Search of the Scent Source in Turbulent Flows

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Abstract:

In our adaptation of the olfactory search scenario by Balkovsky and Shraiman, we consider how smell might disperse in a city grid. Given this environment and a known velocity vector for the wind, we developed an analytical model for the most efficient search strategy for an odor source in a perfect city grid. Monte Carlo simulations show that the distribution of odor patches in a perfect city grid is Binomial. Given a hypothetical "sniffer" robot, we developed three search algorithms to find the source of the scent. After over 50,000 simulations, the active comb strategy was found to have the smallest search times.

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1 Introduction

Certain animals are able to detect odor patches in turbulent flows, and through an innate search strategy; they are able to find the odor sources allowing them to find food, mates, or even to avoid danger. Such animals like the moth and certain species of sharks are able to determine the location of an odor source by using their sense of smell and their ability to determine wind and ocean current directions. These kinds of olfactory search strategies can be useful to humans in order to find hazardous substances such as drugs, chemical leaks, or explosives (Balkoysky). They can even be implemented into robots in order to find these sources of odor in turbulent flows, where the odor is distributed unevenly and in patches whose concentration gradient does not necessarily point towards the source. This is of great interest, especially as it raises the questions of how these kinds of search strategies can be applied in urban settings, where there are buildings that disperse odor flumes differently than in an open field.

In turbulent flow, irregular wind fluctuations cause there to be constant changes in wind magnitude and direction and diffusion due to wind eddies (merrian-webster). These turbulent flows depend on a high ratio of inertial force and viscosity force, known as Reynolds number (MIT). Additionally, when far from the source, the concentration of the odor decreases and the time between detections increases (Balkovsky), which creates the need for elaborate search strategies. The basis for our project is the moth's olfactory search strategy, which been studied by Eugene Balkovsky and Boris I. Shraiman in their paper "Olfactory Search at High Reynolds Number" (2002). This article investigates the statistical aspects of the dispersal of odor in turbulent flows in an open field, and proposes different search algorithms in order to find the most time-efficient search method. Our project will be modeled with the main ideas of this article but applied to a more complex environment.

2 Literature Background

2.1 Modeling the Odor Plume

In order to come up with a simple model, the properties of order plumes must be examined. Odor patches arrive and are detected as bursts, and while the small scale structure of the burst can give some information about distance from the source, it would require much processing. Instead the burst (patch) is treated as a single event. The mean velocity of the wind is set by atmospheric conditions, which do not change on the time scale of odor movement. Odor molecules move according to local velocity with fluctuations around the mean velocity. Ultimately their motion is a random walk being pushing downwind. The fluctuations have a correlation length which can be estimated as the height about ground, and are Brownian at large scales of this length. The diffusion coefficient is given by eddy diffusivity, estimated as Lv_{rms}, where v_{rms} is the root-mean-square of velocity fluctuations. An odor patch stretches and diffuses as it moves which implies that odor patches have a finite lifespan. The probability of a patch to survive for time t in the flow is expected to behave as $e^{-\frac{tv_{rms}}{L}}$.

When considering that relatively long-lived odor patches moving in a random walk are biased downwind, a two-dimensional model can be constructed. Patches start at the source which is set to be at $(0,0)$ on the x,y-plane. Moving downwind means that after every move the patch will have increased along the y-axis and the random walk on the x-axis means there is an even chance (1/3) between moving left, downwind, or to the right. These two rules represent odor dispersion at length scales larger than L. A three-dimensional model is not essential as the random walk and distribution of patches will be unaffected. Figures 1 and 2 below show is a plume that was made by Balkovsky and Shraiman according to those rules. The distribution of patches has the form

$$p(r) = \frac{1}{\sqrt{4\pi D y}} \exp\left[-\frac{x^2}{4Dy}\right] \qquad [1]$$

Where $D = \frac{p_R + p_L}{2}$ is the eddy diffusivity, the boundary of the plume has the form $|x| \sim (Dy)^{\frac{1}{2}}$ and the probability of finding an odor patch when $x \gg (Dy)^{\frac{1}{2}}$ is extremely small.

