Energy Flow in Electrical Grids

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Abstract:

A theoretical and real model of total power flowing up and down a power grid is created using different modeling tools. Original DistFlow ODEs are found by combining the equations for real and reactive power with fundamental Kirchoff's laws. By converting DistFlow ODE's with boundary conditions into an initial value problem with initial conditions shows the solution of voltage with respect to length, and it is able to test not just one length but all length at the single run of the rescaled equation. The graphs drawn by the DistFlow ODEs and rescaled equations, show the desirable and undesirable scenario with multiple stable solutions. The undesirable scenario will cause of power failure and other unexpected troubles.

Content:

1 Introduction and Background

1.1 Introduction

Total power, real and reactive, in an electrical grid is measured as the rate of flow of energy in an electrical grid. Where real power is a combination of current and voltage, both are sinusoidal. If the power load is entirely resistive, current and voltage will reverse polarity simultaneously. When the product of current and voltage is positive in the grid, only real power is transferred. Reactive power voltage and current will be 90° out of phase during each wave cycle. When this happens, the product of current and voltage will be negative. During this phase of the wave cycle, approximately as much power flows up the line as back down.

A power grid or network is traditionally used to deliver electricity from a main producer, a power plant of some kind, to consumers. This model focuses on households and is an interconnected grid consisting of generating stations, transmission lines, and distribution lines. In an ideal case where none of the consumers are also producers the generating stations are large and typically located next to a source of fuel and some distance away from densely populated areas. The transmission lines are used to transport real and reactive power to the subgenerators or substations. The distribution lines are then used to transport power to the consumers. In a non-ideal case an amount, but typically not all, of the consumers will also produce power. This is typically done with the use of solar panels or wind turbines which causes power to flow back up the line at sporadic intervals which can cause power outages or shortages.

A better understanding of the effect self-producers have on the electrical grid is needed as more consumers are also becoming producers. A recreation of the model, results, and a model of the fluctuations of real and reactive power flowing randomly both up and down along an electrical grid for real world application will be created and discussed.

1.2 Background

Real power is positive and represents energy which is consumed. Reactive power is negative energy which is returned to the power source and stored in capacitors and inductors. Total power is the combination of real and reactive power which is represented as a complex number. The equations below, when combined with Kirchoff's fundamental laws, lead to developing the initial model.

$$S = P + jQ \tag{1}$$
$$z = r + jx \tag{2}$$

Where:

S is the apparent power
P is the real power
Q is the reactive power
z is the impedance
r is the resistance
x is the inductance

2 Developing the Model

2.1 DistFlow Equations (Discrete Form)

The system of DistFlow equations for real power, reactive power, and voltage are:

$$P_{k+1} - P_k = p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2} \tag{3}$$

$$Q_{k+1} - Q_k = q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2} \tag{4}$$

$$v_{k+1}^2 - v_k^2 = -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2)\frac{P_k^2 + Q_k^2}{v_k^2}$$

(5)

k = 0,...,N-1 enumerates buses of the feeder (k=consumers)
P_k is real power flowing from bus k to bus k + 1
Q_k is reactive power flowing from bus k to bus k + 1
P_k and q_k are net consumption for consumers
R_k, x_k represent the resistance and inductance of the line element connecting bus k to bus k+1 with boundary condition

PN+1 = QN+1 = 0, $v_0 = 1$

Means the initial voltage is known; the voltage with which the feeder is supplied. Real power and reactive power at the end of the feeder line is zero so all the power supplied in the beginning is being consumed; there is no leftover power in the line.

2.2 Homogenization

The discrete DistFlow equations will be converted into boundary value problem ODEs which would be more convenient to solve for large amounts of consumers (e.g., for a million consumers, three million equations would have to be solved if DistFlow equations were used. With DistFlow ODEs, three equations will be solved.).

When the feeder line is long and the number of consumers assumed large (N>>1), the system of equations can be simplified and represented in the continuous form with limit $N \to \infty$. To solve for the standard homogenization, more assumptions and reduction of parameters are considered, and more simplifications have been done to relate the differences of definition of derivatives ($F_{k+1} - F_k \approx F'(z) l_k / L$). Applying the above equations to relative differences in the DistFlow equations, a set of Ordinary Differential Equations (ODEs) are arrived at in the continuous homogenized form

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2-Q^2}{v^2} \\ q - x\frac{P^2-Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix} \tag{6}$$

with mixed boundary conditions

$V_0 = 1$, $P(L) = Q(L) = 0$

This continuous homogenized ODE is a boundary value problem with mixed boundary conditions. By solving this ODE for known length of feeder line, real and reactive power and voltage along the given line can be evaluated.

