Nonlinear Energy Harvesting

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Introduction

Mathematical modeling is a significant addition to the analysis of energy harvesting systems. While experiments and modeling are able to obtain results on their own, the process of using experimentation to improve models and using models to guide experimentation quickens the discovery process and allows for more intelligent research. The primary benefit that modeling provides is not necessarily the quantitative data provided, but the qualitative information gleaned. One can analyze the model to predict the change in the physical behavior of the system without having to perform experiments, which would be far more costly. Further, by predicting the behavior of the system, it is known whether or not that change to the system is worth investigating further via experimentation or more modeling. In the following, we take a known equation for harvesting energy and analyze the energy harvested when the system is at various angles from the original position. The purpose of this is for application in hand held devices, which are often placed in pockets or purses sporadically, thus knowing how much energy is likely to be generated throughout a day rather than the maximum amount of energy generated.

There is no such thing as a perfect transfer of energy. This theoretical situation is called the “Carnot cycle”, though such a process cannot be replicated. All processes are subject to energy loss to the environment, such as heating or cooling the ambient air, or performing work on the walls of the system. When work is performed on the environment through methods such as walking or driving on a surface, vibrations are released into the surface that eventually dissipate due to friction. One form of energy harvesting is to recapture these vibrations and recapture as much of the lost energy as possible. This can be done with a capacitor or inductor, but also with a piezoelectric material (PM). In the later situation, the vibrations are used to place compressive and tensile forces on the PM. When this is done below the materials Curie temperature, the atoms deform in such a way that a voltage drop forms across the PM, the direction depending on the direction of the deformation [3]. This voltage is normally used to reform the material after the stress is no longer applied, but in an energy harvesting device it is siphoned. A rectifier must be used in such a circuit because the voltage formed by the PM will essentially be AC.

One design is to use an inverted pendulum to harvest the energy. Layers of a PM are placed at the base around a rod, the latter being attached to an inertial mass in order to more easily recapture the vibrations. The rod in this system is clamped, but the elasticity of the rod and PM allow for the inertial mass to oscillate linearly. It has been found, however, that non-linear oscillation was able to better harvest oscillations. This can be achieved by adding magnets with repelling polarities to the system. One is attached to the end of the inertial mass opposite of the PM and one is placed some distance Δ from the pendulum. At some distance Δ_c the magnetic repulsion between the magnets causes the energy potential of the oscillator to turn from having a single potential well to two potential wells. As Δ is decreased further from Δ_c, the energy barrier between the two wells increases. The variation in the potential energy function of the oscillator alters the amount of energy that can be harvested, leading to the idea that there is a set of parameters that will maximize the energy harvested by the oscillator. The creation of the energy barrier causes an increase in the deformation of the PM, resulting in more voltage generated by the system and more energy harvested.

Theory

The model for such a system, originally provided by F. Cottone et al., was modified to be in terms of the angle of the oscillator to normal as follows:

$$m_{eff}\ddot{\varphi} = \frac{dU(\varphi)}{dx} - \gamma\dot{\varphi} - K_v V(t) + D\varepsilon(t) \text{ [1]}$$

where $m_{eff}\ddot{\varphi}$ is the kinetic force of the oscillator. $\gamma\dot{\varphi}$ is the energy dissipated due to the bending of the rod and PM. $K_v V(t)$ is the energy transferred from the PM, where V(t) is given by:

$$\dot{V}(t) = K_c\dot{\varphi} - \frac{V(t)}{R_L C} \text{ [1]}$$

$\sigma\varepsilon(t)$ is the driving force of the oscillator, which is represented as a stochastic process. $\gamma\dot{\varphi}$ and $K_v V(t)$ decrease the energy of the system over time, the latter of which represents the energy harvested by the inverted pendulum. $\sigma\varepsilon(t)$ represents the energy that would normally be lost to the environment but instead drives the oscillation of the inverted pendulum. Although this term does not model the driving force as a wave, as would be the case in real life, it still increases the energy of the system as time goes on, allowing for analysis as to how each parameter effects the voltage generated. The values used for analysis were taken when the force applied was constant rather than varying about a single value due to the randomness of the latter force and the need for consistency.

