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Wave Structures on a Jet Entering the Bulk Liquid

Abstract

This report gives discrete information on the structures of waves as they operate on a jet entering liquid and how to use the experiment to find out the waves. The paper will present results both theoretical and experimental concerning the case. The process will involve a vertical jet entering a deep fluid reservoir. And we will consider the factor regime in which the jet is categorized by a fixed field of capillary waves in a pure water system at its base. If the reservoir is subjected to contamination by a surfactant, then the base of the jet will be void of capillary waves, quiescent and cylindrical. This means that water will find its way to the reservoir like through a rigid pipe. To explain this theoretically, the researcher will match an extensional plug flow upstream of the pipe to the entry pipe flow within it. The results prove the theoretical outcome where the fluid pipe rises on a falling fluid sheet.

Introduction

Flow of interest is seen best in a kitchen sink. A volume flux exiting a tap follows the pressure limited to the diameter of the tap. However, obstructing the stream at a distance will make the flow to form a stationary field of varicose capillary waves on the obstructer. Nevertheless, when the obstructer is dipped in the liquid before insertion to the stream, then capillary waves will start at some distance above it, below which the stream is cylindrical. Checking closely shows that the surface of the cylindrical base of the jet is quiescent. In this report, the research will develop a description of dynamics explaining this phenomenon (Das).

Current research focus majorly on the dynamics of laminar fluid jets together with the propagation of capillary waves on jets. Some scientists researched on the stability of cylindrical fluid jet with no gravity. According to their findings, axisymmetric varicose perturbations having a wavelength greater than the circumference of the jet grew exponentially leading to a jet breakup. Within the same case, experiments with wavelength less than circumference of the jet showed some neutral stability as they propagate along the jet. The theory of Rayleigh also described the same but later extended to describe capillary waves as seen on jets falling freely vertically. The extension also demonstrated the existence of a stationary field of a stable varicose capillary waves on a jet in a situation when the phase speed resembles the speed of a local jet. Sometimes, the fluid viscosity influence focusing on shape of a water jet can be negligible. However, damping may contain an important impact on a stable wave neutrally. In such a case, the wave amplitude will have to decrease exponentially as distance decreases from the disturbance source (Hancock and Bush).

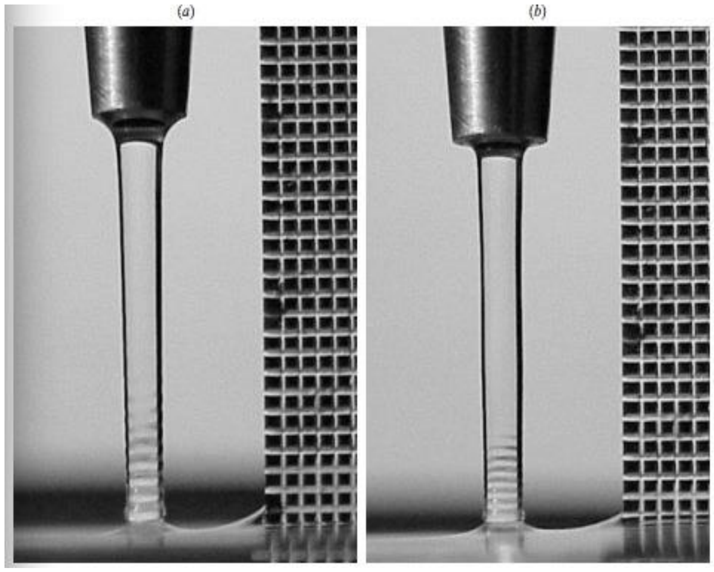


Figure 1: water jet impinging on a pure water reservoir

An axisymmetric fluid jet impinging a deep reservoir cases the excitement of a stationary field of varicose capillary waves by the impact. The evidence of this can be observed at the base of the jet as seen in the above figure one. However, contaminating the reservoir with a surfactant, the surface of the reservoir will be less than the impinging jet. This will lead to the alteration of the flow structure as seen in the below figure two. In such a case, the first process will be the suppression of the varicose capillary waves at the jet's base. This will, however, resume when the experimenter gives some distance over the reservoir. The second process will involve a region of jet surface considered void of capillary waves. The experimenter should ensure that the surface is entirely quiescent. The jet will enter the reservoir like a rigid pipe assuming the name fluid pipe. The first invention of liquid pipe was in an experimental study that considered liquid jets absorbing gas as they impinge on a fluid reservoir (Das).

