Spontaneous Synchronization in Power Grids

Oren Lee, Thomas Taylor, Tianye Chi, Austin Gubler, Maha Alsairafi

Motivation

- Complex Power Grids
- Optimization
- Reliability

What is a Network?

- Generators
  - Nodes
- Transmission lines
  - Matrix
- What is synchronization

$$\dot{d}_1 = \dot{d}_2 = \ldots = \dot{d}_n$$

Building the Grid

- Newton's Second Law

$$Ia = t$$

- Conservation of angular momentum

$$I \frac{d^2 d_i}{dt^2} = t_{mi} - t_{ei}$$

- Conservation of Energy

$$\frac{2H_i}{w_R} \frac{d^2 d_i}{dt^2} = P_{mi} - P_{ei} \qquad H_i = \frac{1}{2} I w^2$$

Network Structure

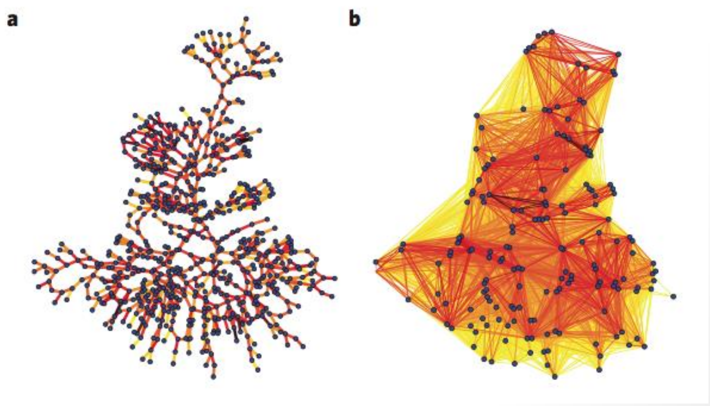
Assumptions?

Physical network vs Effective network

Admittance

$$Y = \frac{\bar{P}}{|V|^2}$$

$\mathbf{Y}_0=(Y_{0ij})$: Y_{0ij} is the negative of the admittance between i and $j \neq i$



Stability condition from the swing equation

Assume $\delta_i = \delta_i^* + \delta_i^{'}; \; P_{ei} = P_{ei}^* + P_{ei}^{'}; \quad P_{mi} = P_{mi}^* + P_{mi}^{'}$

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i^{'}}{dt^2} = \frac{\partial P_{mi}}{\partial \omega_i}\omega_i^{'} - \frac{\partial P_{ei}}{\partial \omega_i}\omega_i^{'} - \sum_{j=1}^{n}\frac{\partial P_{ei}}{\partial \delta_j}\delta_j^{'}$$

Term 1: $\partial P_{mi} / \partial \omega_i = -1/(\omega_R R_i)$

Term 2: $\partial P_{mi} / \partial \omega_i = D_i / \omega_R$

Term 3: $P_{ei}^{'} = D_i \omega_i^{'} / \omega_R + \sum_{j=1}^{n} E_i E_j (B_{ij}\cos\delta_{ij}^* - G_{ij}\sin\delta_{ij}^*)\delta_{ij}^{'}$

Coupled 2nd Order Equations

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \vec{d} \\ \overrightarrow{\dot{d}} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \vec{w} \\ \overrightarrow{\dot{w}} \end{bmatrix}$$

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -PX_1 - BX_2$$

Coupled 2nd Order Equations

$$B = \frac{1}{2H_i} + \frac{D_i}{2H_i}$$

B is the diagonal matrix of elementsβ

$$P_{ij} = \begin{cases} \frac{\omega_R E_i E_j}{2H_i}(G_{ij} \sin \delta^*_{ij} - B_{ij} \sin \delta^*_{ij}), & i \neq j \\ -\sum_{k \neq i} P_{ik}, & i = j \end{cases}$$

Simplifying coupled equations

$$\dot{z}_j = \begin{pmatrix} 0 & 1 \\ -a_j & -b \end{pmatrix} z_j, \qquad z_j \equiv \begin{pmatrix} Z_{1j} \\ Z_{2j} \end{pmatrix}$$

From the different equation, this has the solution

$$(\vec{X}_1)e^{\lambda_1 t} + (\vec{X}_2)e^{\lambda_2 t}$$

How we guarantee stabilities?