The potential energy of the inverted oscillator was derived by integrating the forces acting it. The two considered were the restoring force of the rod and the repulsion of the magnets. The former is well known as –K φ, where K is the effective elastic constant of the pendulum. The latter was approximated by assuming the magnets were small enough to be represented with single points. Under this assumption, the amount of magnetic force being applied in the direction of oscillation was found to be:

$$F = \frac{Q * R * \sin(\varphi)}{(l^2 \sin^2(\varphi) + (R - l * \cos(\varphi))^2)^{3/2}} = \frac{Q * R * \sin(\varphi)}{(l^2 + R^2 - 2 * l * R * \cos(\varphi))^{3/2}}$$

where $Q = \frac{\mu_0 * q_1 * q_2}{4\pi}$. Using the relation $F = -\frac{dU(\varphi)}{d\varphi}$, the potential energy is found to be

$$U(\varphi) = \frac{K}{2}\varphi^2 + \frac{Q}{l * \sqrt{l^2 + R^2 - 2 * l * R * \cos(\varphi)}}$$

For a system with two magnets above the oscillator, each magnet has a distance S from the center line, defined by the position of the oscillator at $\varphi = 0$. This approach results in a potential energy function defined as

$$U(\varphi) = \frac{K}{2}\varphi^2 + \frac{Q_1}{l * \sqrt{l^2 + R^2 + S^2 + 2l(-R * \cos(\varphi) + S * \sin(\varphi))}} + \frac{Q_2}{l * \sqrt{l^2 + R^2 + S^2 + 2l(-R * \cos(\varphi) - S * \sin(\varphi))}}$$

Through the Superposition Principle, the effect of additional magnets on the potential energy function is easily calculated.

The voltage produced at any given moment of time is not representative of the voltage generated by the inverted pendulum at all points of time. This requires that the voltage of the system be evaluated over a span of time. In addition, the direction of the voltage fluctuates depending on the direction in which the beam is bending. Thus, the root-mean-squared voltage (V_{rms}) must be used in order to rectify the AC current being produced and accurately evaluate the voltage produced by the system.

The initial conditions of the system must remain constant in order to compare the V_{rms} produced, save the parameters being varied to analyze their effects. However, the starting angle and the starting energy cannot both remain constant while varying the spacing between the magnets (Δ) and the strength of their interactions (Q). Once double well behavior arises in the potential function, the potential energy at $\varphi = 0$ is no longer equal to the minimum of the system, and thus the inverted pendulum will not start with the same energy for all Δ, Q, and S. Since this starting energy will go towards driving the oscillations, it will artificially inflate the amount of energy harvested by the inverted pendulum. Since the energy being harvested by the system is being analyzed, keeping a constant starting energy was of higher importance than the starting angle and all calculations must be made with the starting angle being at the bottom of the potential well for each set of Δ, Q, and S.

Both systems must produce the same V_{rms} for a given Δ and Q while S=0, or the systems will not be comparable. To achieve this, the total magnet interaction strength was kept constant when transitioning from the single magnet above the oscillator to the double magnet.

Methods

The V_{rms} was first mapped as a function of Δ and Q for a single magnet above the oscillator, which was done by calculating the voltage generated over a 100 second time span for spaced pairs of Δ and Q. The results verified previous work [1] in that double well potentials generate a larger V_{rms} than for a single well. However, the pairs of Δ and Q that generated the largest V_{rms} had potential functions with a single well. These wells had very little variation in the potential energy for angles about $\varphi = 0$.

Following these results, the point at which the potential function transitioned from single to double well behavior needed to be defined. To this end, the equality $\mathrm{U}(\varphi_{\mathrm{min})} = \mathrm{U}(0)$ was produced. This equality holds true so long as the minimum of the potential function is single well, becoming untrue after a given Q surpasses or Δ is decreased past a given value and the potential function becomes double well. Δ was held constant while Q was increased from 0 until the equality was no longer valid, each pair being recorded. The resulting curve can be seen in figure 2(a).