The main goal of this study is to come up with a description of the formation of stationary fields of capillary waves as experienced on a fluid jet as it enters a bulk liquid. Normally, the velocity of the jet, its height, stream size together with the fluid's surface tensions affects the waves.

Potential Applications

Understanding capillary waves allows us a better understanding of surface tension as a whole. Surface tension can be a difficult measurement to make, but observing the way that the waves react to changing conditions can give us insight.

For example, surface tension is currently being used in some exciting projects, including attempts to communicate with dolphins. A high surface tension allows sound waves to create a unique imprint on the water that mimics the type of communication that dolphins use between themselves. These imprints, made up of intricate waves, can be captured on film and made into images. These images are then able to be shown to dolphins, who seem to understand them. Study of surface tension will give scientists a greater understanding of how dolphins, and perhaps other water species as well, are able to create and receive these intricate images even in non-ideal water settings where the surface tension may not be as high.

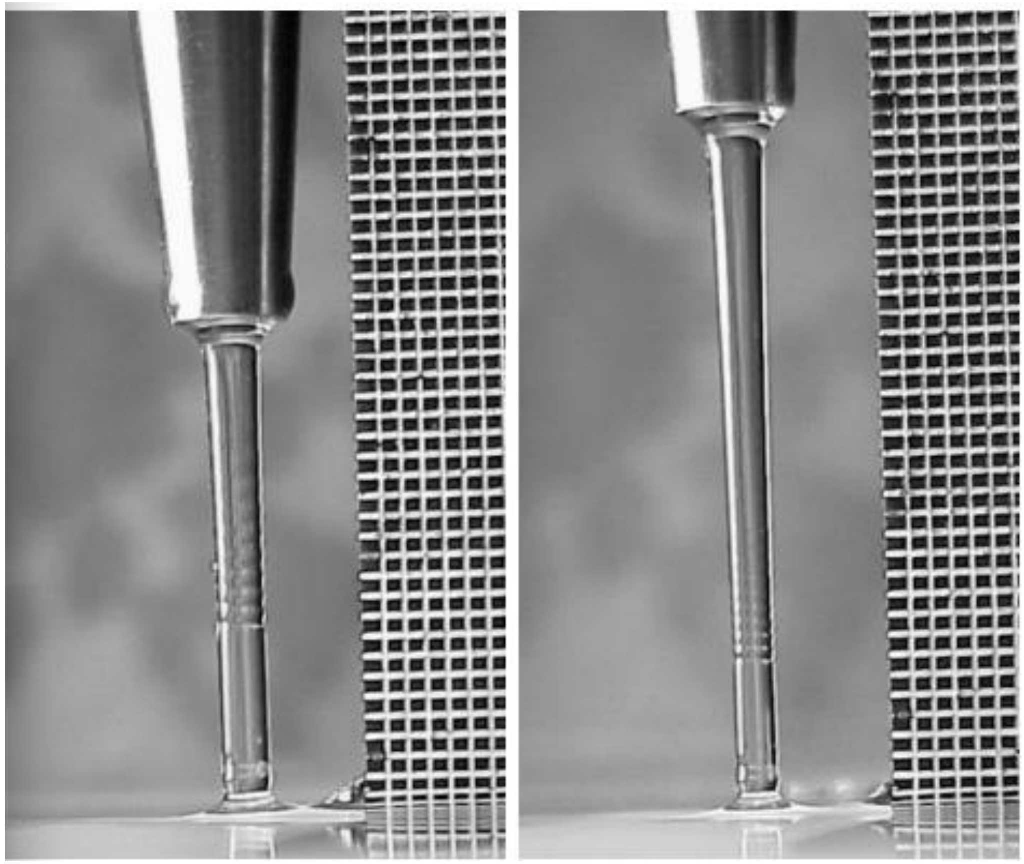


Figure 2: stagnant fluid pipes positioned at the base of water jets as they impinge on a reservoir, which is contaminated with Ivory dish detergent (Hancock and Bush).

Physical picture

The figure illustrated below, figure three, the experimenter considers a laminar vertical water jet with viscosity, surface tension and density generated through release of a certain volume flux from an outlet with a given radius. The attribute Reynolds number given is very large. The jet, on the other hand, will evolve because of gravitational acceleration, *gz*^ (*z*^ is a unit vector