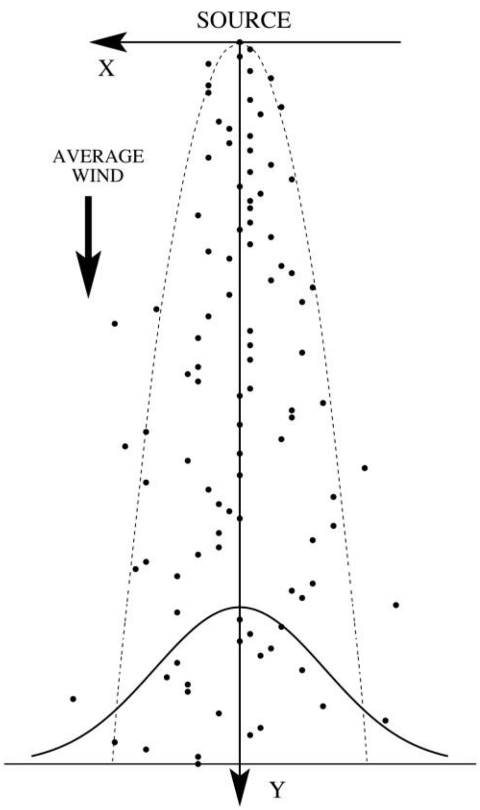


Fig. 1. The model odor field (dotted) and probability density function of patch distribution. (Balkovsky, Shraiman)

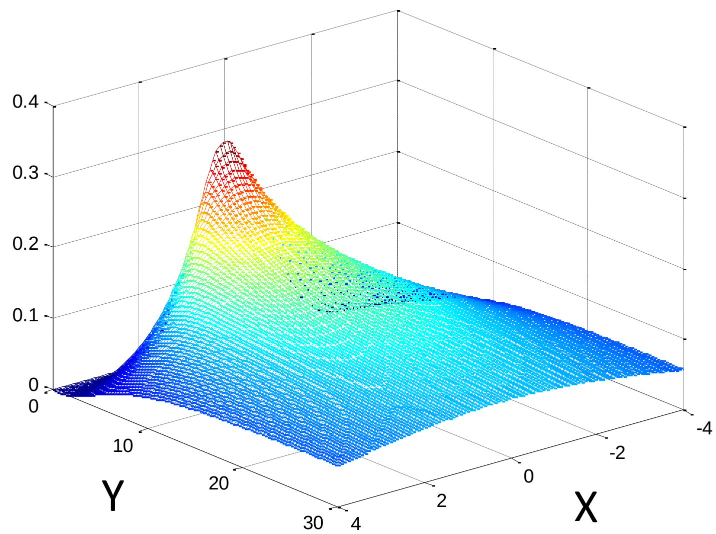


Fig. 2. The probability distribution function. Red indicates the most probable location of odor plume

2.2 Strategies

To ensure the odor-sensitive robot is designed to travel on the most time efficient path, and therefore uses the best possible search algorithm, in addition to modelling the behavior of the plume Balkovsky and Shraiman also developed a mathematical model and a series of strategies the robot might select. This search algorithm is relevant to any time-sensitive odor search such as sniffing for a bomb in in a crowded area. Before beginning the model, specific rules were established in order to maintain a structured and realistic model. Assuming the robot is not aware of the target prior to the first detection of odor, the robot does not begin searching until it gets the first whiff. To simplify modelling the robot's movement, each time step it is assumed to move one lattice step along the x or y-axis, and only travels upwind towards the source.

The first strategy examined by Balkovsky and Shraiman is a passive strategy. In this approach, the robot simply waits at a site until it detects an odor patch, moves to the location of the last patch and remains stationary until detecting another patch, repeating the process until eventually locating the source.

The second strategy described by Balkovsky and Shraiman involves a more active process of locating the scent source. Once a scent source is detected, the robot moves a unit upwind towards the direction that the scent patch was detected. From there, the robot begins a triangular zigzagging motion until the next scent patch is located, where the process is then repeated. The amplitude of each successive crosswind movement increases linearly as seen in the Figure 3 to right, ensuring that the robot covers every possible location of previous scent patches.