2.3 Rescaled & Simplified Form

The original DistFlow ODEs can be rescaled and simplified into an initial value problem. The difference between boundary value problems and initial value problems is that boundary value problems have conditions given for integration on both sides of the feeder line, while initial value problems have only one condition given for one side of the feeder line. Therefore, solving initial value problems is easier than solving boundary value problems.

Assuming that p = constant, then define new dimensionless variables ρ, τ, υ and s to represent P, Q, voltage, v, and position along the feeder, z, in term of s.

$$s = \frac{\sqrt{|p|r}}{v(L)}(L-z) \qquad \rho(s) = \sqrt{\frac{r}{|p|}}\frac{P(z)}{v(L)}$$

$$\tau(s) = \sqrt{\frac{r}{|p|}}\frac{Q(z)}{v(L)} \qquad v(s) = \frac{v(z)}{v(L)} \tag{7}$$

Dimensionless Form Proved as following:

$$\textit{Example of } \rho(s) = \sqrt{\frac{r}{|p|}}\frac{P(z)}{v(L)} \quad \textit{units of } [r] = \Omega \qquad [p] = \frac{v^2}{\Omega} \qquad [P] = \frac{v^2}{\Omega} \qquad [v] = V$$

$$\textit{Then} \quad [\rho] = \sqrt{\frac{[r]}{[p]}}\frac{[P]}{[v]} \quad \rightarrow \quad \sqrt{\frac{\Omega}{|\frac{v^2}{\Omega}|}} * \frac{\frac{v^2}{\Omega}}{V} \quad \rightarrow \quad \frac{\Omega}{V} * \frac{V}{\Omega} = 1$$

There is no unit in ρ equation, thus this is a dimensionless variable. The same method proved τ and υ are dimensionless variables.

When rescaling and simplifying the boundary value problem into an initial value problem, the rescaling method changes the position of the feeder line and the end of the feeder line. Whether voltage at the end of feeder line is the same as voltage at the beginning of the feeder line can then be calculated.

The following equation illustrates the rescaled DistFlow ODEs

$$-\frac{d}{ds}\begin{pmatrix}\rho\\ \tau\\ \upsilon\end{pmatrix} = \begin{pmatrix} sign(p) - \frac{\rho^2 - \tau^2}{\upsilon^2} \\ A - B\frac{\rho^2 - \tau^2}{\upsilon^2} \\ -\frac{\rho + B\tau}{\upsilon} \end{pmatrix} \tag{8}$$

With initial conditions

$$v(0) = 1, \rho(0) = \tau(0) = 0$$

To determine whether the original DistFlow ODEs are related to the rescaled DistFlow ODEs, simply use the dimensionless variables in equation (7) in equation (8). .

From the original DistFlow ODEs, equation (8) is proposed to be an initial value problem. Integrate from equation (8) to obtain s_* where s_* represent some stopping points. The result is ρ(s_*), τ(s_*), υ(s_*). Then re-compute L, P(0), Q(0) and v(L) using the following equations:

$$L = \frac{s_*}{v(s_*)\sqrt{|p|r}} \qquad v(L) = \frac{1}{v(s_*)}$$

$$P(0) = \frac{\rho(s_*)\sqrt{\frac{|p|}{r}}}{v(s_*)} \qquad Q(0) = \frac{\tau(s_*)\sqrt{\frac{|p|}{r}}}{v(s_*)} \tag{9}$$

Overall, the computation appears to be efficient and reliable. Consider rescaling a non-dimensional system each time when solving boundary value equations by selecting certain L (length of the feeder line) because the boundaries need to be specified in order to solve the equations. The solution of these equation domains are given without length. On the other hand, when the problem is rescaled to create an initial value problem, as the initial value problem is integrated to test every single length along the feeder line which also means solving every possible boundary value problem up to that length. Thus, not only is the initial value problem easier to solve than a boundary value problem, but also testing not only one length, but all lengths at the single run of the equation is possible.

3 Theoretical Results

Two cases are included in this section. The re-scaled equation and Ordinary Differential Equations (ODEs) are used to produce the graphs by Matlab. The solutions are used to calculate the end voltage and power utilization. All the cases are based on r=x=1, and q=p/2.

Case 1 is a standard situation. Real power and reactive power are both negative along the feeder line which means there is consistent consumption for each consumer. Figure 1 shows the relation between the end voltage and the length of the line. The parameters p and q are -1 and -0.5 which implies constant power consumption. Figure 1 shows an interesting phenomenon –a maximum length around L=0.6. The length of the line is limited because there is a limitation of power consumption along the feeder. The amount of power drawn from the system cannot exceed the threshold dependent on the system characteristics.