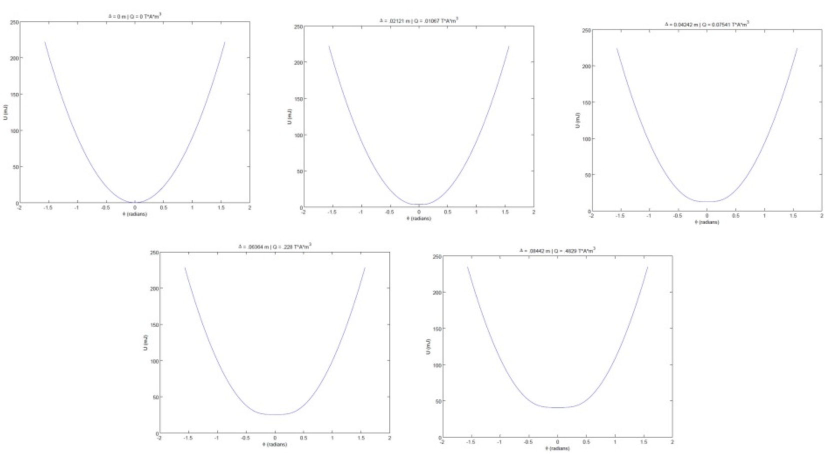


Figure 1: The potential energy functions for ideal pairs of Δ and Q. The pairs, from left top to bottom right, are as follows: (0,0), (.02121, .01067), (.04242, .07541), (.06364, .228), (.08442, .4829)

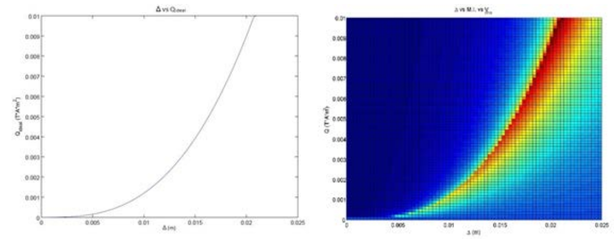
The V_{rms} produced for pairs of Δ and Q along this curve were calculated with constant force, with each simulation being run for a 400 second time interval with 50 simulations each with a varying force. This force was scaled by a value between 0 and 2 based on a Gaussian distribution centered on 1.

Following, the V_{rms} for pairs of Δ and Q slightly larger and smaller than those found on the curve were tested for the V_{rms} produced. This was done in order to test how accurate using the above equality is for calculating the greatest V_{rms} produced.

For simulations where two magnets are placed above the inverted pendulum, the V_{rms} is a function of three variables: Δ, Q, and S. Since S is the new variable introduced in this system, the V_{rms} was calculated twice, once with constant Δ and once with constant Q. The results were then analyzed to see the effect of S on the V_{rms}.

Results

When comparing the curve generated by the equality $U(\varphi_{min)} = U(0)$ to the calculated results it becomes clear that the results follow the curve. Along the curve generated by the equality $U(\varphi_{min)} = U(0)$, it was found that for increasing values of pairs Δ and Q, the V_{rms} generated increased. Thus, there is no ideal pair of Δ and Q for which the most voltage is generated. Rather, the limitation is on the physical constraints of the location for which the inverted pendulum is intended to be used. However, these pairs simply provide the most consistent results. By varying Δ and Q about the point (.07, .2942) it was found that increasing Q slightly greatly increases the V_{rms}. However, the standard deviation also greatly increases. Thus the curve resulting from $U(\varphi_{min)} = U(0)$ does not produce the greatest V_{rms} for the system, although it does provide the most consistent results. Depending on the application of the inverted oscillator, stability or voltage produced may be of greater importance.



a b

Figure 2: a) The curve resulting from the equality $U(\varphi_{min)} = U(0)$ and b) the plot of Δ vs Q with colors representing the V_{rms} produced. Blue corresponds to low V_{rms} and red corresponds to high V_{rms}

Δ (m)	Q (T*A^2*m^3)	$V_{rms, ave}$ (V)	Standard Deviation
0	0	0.0012	0.000011552
0.007071	0.000435	0.0044	0.00020404
0.01414	0.003315	0.0062	0.00029936
0.02121	0.01067	0.0071	0.00043506
0.02828	0.02423	0.0082	0.00048111
0.03535	0.04541	0.0091	0.00052889
0.04242	0.07541	0.0099	0.00049762
0.04949	0.1153	0.0108	0.00067839
0.05657	0.1659	0.0108	0.00064192
0.06364	0.228	0.0114	0.0007905
0.07	0.2942	0.0117	0.00064062
0.07739	0.384	0.0121	0.00078317
0.08442	0.4829	0.0128	0.00067265

Table 1: The average V_{rms} for pairs of Δ and Q along the curve resulting from the equality $U(\varphi_{min)} = U(0)$

Δ (m)	Q (T*A*m^3)	$V_{rms, ave}$ (V)	Standard Deviation
0.07	0.2942	0.0117	0.00064062
0.07	0.2992	0.0232	0.0051
0.07	0.2982	0.0076	0.0005606
0.065	0.2942	0.0019	0.00018046
0.075	0.2942	0.0028	0.00023107