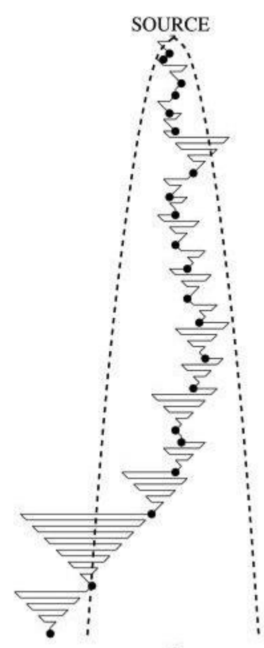


Fig. 3. Triangular Active Search Strategy

Finally, the third and most effective strategy that Balkovsky and Shraiman describe is a modification of the “active search” and of the second strategy. Instead of spending time searching the entire triangular-region of possible odor locations, the robot focuses on areas in which the probabilities are highest to encounter the next scent patch. In this case, the amplitude of each successive crosswind movement increases proportionally to the square of the distance from which the previous scent patch was located. This results in a parabolic search pattern with a greater net upwind velocity then the previous strategy (Figure 4).

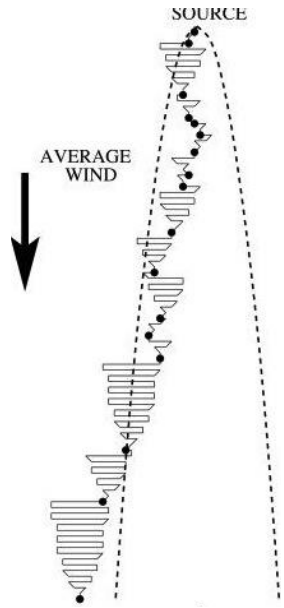


Fig. 4. Parabolic Active Search Strategy

2.3 Results and Conclusions

The passive strategy is the least efficient of the strategies, but will always lead to the source. The probability distribution function for this method is analytically calculated to be

$$\rho(t) = \frac{1}{\sqrt{2\pi\Delta}} \exp\left\{-\frac{(t-t_s)^2}{2\Delta}\right\} \quad [2]$$

$$t_s \propto y_0^{\frac{3}{2}} \exp\left(\frac{x_0^2}{4Dy_0}\right), \Delta \propto y_0^2 \exp\left(\frac{x_0^2}{2Dy_0}\right)$$

where (x_0, y_0) is the robot’s initial position, Δ is the probability distribution function variance and t_s is the typical search time. Further analysis of this probability distribution function shows that the search time will increase exponentially outside of the parabolic region, where $x_0 > 4Dy_0$ and where encountering an odor plume is least probable. This implies that the passive search is a poor strategy for robots located outside of the parabolic region and explains the distant robot’s tendency to get trapped outside of the parabolic region. Robots inside the parabola have a larger search time following the passive method than a robot actively searching for the scent due to the low probability of an odor patch travelling to one specific location, especially far downwind of the source. Obviously, in a dangerous situation like a bomb threat, the inefficiency of this method is far from ideal, though it does demonstrate the significant improvement to the algorithm caused by actively searching.

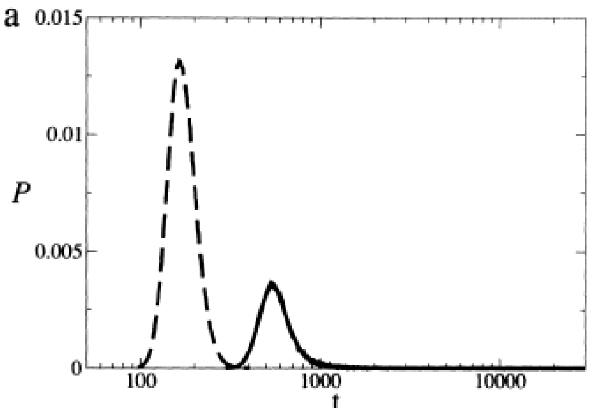
The probability distribution function of both of the “active search” strategies is

$$\rho(t) = \frac{1}{4\sqrt{\pi \mathrm{bt}}} \exp\left(-\frac{(t-t_s)^2}{2bt}\right)\left(1+\frac{t_s}{t}\right) \qquad [3]$$

The typical search time is dependent on the type of active strategy. The second, triangular active search strategy yields a typical search time of $t_s = a y_0^{\frac{5}{4}}$. The benefit of this strategy is that the typical search time is independent of the initial crosswind position of the robot. The downside, however, is the fact that this triangular movement toward the scent source spends time searching areas of low probability of odor particle detection.