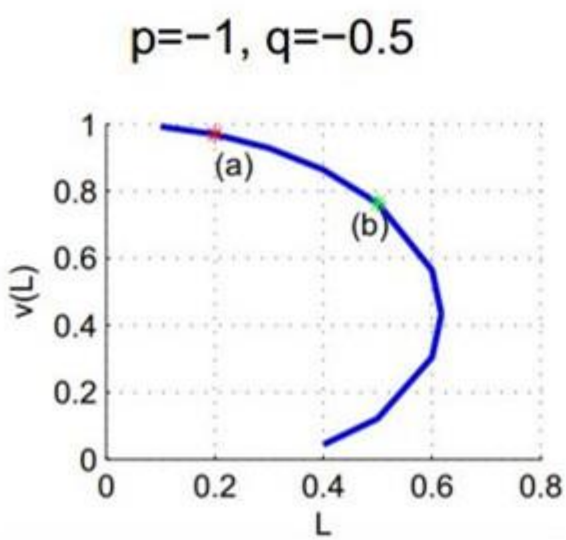


Figure 1: Voltage at the end of the line versus the length of the line, for p=-1 and q =-0.5

Figure 2 shows the trade-off between the power utilization of the feeder. The line decreases slowly at the beginning and the voltage drops at the end of the line. Since the parameters and conditions of the system are the same as Figure 1, there is the same limitation of the length with the constant consumption. Thus, the length to the maximum value is also around 0.6.

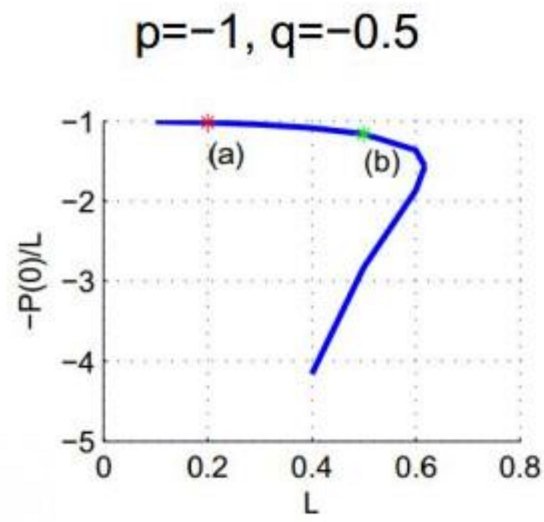


Figure 2: Power utilization versus length of the line, for p=-1, q=0.5

Figures 3 and 4 show the relation between voltage and position along the line. Figure 3 shows the shape of the voltage curve which is decreasing during the entire interval because when the power decreases along the line, the voltage will also decrease. The feeder is longer so the rate of consumption is higher. Figure 4 shows the real and reactive power along the line for lengths L=0.2 and L=0.5. Figure 4 is also decreasing across the entire interval, and at the end of the line, real and reactive power equal 0. Since the boundary value is P(L)=Q(L)=0, all power will be consumed and become 0 in the end.