By ensuring the REAL part is negative in both terms

Finding Stability from Matrix

- From ODE's, we can find requirements for stability.
- Lyopanov Exponents

$$l_{j\pm} = -\frac{b}{2} \pm \frac{1}{2}\sqrt{b^2 - 4a_j}$$

Keeping λ Negative

- Need to keep all Re(λ) < 0
- Alter function to $\Lambda = \max(\mathrm{Re}(\lambda)) \leq 0,\ j \geq 2$
- J=1 is a null eigenvalue
  - Related to shift in all phases

- How can we tune variable α to keep λ negative?

Function Behavior

- For $\alpha<0$
  - $\Lambda>0$, not usable
- For $\alpha>0$
  - $\Lambda<0$, usable and decreases for increasing α
- For $0\leq\alpha\leq\beta^2/4$
  - Λ reaches minimum at $\alpha=\beta^2/4$
  - Imaginary part is added past this value

What value for α?

- α is based on eigenvalues of the matrix P
- It is dependent on properties of the generator in the system
- We want the smallest non- zero value of α
  - This will increase stability
  - Choose α_2
  - α_1 is null, we ignore this value

Parameters for changes in demand

$$b = b_{opt} \circ 2\sqrt{a_2}$$

- Droop Parameter (Off-line optimization)

$$R_i = \frac{1}{4H_i\sqrt{a_2} - D_i} \qquad i = 1,\ldots,n$$

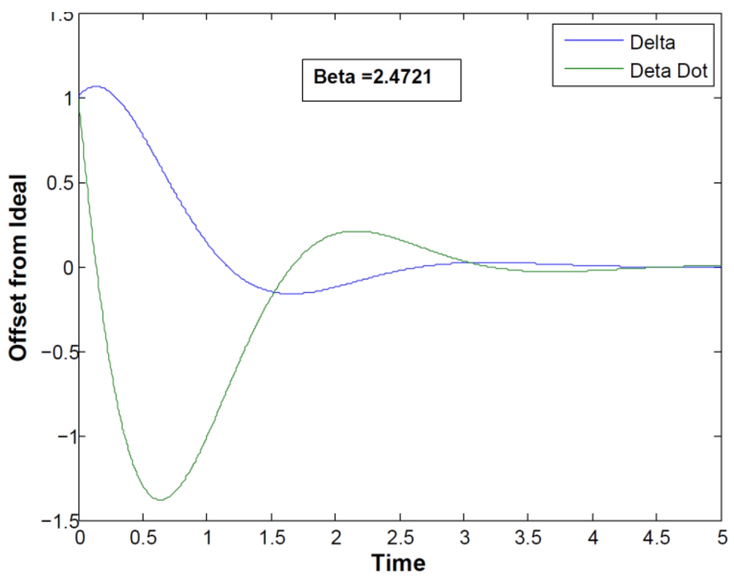
- Damping Coefficient (Online optimization):

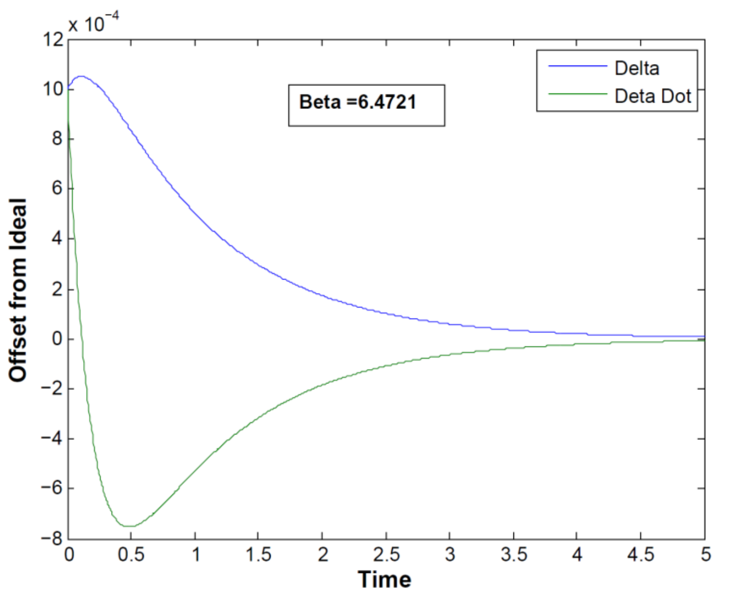
$$D_i = 4H_i\sqrt{a_2} - \frac{1}{R_i}, \qquad i = 1,\ldots,n$$