that points downward). It will then impinge itself on the deep surfactant laden reservoir categorized by the surface tension. The concomitant surface-tension gradient will attract a surfactant up the jet until it balances together with the viscous and Marangoni stress. In normal occasions, surfactant has a significant effect on the elasticity to the interface. This suppresses the capillary waves. On the same line, the capillary waves are normally suppressed within the pipe but resume on the surface of the pipe. Within the same procedure, the whole process using Vertical Marangoni stress also suppresses the motion of the extensional surfaces that is associated with the plug flow. This leads to the production of a static surface generated at the bottom of the jet and a fluid pipe (Das).

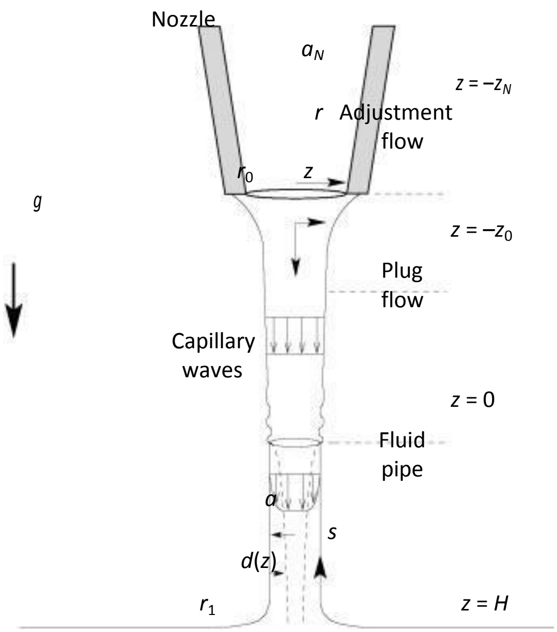


Figure 3. A schematic illustration of the flow of interest, which may be described in terms of three distinct regions: (a) adjustment flow ($-z_N < z < -z_0$); (b) extensional plug flow ($-z_0 < z < 0$); and (c) the fluid pipe ($0 < z < H$).

The laminar entry of the flow pipe has a boundary layer with a certain thickness growing along the sides of the pipe in the inlet region. This happens until the thickness can be compared to the radius of the pipe. According to Mohanty and Asthana, the stream wise extent of the inlet region is calculated as L_F 0:072 *aRe* with $Re= aV/v$ being the Reynolds number. The speed and the radius of the pipe calculate this number. In this experiment, a was taken to be approximately 0.1 cm, as V to be (30-100) cm s^{-1}, and v equally to 0.01 cm^2s^{-1}. This means that Re will be approximately 300 as L_F will be approximately (2-7) cm. considering that the maximum height of the pipes containing the fluid was 2 cm, this means that the flow in the pipe will be that within the inlet region of entry pipe flow.