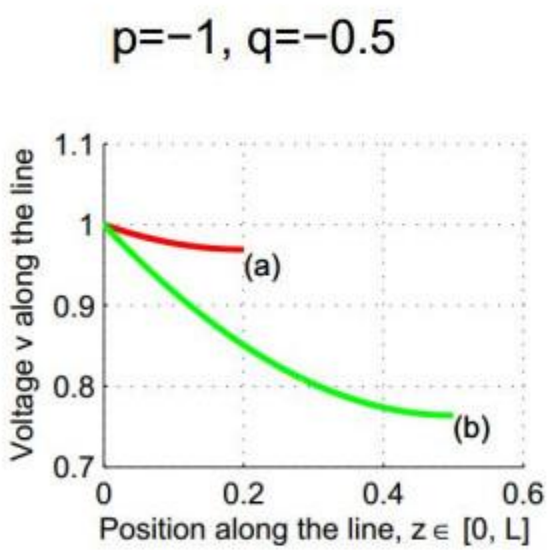


Figure 3: Voltage along the feeder L=0.2 and L=0.5, for p=-1 and q=-0.5

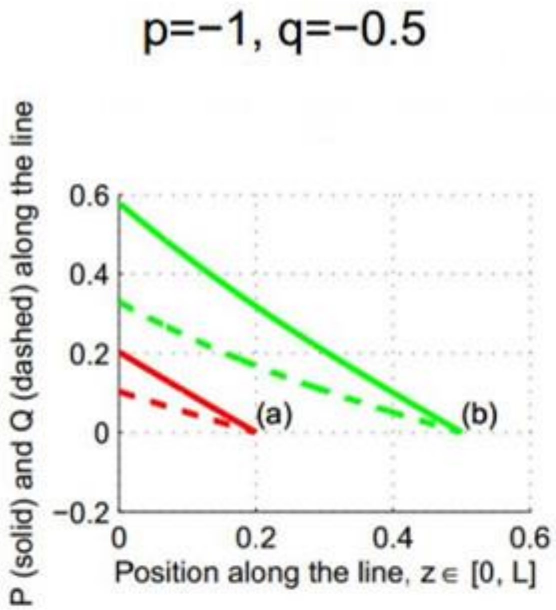


Figure 4: Power along the line, L=0.2 and L=0.5, for p=-1 and q=-0.5

Figures 5 and 6 display a similar graph to Figures 1 and 2. The graphs have the same value p; however, q has been changed from -0.5 to 0. This means there is no more reactive power consumption along the feeder. Thus in Figure 5, the nose-curve shifts to the right a little. In Figure 6, the rate of consumption is smaller. The maximum length is longer than the previous one, around L=0.7.

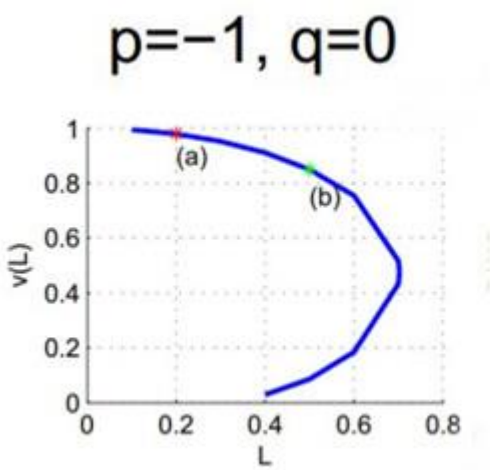


Figure 5: Voltage at the end of the line versus the length of the line, for p=-1 and q=0

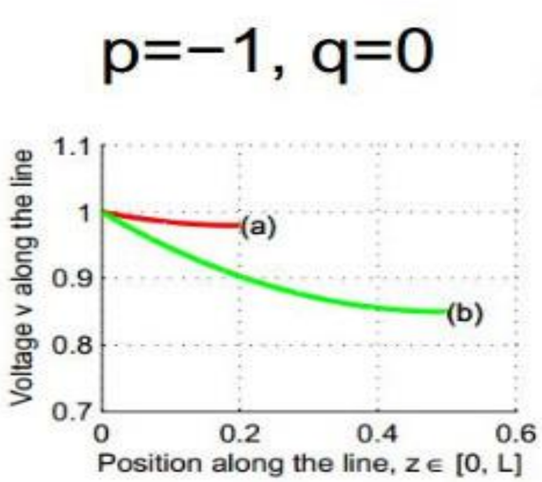


Figure 6: Voltage along the line, L=0.2 and L=0.5, for p=-1 and q=0

Case 2 considers all consumers are producing power, so real power and reactive power were set to be positive and constant along the feeder. Figure 7 shows the relation between the end voltage and the length of the line. The parameters p and q are 1 and 0.5. The graph shows an interesting phenomenon that the line is increasing during the maximum length, but decreasing at the end. After the last producer, there is no more power being produced which means the last producer will also consume the power so the line decreases in the end. Furthermore, the graph shows all buses will produce and consume power at the same time.

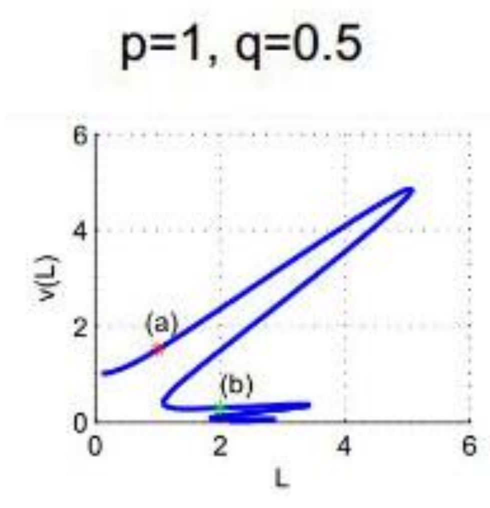


Figure 7: Voltage at the end of the line versus the length of the line, for p=1 and q =0.5

Figure 8 shows an interesting curve. For the red line, the graph is the standard situation. Since the buses are producing power, the voltage is increasing and power flow is negative. The green line is not a typical linear line. During the interval [0,0.5] and [1.6,2], the graph is increasing and power flow is negative. However, between 0.5 and 1.6, the graph is decreasing and the power flow is positive. Since ODEs have different solutions, the graph will be different. The green line shows the power flow directions will reverse several times along the feeder during the day. This is an uncertain situation, and may cause some problems, such as power failure suddenly. Thus, the graph shows a result with a hard to control and dangerous behavior.