Testing Methods

- MATLAB script written from a simplified coupled equations.
- Assumed β equal for all nodes
- Arbitrary values for α
- 5 node system
- Solved using ODE45 function
- Integrated δ solution for varying β
- Repeated multiple times

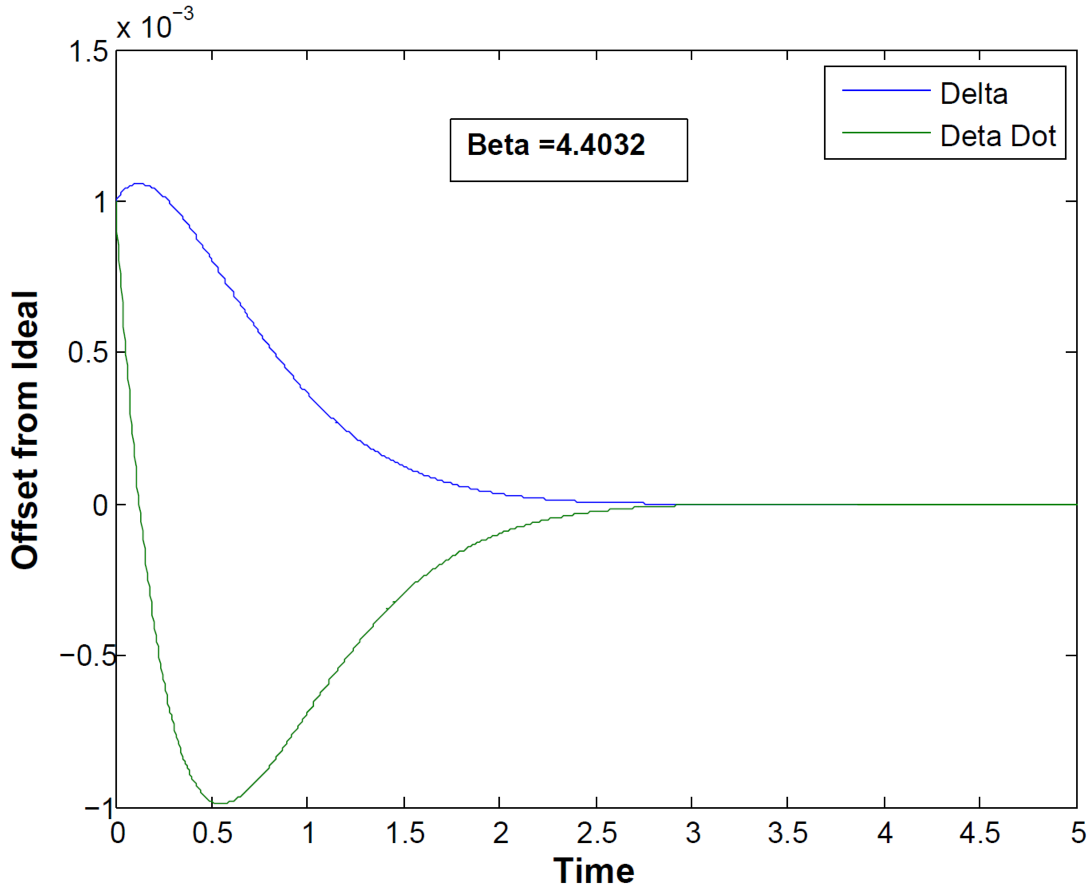