The next step is to deduce a simple scale indicating the dependence of the height of the fluid pipe on the leading system parameters. To balance both the viscous and Marangoni stresses using the pipe surface will give the following:

$$pv\frac{V}{\delta H} \approx \frac{\Delta\sigma}{H} \qquad 2.1$$

$\Delta\sigma = \sigma 0 - \sigma 1 \; and \; \delta H$ is the boundary layer giving the thickness at the basement of the fluid pipe. The assumption made is that the thickness of the boundary layer increases with the distance measured *z* from the inlet in accordance to the following equation

$$\frac{\delta}{a} \sim \left(\frac{vz}{a^2 V}\right)^{\frac{1}{2}} = \left(\frac{z}{aRe}\right)^{\frac{1}{2}} \qquad 2.2$$

Substituting 2.2 into 2.1 will get

$$\frac{H}{a} \sim \frac{1}{Re}\left(\frac{\Delta\sigma}{pvV}\right)^2 = \left(\frac{Re}{W_d^2}\right) \qquad 2.3$$

Where $W_d = paV^2/\Delta\sigma$ represents the dynamic Weber number. Converting it to the dimension form, the results come as follows

$$H \sim \frac{(\Delta\sigma)^2}{p\mu V^3} = \frac{(\Delta\sigma)^2 \pi^3 a^6}{p\mu Q^3} \qquad 2.4$$

The above shows an increase in the height of the pipe as one differentiate the surface tension together with the pipe radius. Conversely, it decreases in the same level with the fluid viscosity together with the volume flux. A discrete theoretical explanation of the fluid pipe requires other aspects of combined influence of both free pipe surface and gravity on the boundary layer dynamics. Figure five helps in explaining the above concept.

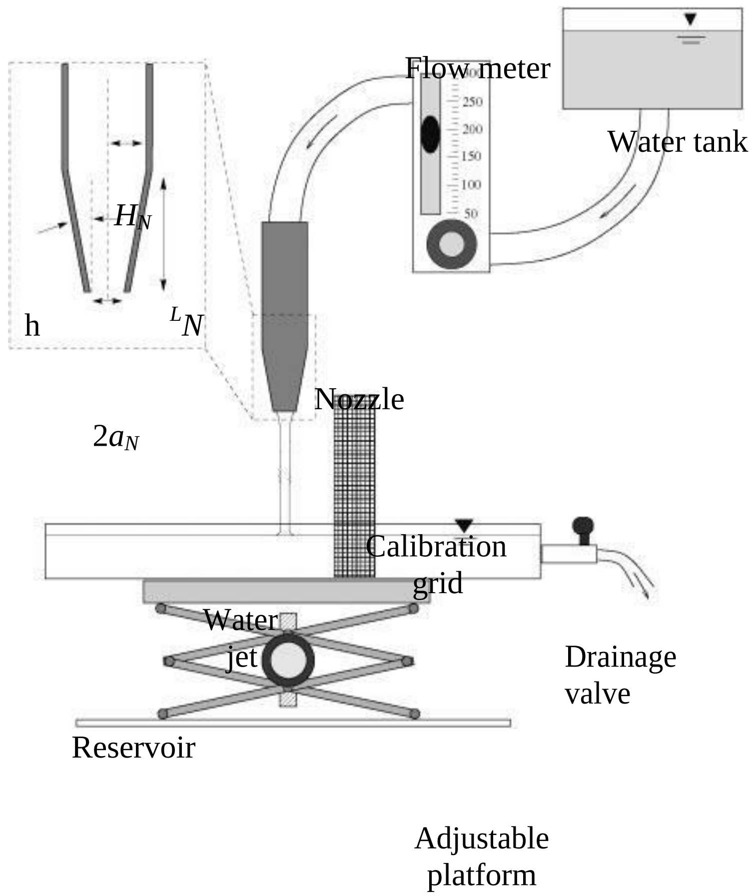


Figure 4. A schematic illustration of the experimental apparatus. The nozzle specifications (L_N = 3:0 cm, H_N = 0:50 cm, a_N = 0:15 cm, = 6:7) were chosen to minimize the extent of the adjustment region.