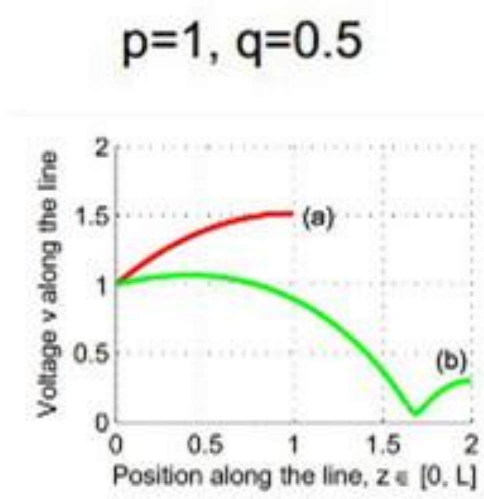


Figure 8: Voltage along the feeder with different length, for p=1 and q=0.5

4 Reproduction of Previous Results

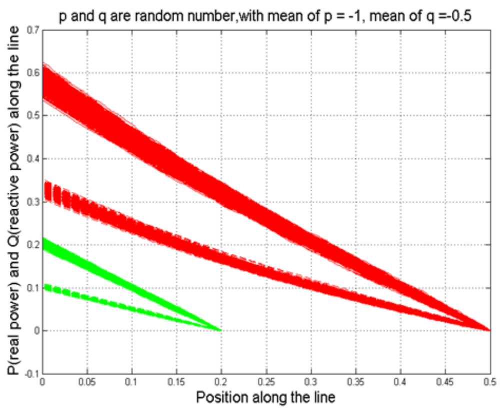
5 Results

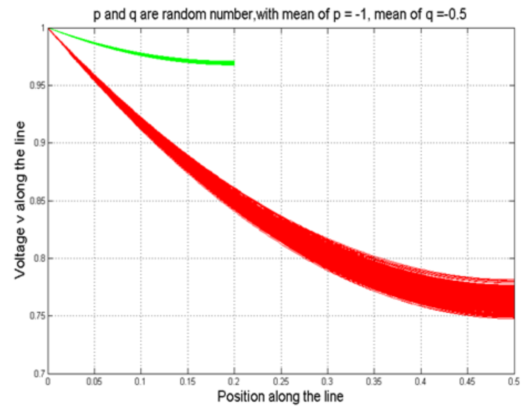
5.1 Adding Stochasticity

Throughout the previous results, the power consumption p was assumed to be a random number about -1, and q was constant -0.5 along the line. The Wiener process was used in the boundary value conditioned ODE's. However, this is not the case for how people use electric in the real world since every customer does not use power based on the neighbors. For the previous work, the power consumption rates between each house are independent, but the power consumption at House 2 depends on the power consumption at House 3 which does not make sense. Thus in this paper, more practical method of adding stochastic in the boundary value conditioned ODE's will be used.

Identically independently distributed noise (IID noise) was chosen because it will add noise to each parameter more practically than a power distribution system might experience. First, the mean of p was set as -1, and the mean of q as -0.5. The same mean was used as in previous work as a convenience to compare two works. Random values of noise were added to parameter p and q. Functions of add IID noise are p= -1+ 0.3*random numbers and q=-0.5+0.15*random numbers. This choice for power consumption allows p and q in each house as independent and will not affect each other.

For the calculation, 200 steps were chosen in the process which means the power line can be divided into 200 steps of equal length, with a power consumption p0+noise and q0+noise at the nth step along the line. The p and q were substituted in the boundary value problem. Matlab was used to get the graph of all the solutions. In each simulation, a distinct real and reactive power consumption profile was created, the boundary value problem was solved for the two lengths L=0.2 and L=0.5, and the solutions were plotted.





Figures below depict the aggregated results of 1000 simulations, as described above, along with the averaged solutions.

Each green and red line represents an individual solution to the boundary value problem for distinct, randomly generated power consumption profiles where length equals 0.2 and 0.5. As seen on the graph, 1000 lines gather tightly and each line is a little bit shaky. Since noise was added to both p and q, the boundary value solutions are harder to solve; thus the lines are not fluent any more.