Test Results



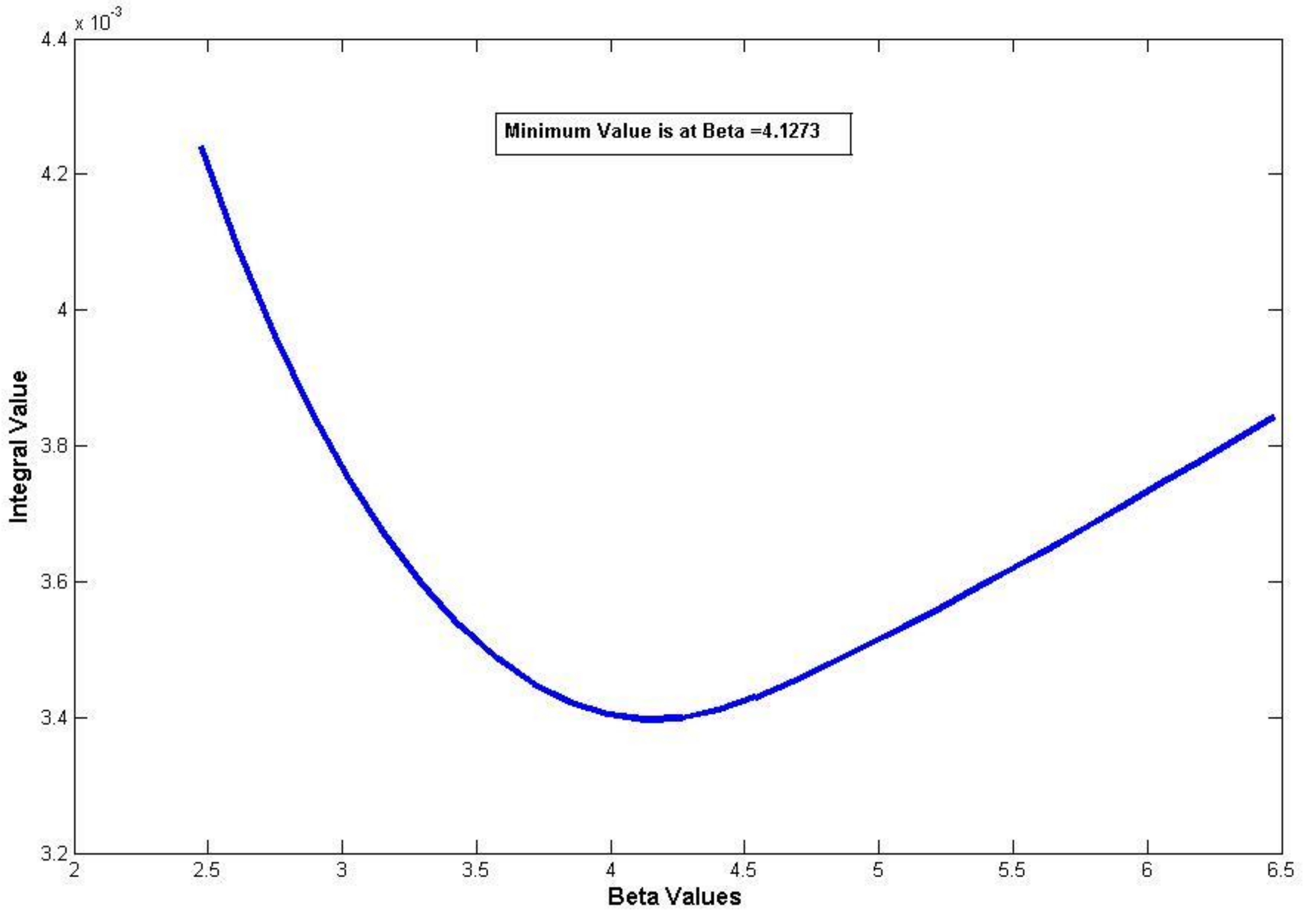


Test Results

- Optimal Beta



Test Results



Conclusions

- Optimal Beta differs slightly from the expected value from the literature
- Can be explained by the randomness added to alpha values
- Within 10% range

Questions?