The typical search time for the third, $t_s = a_2 y_0^{\frac{7}{6}}$, decreases during the parabolic active search: since the robot saves time by neglecting the statistically unlikely areas to find the odor plume. This search time shows that the modified “active search” strategy is less dependent on the downwind initial position of the robot. The typical search time is also independent of the initial crosswind position of the robot. The major drawback of this strategy comes in the fact that there is now a small possibility of the odor patch being missed due to not searching every possible scent location like in the triangular movement of the previous method.

To determine which strategy is the most effective, Balkovsky and Shraiman obtained numerical data through Monte Carlo simulations and overlaid plots of the histograms and typical search times for each search strategy (Figure 5). The modification of the “active” search strategy with the parabolic movement is shown to be the most efficient.



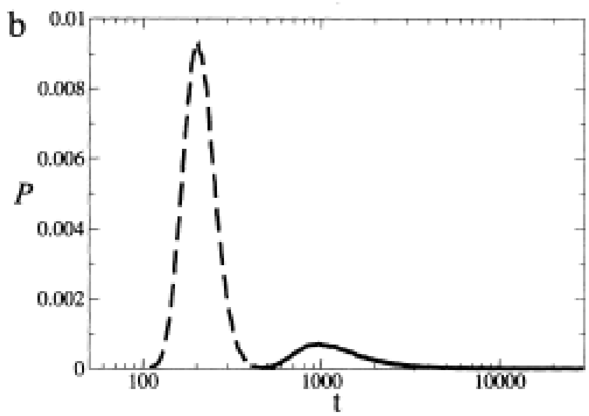


Fig. 5a. The histogram of the passive (solid line) and active (dashed) search strategies with initial robot position at (0, 50).

Fig. 5b. The histogram of the passive (solid line) and active (dashed) search strategies with initial robot position at (10, 50).

3 City Model

3.1 Model Assumptions

We began by defining the city grid as composed of equally spaced, tall, square buildings. Tall is defined as high enough from street level such that the more turbulent airflow above the buildings does not significantly affect the air flow at street level. The wind is presumed to flow in channels, such that on any given street, the wind flow can be approximated by the horizontal and vertical components of the wind vector. This assumption is sound given that the greater wind vector never runs parallel to one of the city streets. The wind must always come in at an angle; if the wind were parallel to one of the streets it would flow perpendicular to the faces of the buildings, this would result in eddies forming behind the structures, which would require an entirely separate analysis.

The scent source is regarded as a point in the center of one intersection, and releases one odor patch for each time interval. The odor patch travels in the direction of the component wind vector that affects it. When an odor patch reaches an intersection, the probability that it travel horizontally or vertically down a street is weighted by the velocity of that given wind flow. Additionally, the wind velocity is split into different magnitude x and y components depending on the angle of the wind. We assume that the robot is equipped with sensors that can tell the wind vector, the direction at which the odor patch hits it and where the buildings and streets are located.

3.2 Monte Carlo Simulations

Some preliminary Monte Carlo simulations were run to determine if odor distribution followed the same Gaussian distribution like in Balkovsky and Shraiman's models (Figure 6). In this specific simulation the wind vector points to the Northeast at a 45° angle from horizontal. It can be seen that the odor patches follow a parabolic distribution similar to the one presented in the moth scenario. This indicates that the assumptions made for the Monte Carlo simulation are accurate. More Monte Carlo simulations will be run to test the analytical models that we will derived.