Experimental Method

This study will consider four cases, which include the following:

1. a fluid jet entering a reservoir that contains the same fluid
2. a fluid jet entering a reservoir that contains a fluid with a smaller surface tension
3. a fluid jet entering a reservoir that contains a fluid with a greater surface tension
4. A fluid jet meeting a fluid (air) having no surface tension.

To handle this situation, the experiment will consider two forms of formulas: simple formula and Fourier transform formula

$$\omega^2 = \frac{\sigma}{\rho+\rho_1}|k^3|$$

Where

ω is angular frequency

σ is surface tension number

p is density of the water

ρ_1 is density of the air, in this formula, it is nearly equal zero

K is wavenumber and it is equal

Simple formula

V is phase velocity, also said linear velocity, is $\frac{\omega}{k}$

$$v^2 = \frac{\omega^2}{k^2} = \frac{\sigma}{\rho}|k| = \frac{\sigma}{\rho} * \frac{2\pi}{\lambda}$$

$$\mathbf{v} = \sqrt{\frac{\boldsymbol{\sigma}}{\boldsymbol{\rho}} * \frac{\mathbf{2}\boldsymbol{\pi}}{\boldsymbol{\lambda}}} = \frac{\boldsymbol{a}}{\sqrt{\boldsymbol{\lambda}}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

Fourier transform formula

$$-(i\omega)^2 = \sigma/\rho \ (ik)^3 i$$

$$\partial/\partial t = i\omega$$
$$\partial/\partial x = ik$$

$$-\partial/\partial t^2 = \sigma/\rho \ \ \partial/\partial x^3 i$$

$$-U_{tt} = \frac{\sigma}{\rho} i \, U_{xxx}$$

$$-\frac{\partial}{\partial t}^2 \, \hat{U}(k,t) = \frac{\sigma}{\rho} i \, (-i\omega)^3 \, \hat{U}(k,t)$$

$$\hat{U}_{tt}(k,t) = \frac{\sigma}{\rho} k^3 \, \hat{U}(k,t)$$

$$\hat{U}_{tt}(k,t) - \frac{\sigma}{\rho} k^3 \, \hat{U}(k,t) = 0$$

$$\hat{U}(k,t) = A_1 \, e\sqrt{\frac{\sigma}{\rho} \, k^3} t + A_2 e^{-}\sqrt{\frac{\sigma}{\rho} \, k^3} t$$

$$\mathbf{U(x,t)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{U}(k,t) \, e^{-ikx} dk$$

The procedure about this experiment and how to make measurement

Equipment for this experiment

1. cup with volume
2. stopwatch to measure time
3. rule to measure the length of waves
4. Camera to get pictures

Procedures of the experiment

1. measure the volume of the cup to know the capacity of the water
2. use the stop-watch in measuring the time
3. after recording the time on the flux, move the cup until the waves are seen clearly
4. put the ruler on the side then use a camera to take pictures of the outcome
5. repeat the above steps many times then take pictures for different length of waves

Procedure of measuring wavelength and cross sectional area

1. use a ruler in finding out the length of the waves in pictures
2. use a ruler again to get the proportion about the ruler in pictures
3. use the proportion in finding out the ideal length about the waves
4. repeat the steps above and find the radius about the cross sectional in different high
5. after the above steps, use the measurements to perform various tests
    a. time
    b. cup volume: 4.93x10^6 mm^3
    c. v = ((diameter/2)^2) * pi * height of cup

d. wave

e. a: water wave cross section area

Get the flow value

Flow = volume of cup/time

time (sec)	flow (mm^3/s)	r (mm)
105.4	4677.4	1.6
110.4	4465.6	1.5
99.5	4954.8	1.7
109.3	4510.5	1.5
105.5	4672.9	1.6
110.4	4465.6	1.5
110.9	4445.4	1.5
107.3	4594.8	1.5
108	4564.8	1.5
107.3	4594.6	1.5
109.8	4489.9	1.6