Table 2: The average V_{rms} for pairs of Δ and Q slightly off the curve resulting from the equality $U(\varphi_{min)} = U(0)$

The system with two magnets above the oscillator was shown to produce a greater voltage at some pair (S, Δ) and (S, Q). When Δ was held constant, it appears that there is no ideal pair (S, Q) that produces the greatest V_{rms}. This is possibly a result of the boundary conditions imposed on the calculations. However, there is a maximum for individual Q, with an S greater than zero that produces the most voltage. When Q is held constant, there is an ideal pair (S, Δ) that maximizes the V_{rms}. There are local maxima near the global maximum and the V_{rms} quickly decreases as one increases S or Δ from this ideal pair. The maxima in both of these cases will, of course, vary based on the Δ and Q respectively.

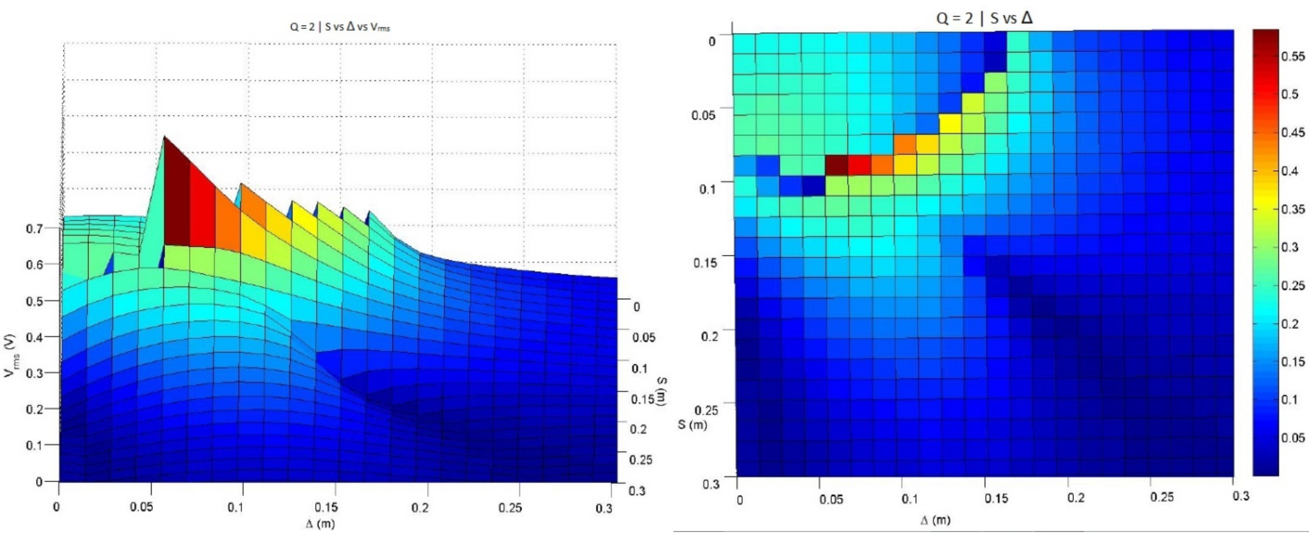


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Figure 3: a) The surface plot for Q = 2, S vs Δ vs V_{rms} and b) the surface plot (a) viewed from above, S vs Δ