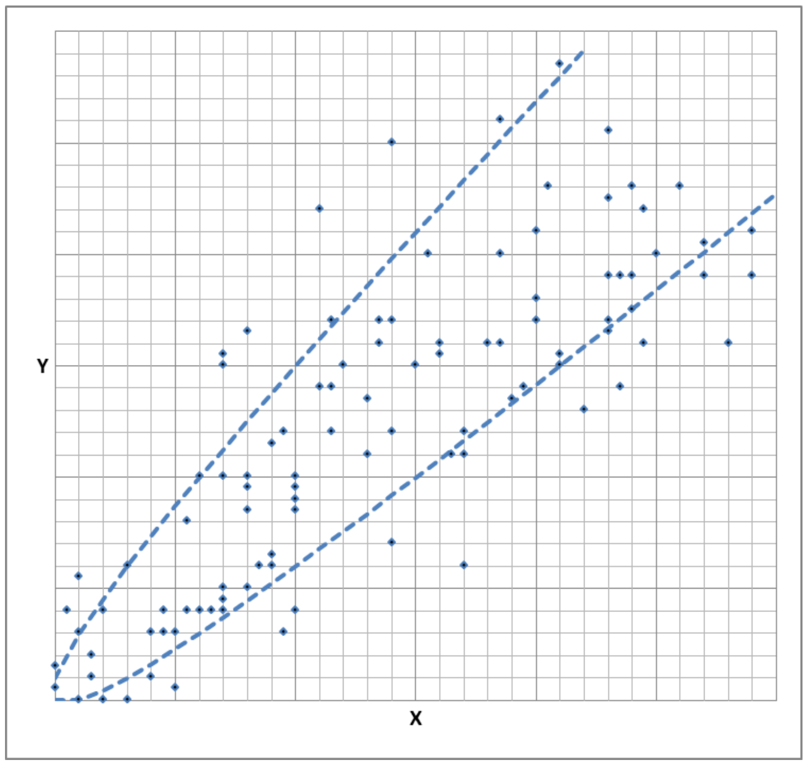


Fig. 6. Monte Carlo simulation with wind vector pointing from the Northeast

3.3 Analytical Model

In the city grid, buildings can affect the propagation of wind and therefore make detecting a scent source more difficult. Additionally, the distribution of the odor patches is no longer Gaussian due to wind obstructions, which requires us to find a probability distribution function that will model the wind distribution in the urban setting. We found that the binomial distribution does just this:

$$f(N,n) = \binom{N}{n} p^n q^{N-n} = \frac{(\mathrm{N})!}{\mathrm{n}!\,(\mathrm{N}-\mathrm{n})!} p^n (1-p)^{N-n}$$

The probability density function of the discrete binomial distribution gives the probability of getting *n* success in *N* trials, with *p* being the probability of success and *q* that of failure. We adapted the original binomial PDF to fit our model. First, we identified *p* as the probability of moving to the right, and *q* or *1-p*, as the probability of moving up in the city grid. The variable *N* is now equal to number of times step, which will be *x* + *y* since the robot can only move once in the x or y direction at each time step. This yields the new probability density function provided below:

$$f(x,y) = \binom{x+y}{x} p^x q^y = \frac{(\mathrm{x}+\mathrm{y})!}{\mathrm{x}!\,\mathrm{y}!} p^x (1-p)^y$$

Microsoft Excel was then used to visualize the binomial probability density and the cross section of the odor distribution (Figure 7).

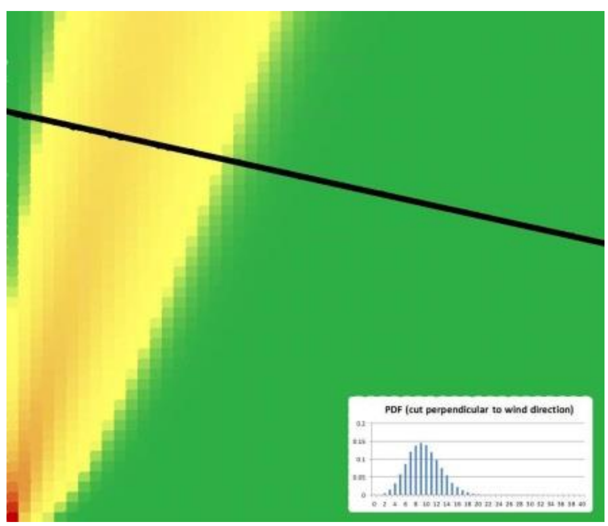
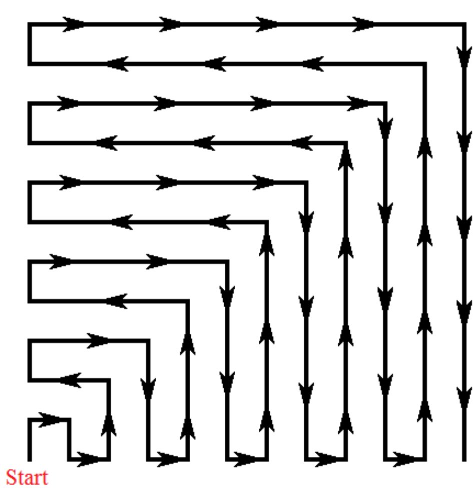


Figure 7: The odor plume with wind direction at 75º North of East and cross section of the distribution