Get the value of velocity

Velocity = flow/area of wave section

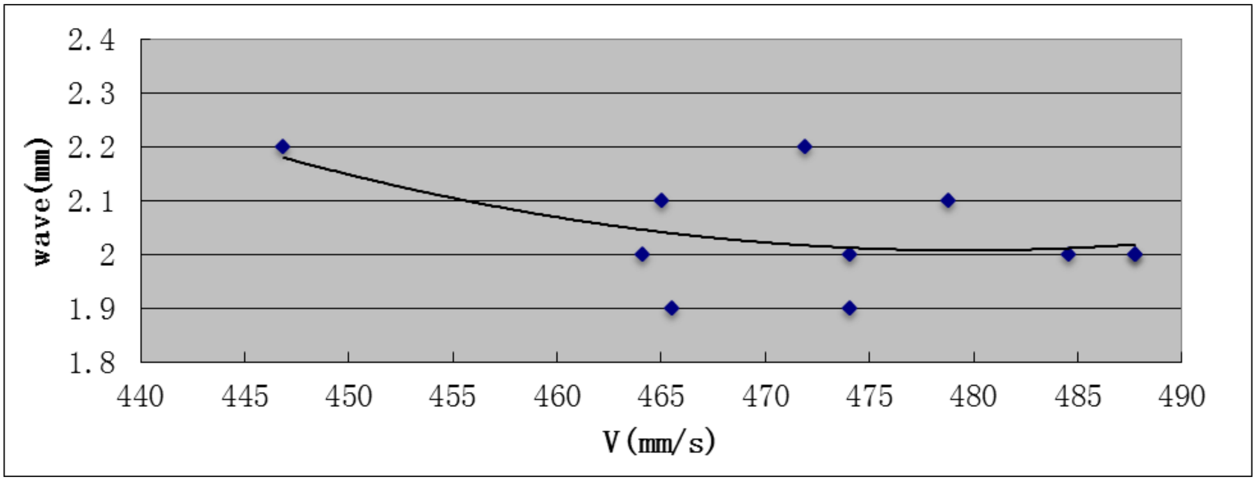
time (sec)	flow (mm^3/s)	r (mm)	velocity (mm/s)
105.4	4677.4	1.6	465.5
110.4	4465.6	1.5	474.1
99.5	4954.8	1.7	464.1
109.3	4510.5	1.5	478.8
105.5	4672.9	1.6	465.1
110.4	4465.6	1.5	474.1

110.9	4445.4	1.5	471.9
107.3	4594.8	1.5	487.8
108	4564.8	1.5	484.6
107.3	4594.6	1.5	487.7
109.8	4489.9	1.6	446.9

Relationship between velocity and wave

velocity (mm/s)	waves (mm)
465.5	1.9
474.1	1.9
464.1	2.0
478.8	2.1
465.1	2.1
474.1	2.0
471.9	2.2
487.8	2.0
484.6	2.0
487.7	2.1
446.9	2.3

The graph of wave vs. velocity



Calculate the value of a

Velocity (mm/s)	Waves (mm)	Surface (N/mm)
465.5	1.9	$6.55*10^{-5}$
474.1	1.9	$6.79*10^{-5}$
464.1	2.0	$6.86*10^{-5}$
478.8	2.1	$7.64*10^{-5}$
465.1	2.1	$7.23*10^{-5}$
474.1	2.0	$7.16*10^{-5}$
471.9	2.2	$7.78*10^{-5}$
487.8	2.0	$7.56*10^{-5}$
484.6	2.0	$7.47*10^{-5}$
487.7	2.1	$7.94*10^{-5}$
446.9	2.3	$7.29*10^{-5}$

Use the function of experiment

$$\mathbf{V} = \sqrt{\frac{\sigma}{\rho} * \frac{2\pi}{\lambda}} = \frac{a}{\sqrt{\lambda}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

The result of a will be as follows:

From the function of experiment, we can get the result of a have a range.