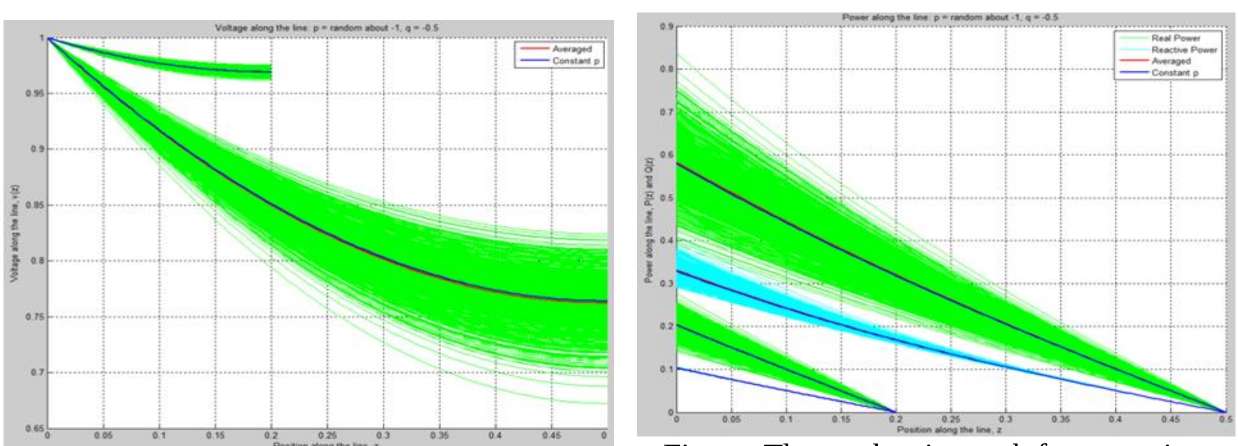


Figure: The stochastic graph from previous work using Wiener process, random p around -1 and q=p/2.

From the previous work (left graph), the range of voltage is wider the current work on the right graph for several reasons. First, the previous work included a larger variance; thus the three times of standard deviation is larger. It is highly possible that p in the previous work could be positive number. But in the current work, the standard deviation was set at 0.3 so it is less possible positive numbers will appear in p. Second, since the Wiener process was used in the previous work, and the power consumption is based on a previous house, the following house will be larger and larger if the previous house gets a positive p. Although the power consumption rate is independently random, the whole power consumption will not decrease or increase rapidly. So from the previous graph, some lines below or upper the whole area which will not happen in the current project. Noise was added to p and q, which are both independently random, so each house will not affect other houses along the line.

5.2 Adding Variance

Studying the relation between the variance of the power consumption parameter and the variance of the results is helpful to compute a safety range in order to avoid the voltage shortage at the end of the feeder line. To do this, the parameter for length of the line, $L = .5$, was fixed and eight different log values of the variance of p were picked. Then for each single value chosen for p, the variance of the voltage at the end of the feeder line were calculated over 200 individual simulations using the same method previously introduced. Figure (number) shows a scatterplot of these results.