3.4 Search Strategies

Three different search strategies were created. One is the *"Passive"* strategy, similar to the passive strategy found in the article by Balkovsky and Shraiman: here the robot will simply wait at one location until it catches an odor patch, and will then move in the direction from which the patch came from. The robot will move one grid spot over and resume waiting. The *"Comb"* strategy utilizes a zigzagging pattern, like that of the moth, and it centers its search on the wind gradient. The pattern widens its search distance after each zigzag, and resets the pattern each time it catches a new odor patch. The last search method is the *"Center,"* that casts the same distance in the *x* and *y* direction, and when it detects one scent patch it continues its search pattern before moving to that location. If several scent patches are detected, the robot will choose the farthest detected points (one on each axis) and will move to this coordinates as its new starting position and start casting again. In the case that a scent patch is not found, the robot will travel up-wind by one *x* and *y* unit.



a

Figure 9a: The "Comb" strategy with the wind at a 45º angle

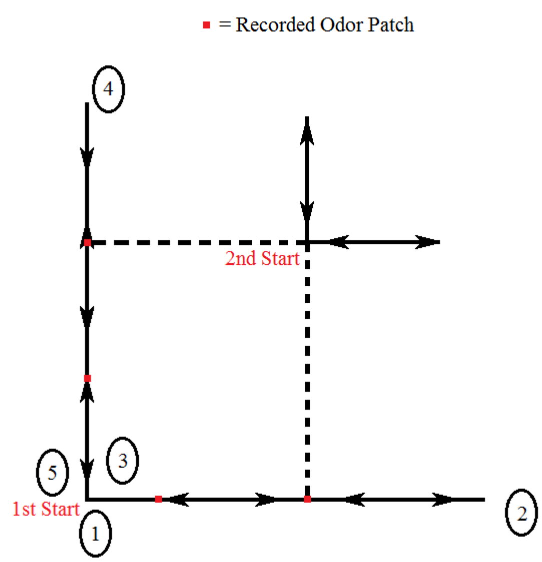


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Figure 9b: The "Center" strategy with the wind at a 45º angle

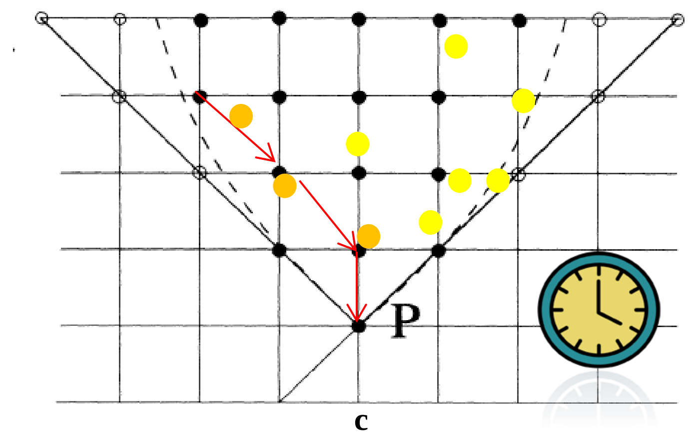

Figure 9c: The “Passive” strategy

4 Model Results:

Different simulations were run on the computer program Python to determine which of the three search strategies (active comb, active center, or passive) would be the most effective. These simulations were done for different wind vectors, and all had the initial condition inside the odor plume.

Here we have the data found for the different strategies with wind vector of 45º and 75º degrees, and the averaged search time of 50,000 simulations, as well as the number of failures to find the scent source, which indicates that the search took longer to find the source than the computer could compute. It should be noted that the average search time did not include the failed trials.

Search Type	Wind Vector	Average Search Time (50,000 Runs)	Failures (Out of 50,000)
Passive Search	45°	813.0 Time steps	116
	75°	353.6 Time steps	8
Active "Comb" Search	45°	208.2 Time steps	0
	75°	114.6 Time steps	0
Active "Center" Search	45°	822.3 Time steps	0
	75°	848.8 Time steps	55

Table 1: The average search times for each strategy at 45° and 75° angles from horizontal, as well as the number of program failures

From the table we can see that the "Passive" and "Center" search strategies both take a large amount of time to find the odor source; the passive search is particularly slow at winds near 45°, and had the most missed runs of the three search strategies. The "Comb" search strategy had the shortest time averages for both wind vectors, and always found the scent source, which is likely because it moves towards the source regardless of the detection of odor patches. Overall, we saw that the active "Center" method moved towards the source at the slowest rate, and was most likely to not locate the scent source very close to nearly-vertical and horizontal wind vectors, which we can observe from the data table where 75° wind vector had 55 failures, while the 45° had none. The slow rate of the "Center" search is due to the robot casting far outside the plume before returning and finding an odor patch.