From 20.74 to 22.8

Use the function of experiment

Velocity = a / (wave) ^0.5

Get the average value of a, a = 21.338

$$\mathbf{v} = \sqrt{\frac{\sigma}{\rho} * \frac{2\pi}{\lambda}} = \frac{a}{\sqrt{\lambda}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

Error Bar

$$v = \frac{V}{t\pi r^2}$$

$$\ln v = \ln V + \ln t + \ln \pi + 2\ln r$$

$$\frac{\Delta v}{v} = \frac{\Delta V}{V} + \frac{\Delta t}{t} + 2\frac{\Delta r}{r} \approx 0.04$$

$$\Delta v = v \times 0.04 = 18.8$$

$$v = 470 \pm 18.8$$

$$\sigma = \frac{v^2 \rho\lambda}{2\pi}$$

$$\frac{\Delta\sigma}{\sigma} = 2\frac{\Delta v}{v} + \frac{\Delta\lambda}{\lambda} = 0.08$$

$$\Delta\sigma = 7.29 \times 0.08 \approx 0.6$$

$$\sigma = 7.3 \pm 0.6$$

So:

Surface tension = density of water * a^2 / 2 pi

In addition, the surface tension is about 72 N/mm.

Experiment error

1. personal error entailing taking photos, feeling and reflection from the photos taken, performing the calculations)
2. the temperature both room and working condition might give a wrong value
3. the lighting system which might give a wrong impression of the selected value

Conclusion

From the above experiment, one can deduce the relationship existing between temperature and surface tension. When the temperature of the liquid increases the molecular motion is enhanced. This means that there will be an increase in the average distance between the liquid molecules leading to a decrease in the attraction between the liquid molecules. The major assumption of this experiment was that the surface tension is inversely proportional to temperature. Judging from the results of the experiment, they resembled that of Hancock and Bush that investigated information on capillary waves.

Future Research

One can perform the same experiment but use a different method by the name capillary rise method to know the relationship between temperature and surface tension. In doing this, one can perform the following procedure:

- Use a circular capillary to dip into the water
- Water wets the walls and rises in the capillary to get a balance level
- The surface is a hemispherical concave
- On the surface, there is an equilibrium with gravity force and surface tension
- Derive the equation of surface tension from these balance forces
- Compare the result of experiment to verify the relationship between temperature and surface tension

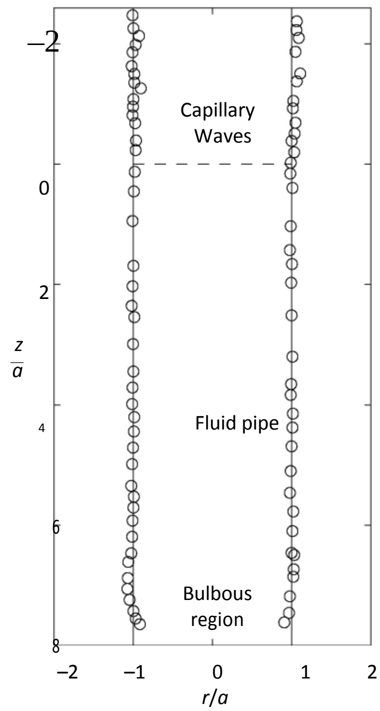


Figure 5. A typical profile of a water jet impinging on a reservoir contaminated with Ivory detergent. The radius of the fluid pipe is constant within the experimental error indicated by the radii of the circles. Capillary waves are evident above the pipe, while an underlying bulbous region joins the jet to the reservoir. Here $a = 0{:}11$ cm and $Q = 2{:}55$ cm^3 s^{-1}.

Work Cited

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Hancock, M attew J. and John W. M. Bush. "Fluid pipes." *J. Fluid Mech.* (2002): vol. 466, pp. 285-304.