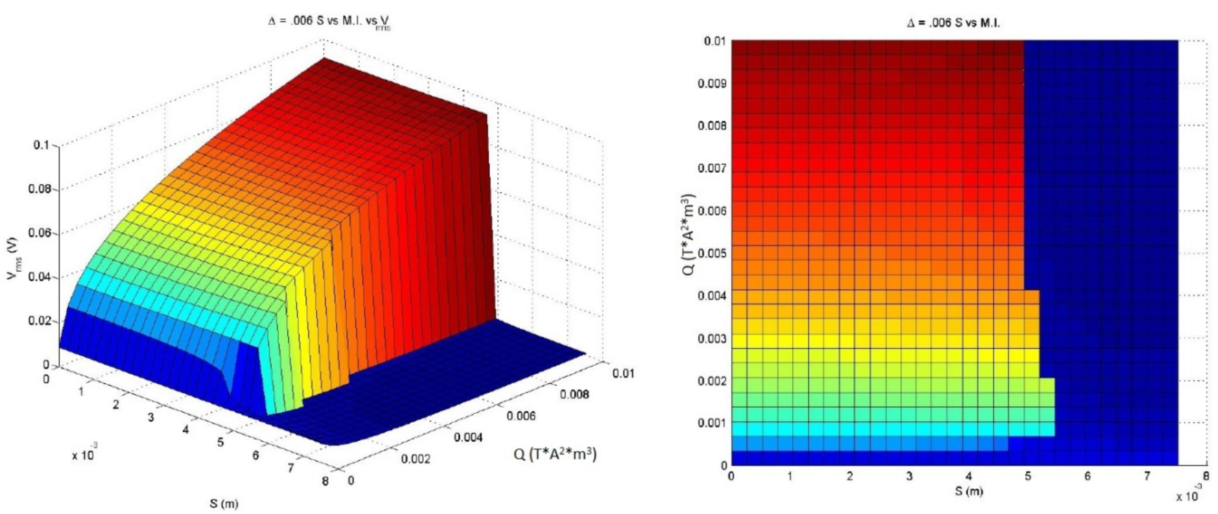

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Figure 4: a) The surface plot for Δ = .006, S vs Q vs V_{rms} and b) the surface plot (a) viewed from above, S vs Q. The maximum V_{rms} is not shown

Conclusions

It has been shown that adding magnets to and above the inverted pendulum increases the energy harvested from an applied force. There is no ideal pair of Δ and Q that will produce the most voltage; rather, V_{rms} increases with Δ and Q, and every Δ has a Q at which the most voltage will be generated. The curve generated by the equality $\mathrm{U}(\varphi_{\mathrm{min})} = \mathrm{U}(0)$ was shown to map pairs (Δ, Q) that produced consistent and significant V_{rms}. By slightly increasing the magnetic intensity from these values, the V_{rms} produced by the system increased, showing that the equality does not produce pairs of (Δ, Q) that maximize V_{rms}. However, the increase in Q also increased the standard deviation of the V_{rms}, showing that the pairs are unstable and inconsistent in the voltage they produce compared to the pairs produced by the equality. As a whole, though, the system was shown to be self-averaging, ie the system will produce the same voltage regardless of how large the time scale provided the time scales are sufficiently large. This is essential to implementing such an oscillator, as the electronics which it will be powering or the batteries which it will be charging must be designed around the voltage produced by the system.

When adding a second magnet above the oscillator, the V_{rms} was found to increase. When Q is held constant, there is a distinct pair of values (S, Δ) that produces the greatest V_{rms}. This value is greater than for S=0, showing that the additional magnets increases the efficiency of the energy harvesting inverted oscillator. However, this was a maximum for a single value of Q. If Q is altered, then the ideal pair of S and Δ will change. There was no ideal pair (S, Q) that maximizes the voltage produced, though this is likely a result of boundary conditions. However, when both Δ and Q were taken to be constant, there was a non-zero S that produced a maximum voltage.

Overall, the main constraint on the efficiency of the energy harvesting inverted oscillator is the physical space and materials available for the oscillator. There is not an ideal set of Δ, Q, and S that produces a maximum voltage, only sets that produce local maxima.

Future Work

A relationship between Δ and Q that results in the maximum V_{rms} has yet to be found can be of vital importance to maximizing the energy harvested by the inverted pendulum. Although the current relationship provides a close approximation, the large increase in V_{rms} warrants future work to derive this relationship. Further, the consistency of the voltage generated by the ideal (S, Δ) in the system with two magnets above the oscillator has not been determined. Refinement in the precise value of this maximum and determining the stability of it must be done in order to determine whether it is truly able to generate more energy than the system with a single magnet above the oscillator.

Given the benefits of having two magnets above the oscillator rather than one, looking at placing additional magnets in the system is a promising avenue of future work. Do to the similarity in calculations and the Superposition Principle, this can be easily calculated once the foundation work is complete. Further, altering the arrangement of the magnets may yield an increase in the amount of energy harvested.

In addition, the current driving force, while sufficient for the work done, can be improved to better model the waves that will be driving the oscillation. Altering the driving force to better match the vibrations produced by walking, wind, and cars will allow for better prediction in the efficiency of the inverted oscillator.

Cited Sources

1) Cottone, F. , Vocca, H. , Gammaitoni, L. , *Nonlinear Energy Harvesting* Physical Review Letters **102** (2009) 1-4