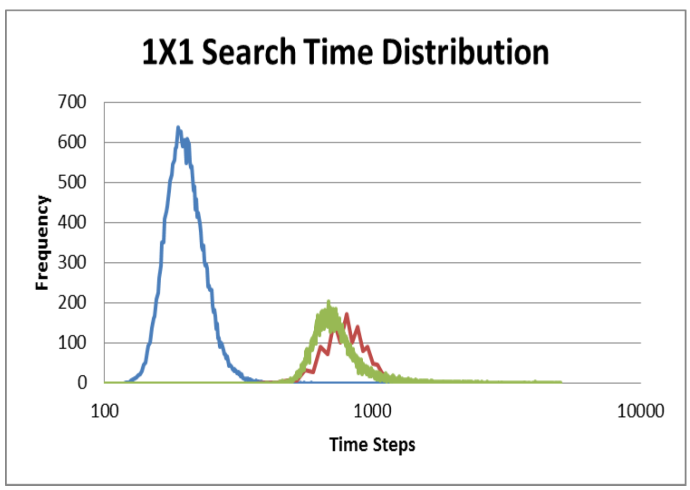

Figure 10a: Red: "Center" method, Green : "Passive" method, blue : "Comb" method at a 45° angle from horizontal

From the data table passive frequency plots were created (Figures10a and

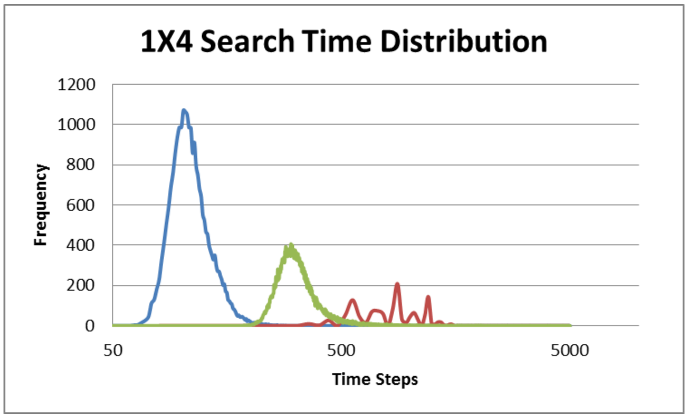

Figure 10b: Red: “Center” method, Green : “Passive” method, blue : “Comb” method at a 75° angle from horizontal

10b). The “Passive” method consistently performed better than the “Center” method for inside of plume searches, while the “Center” method has the advantage if the robot starts outside of the plume because it actively looks for odor patches and will move into the plume through its casting algorithm. The “Comb” method performed best overall. It should be noted that the search times should not be compared for different wind vectors because of the different initial starting positions of the robot.

5 Conclusions/Future work

In general, we concluded that out of the three search strategies the active “Comb” search would be the most efficient choice when looking for a scent source in a rural environment with building obstruction. The comb average search time was less than half the other two strategies, and always found the odor source.

One thing that could be added to the problem for future analysis is the use of multiple searching robots. While this would not affect the Comb search very much, the effects on the “Passive” and “Center” Searches would be more significant. Using the passive search, the robots would be able to form a line to increase the chances of encountering a scent patch. The “Center” search would similarly become much more effective as the casting time would be significantly reduced.

Another problem that could be addressed in future work is the linearity of the city grid. In our model, we assumed a perfectly even city grid of evenly spaced buildings and streets. Adaptations of this could include making some buildings larger as well as streets that change direction or have dead ends.

5 Potential Applications

Our results could be applied to many scenarios. Searching for explosives has many military applications from identifying IEDs to locating and disarming explosives in a civilian area. A city grid like this could also be adapted to represent a forest with the modeling giving insight into animal search patterns with obstacles like large trees or bodies of water. A third possible application would be for the search of drugs or chemical leaks.

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