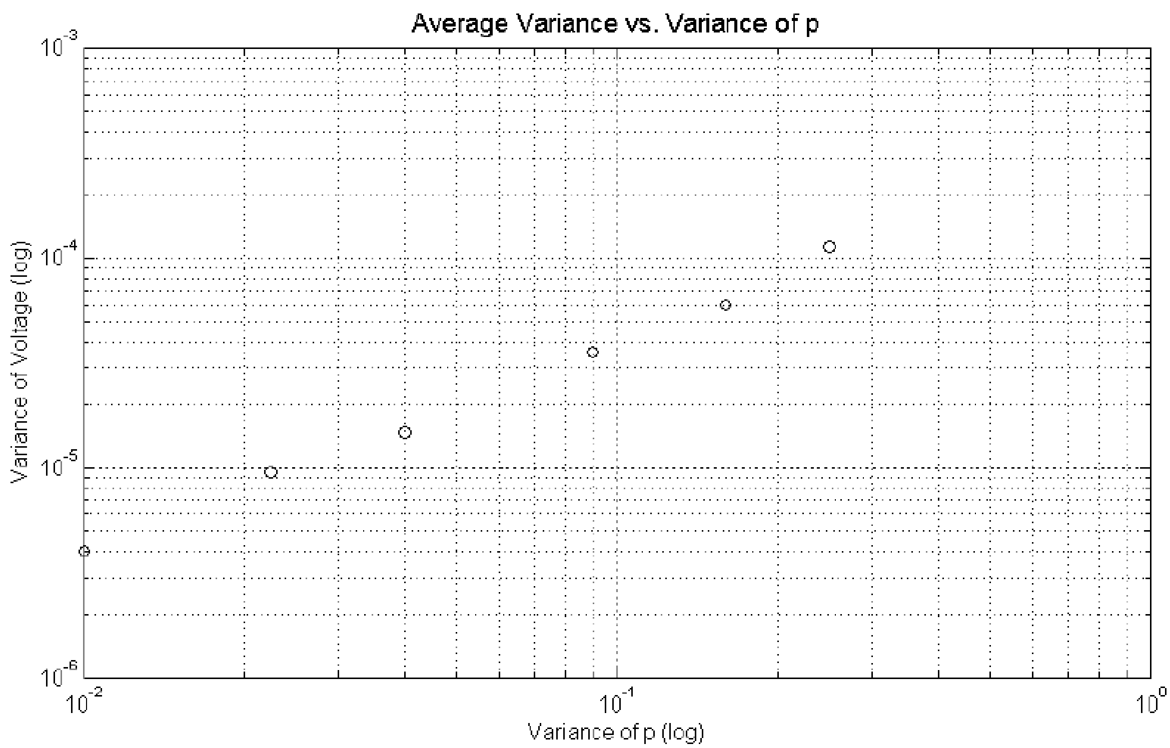
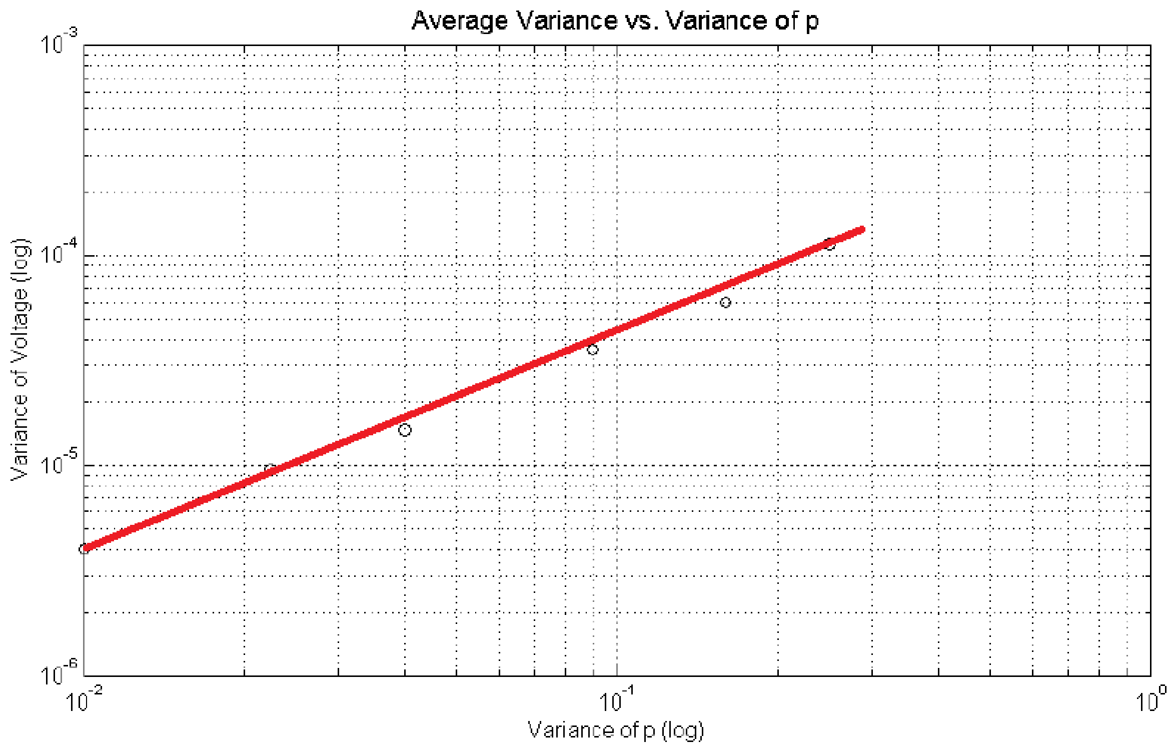


Figure: Here, the sample variance of the voltage value, S_v^2, was computed using the sample variance formula: $S^2 = \frac{\sum(X-\overline{X})^2}{n-1}$.

From the plot, eight dots are perfectly connected by a straight line, as indicated on the figure.



Then a reasonable assumption was made that this straight line follows a common linear form of $y = a \cdot x + b$, where a stands for the slope of the line, and b stands for the deviation. By substituting the variables that y and x represents into the equation, it will then be:

$$\ln(S_v^2) = a \cdot \ln\left(\sigma_p^2\right) + b$$

With a few steps of transformation, the equation ends up with the form

$$S_v^2 = C \cdot (\sigma_p^2)^a$$

Where C is a constant representing the value of e^b. From the scatterplot in Figure, the line has a slope of approximately 1, which is the value of a. Then the equation here would be

