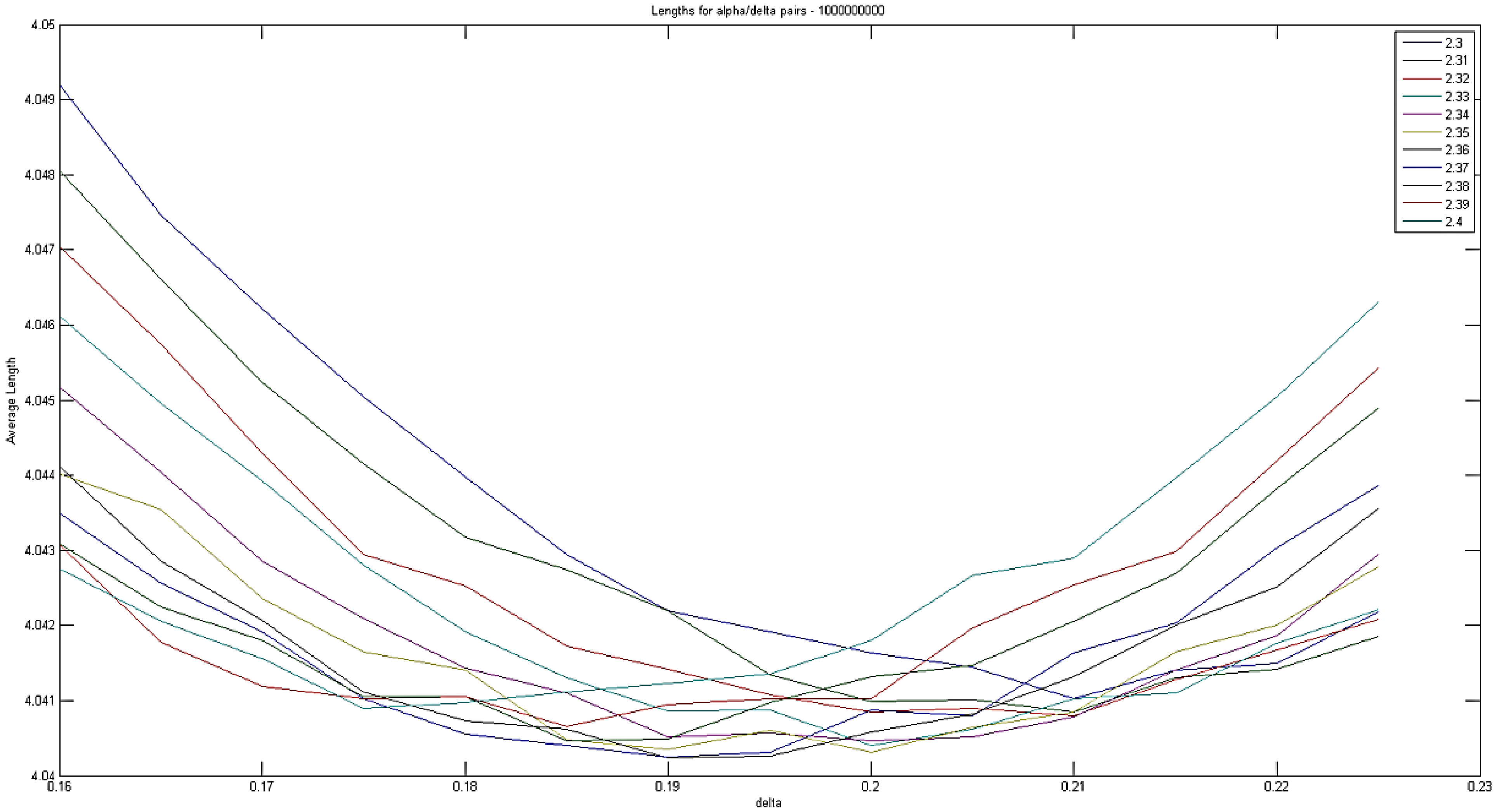


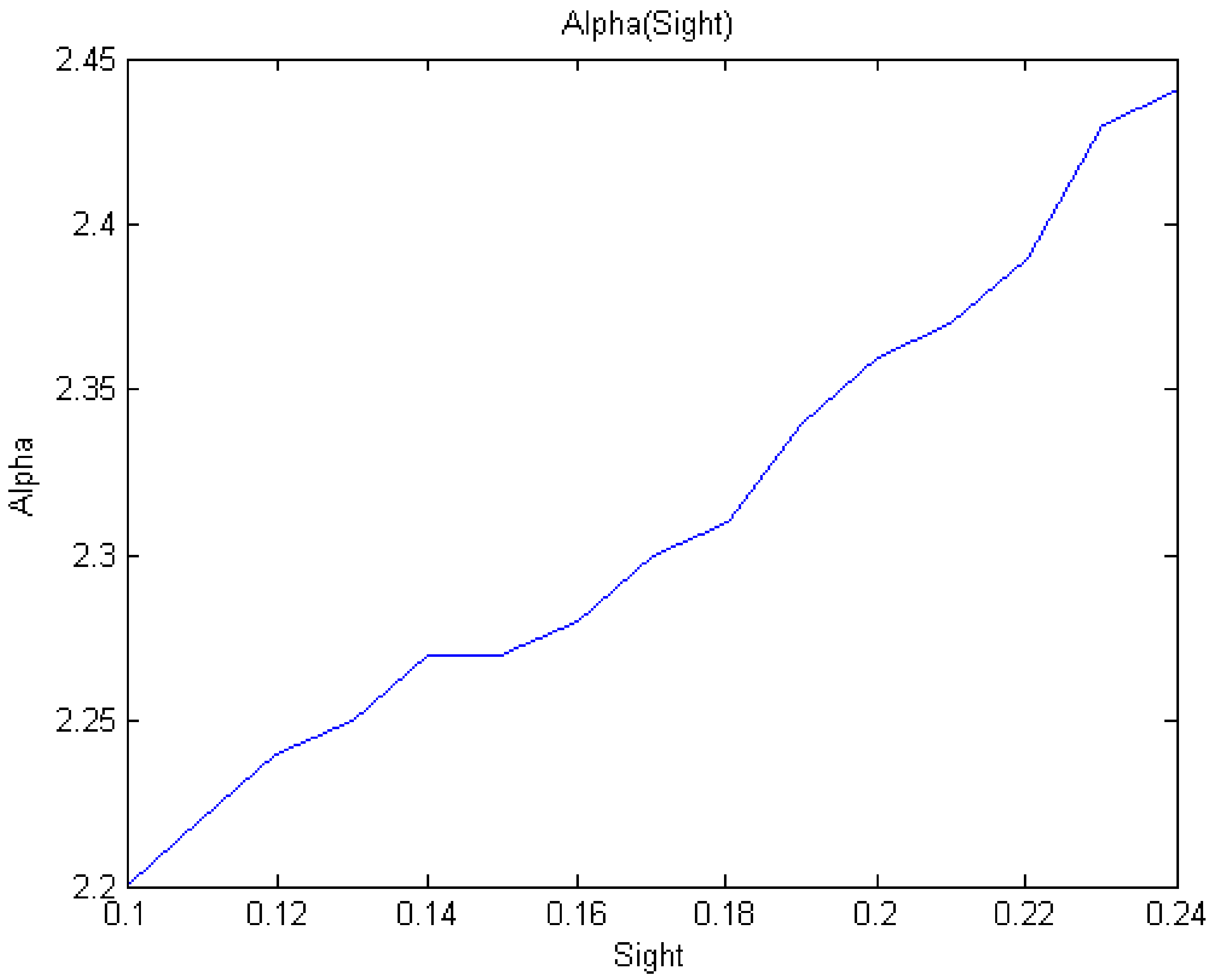
With Sight Parameter:
N = 1,000,000



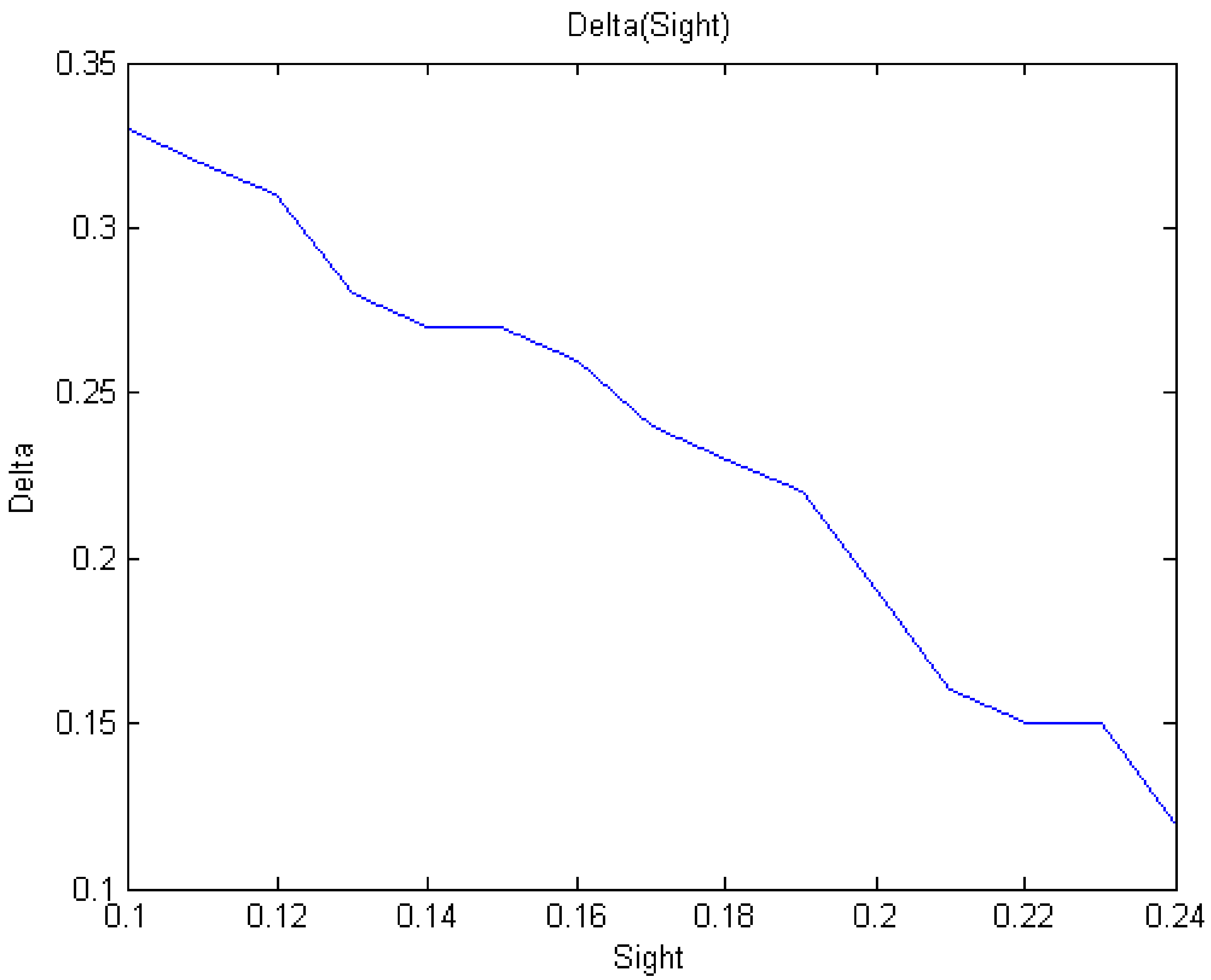
Simulations with Sight

Optimal -> α = 2.36; Δ = 0.19; L = 4.04024 : N = 1,000,000,000

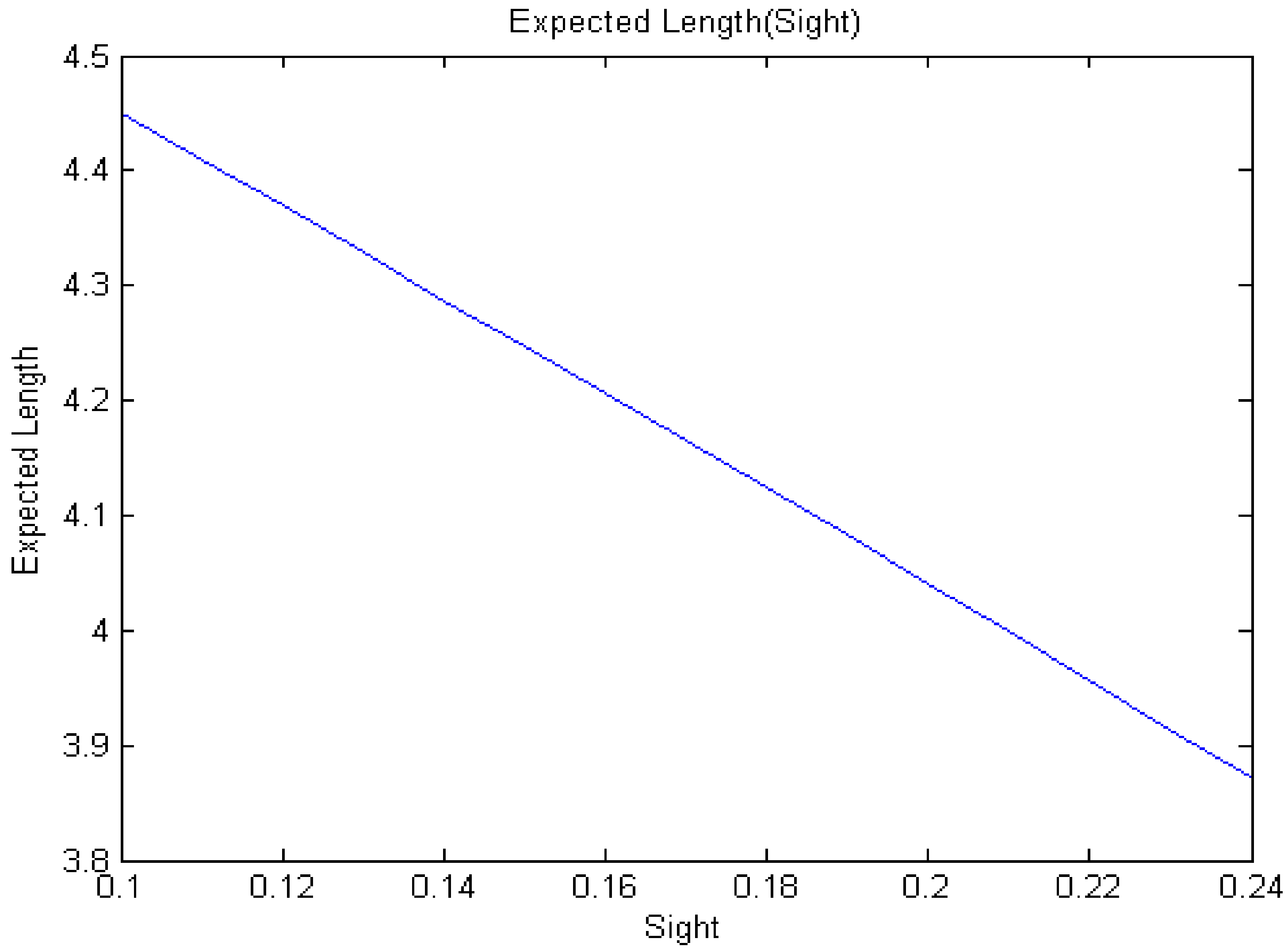




Alpha(Sight)
2.45
2.4
2.35
2.3
2.25
2.2
Alpha
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
Sight



Delta(Sight)
Delta
Sight
0.35
0.3
0.25
0.2
0.15
0.1
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24



Expected Length(Sight)
4.5
4.4
4.3
4.2
4.1
4
3.9
3.8
Expected Length
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
Sight

Conclusion

- Optimal Values:

	Preliminary	Sight in
Sight	-	0.2
Alpha	2.1	2.36
Delta	0.5	0.19
L	4.7932	4.04024

- General Trend as sight increases
  - Alpha increases
  - Delta decreases
  - Length decreases
- Resolution and dependence

Conclusion

- Monte Carlo simulations are used since an analytical solution to the model cannot be found
- Added sight parameter makes problem more realistic
- Additional sight parameter makes results more varied such that a higher number of simulations are needed to obtain precise results

Questions?

Image Sources:

Art.com

http://mathfaculty.fullerton.edu/mathews/n2003/montecarlopi/MonteCarloPiMod/Links/MonteCarloPiMod_lnk_2.html