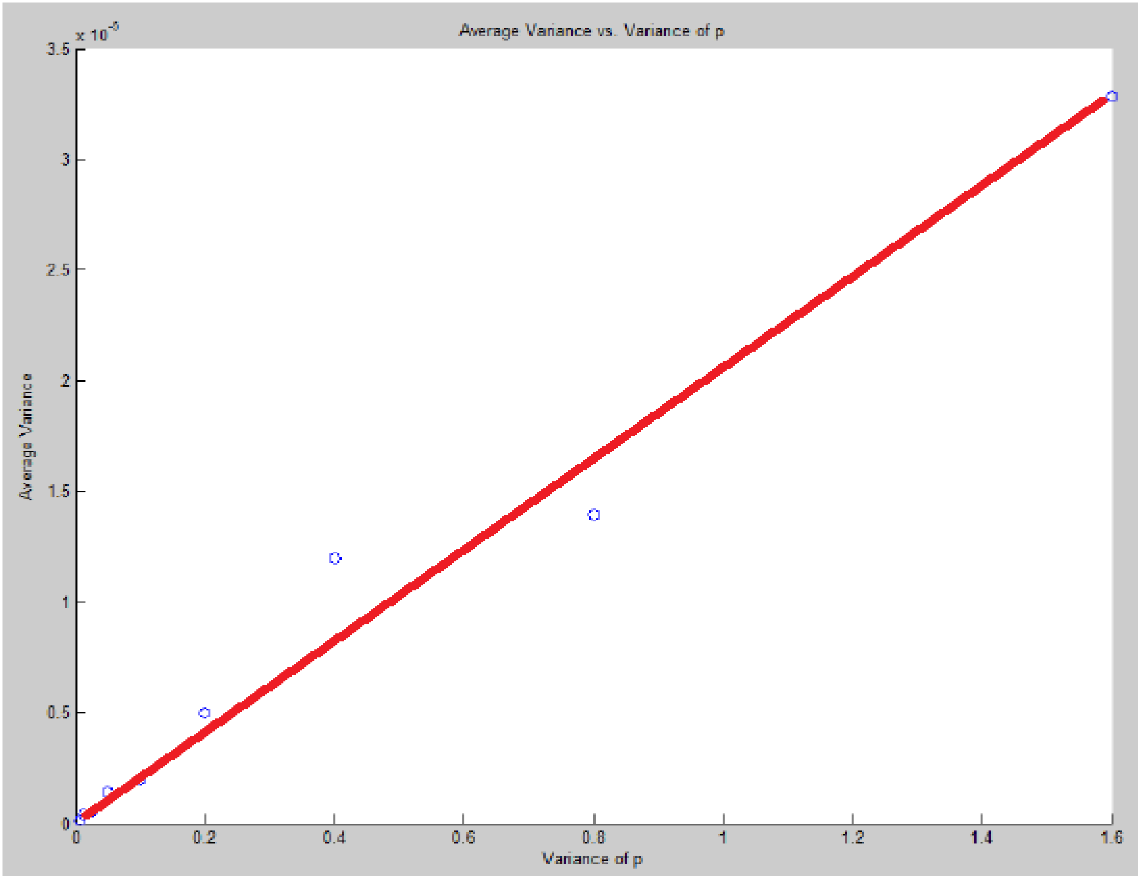
$$S_v^2 = C \cdot \sigma_p^2$$

And since C is a constant,

$$S_v^2 \sim \sigma_p^2$$

is derived which indicates the variance of the voltage value is directly proportional to the variance of the power consumption.

However, after the comparison between the results of the current work and the previous work, this proportion may vary. The above figure shows the previous work using the same method.



The above figure: The scatterplot also shows the relation between $S_v^2$and σ_p^2, but under the condition that a Wiener Process is the type of stochastic process introduced into the system.

Obviously, the previous results also form a linear relation between these two variables, and therefore will also have the same form as in Figure ___. However, as the graph indicates, the slope of the line is approximately 2, not 1, which lead to the difference in the results that provides the relation between the variance of voltage value and the variance of power consumption. It will be, thereby $S_v^2 \sim (\sigma_p^2)^2$ in the previous case, there will be a distinct result from the current work, that the variance of the voltage value is currently proportional to the square value of the variance of the power consumption.

6 Conclusion

The major points of the current work can be summarized as follows:

- The randomness introduced into the system using the I.I.D. process that creates a small perturbation on the power consumption in this model is proven to have very little impact in terms of varying the solutions of the initial value problem. Thereby, assuming constant power consumption, as what Wang, Turitsyn and Chertkov suggested, is statistically valid.

- The exact pattern of how the variations of the power consumption will contribute to the variance of the voltage value is dependent on the specific model and type of stochastic process used to imitate the real life situation in the simulations. It must be taken into consideration before computing the safety range for voltage drop to avoid any voltage blackouts at the end of the feeder line using this method.

7 Acknowledgements

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