Synchronization in Electrical Power Network

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Introduction

Power grids are vast complex networks that make up a large part of an infrastructure. It is crucial to consumers to have a reliable power grid that is also as efficient as possible. Many precautions are taken, and operators are hired to maintain reliability; however, seventy-five percent of power outages are caused by operator errors. These errors can be avoided by implementing automatic adjustments based on models of the grid system. Our model analyzes the stability of a power network with a series of 5 generators with randomness added to the generator characteristics. We then found the requirements for optimal system stability.

Synchronization

Synchronization is important because it ensures quality, reliability, as well as optimizing efficiency within the system. By synchronizing generators within a grid, it ensures no destructive interference occurs. This then causes a more consistent energy supply to the consumers since all generators are working in unison. Finally, not only will the grid not have destructive interference, constructive interactions will occur which increases the total power the grid can produce which optimizes the grid.

It is first important to define what a synchronized state is. In a system containing n generators, synchronization is defined by:

$$\dot{\delta}_1 = \dot{\delta}_2 = \dot{\delta}_3 = \cdots = \dot{\delta}_n$$

Where $\delta_i = \delta_i(t)$ represents the rotational position of the i^{th} generator and dot represents a time derivative. This definition of synchronization is important as it ensures the properties discussed above about the system.

The power grid can then be broken down into two pieces, the generators and the transmission lines between generators. This characterization can fully define the grid as it allows us to study both the dynamics of each generator individually, as well as the effects it causes on the other generators (with the alterations caused by transmission lines).

Each individual generator follows the law of conservation of angular momentum. This equation describes the motion of the generators at the most basic way possible:

$$J_i \frac{d^2\delta_i}{dt^2} = T_{mi} - T_{ei}$$

Where J_i is the moment of inertia of the i^{th} generator and T_{mi} and T_{ei} are mechanical torque created and electrical torque out of the i^{th} generator respectively. This equation can be converted into an energy balance by multiplying both sides by ω_i, the angular frequency of the i^{th} generator. It can then be rewritten to:

$$J_i \omega_i \frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei}$$

By recognizing that $\frac{J_i\omega_i^2}{2}$ is the kinetic energy of the system and assuming that the system is operating very close to synchronization where ω_R is the reference frequency, we can rewrite a final time to:

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei}$$

Where H_i is the inertia constant of the i^{th} generator. This equation gives some initial insight into the dynamics of the system. When the system is in equilibrium, power created equals power demanded, and $\dot{\delta}_i$ is close to ω_R it is sufficient to assume we are in a synchronous state. However if P_{ei} becomes too large, the angular momentum of generator must increase in order to compensate until P_{ei} returns to an equilibrium state.

We would like to focus on the short-term stability of these power network systems. To begin, we linearize the swing equation given earlier to:

$$\frac{2H_i}{\omega_r}\frac{d^2\delta_i'}{dt^2} = \frac{\partial P_{mi}}{\partial \omega_i}\omega_i' - \frac{\partial P_{ei}}{\partial \omega_i}\omega_i' - \sum_{j=1}^{n}\frac{\partial P_{ei}}{\partial \delta_j}\delta_j'$$

To do this we make some assumptions including that the effect the mechanical power has on the phase is negligible. The first term on the right corresponds to the droop equation, a common equation in power control. The second term follows a damping equation. The third term comes from the structure of the network and the interactivity between the generators.

From here, we take this series of equations and simplify them down to a coupled set of 2n equations:

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = -PX_1 - BX_2$$

Where the matrix P is a zero-sum matrix that is composed from the third term in the linearized equation and some leftover constants, and B is a diagonal matrix with elements $\beta_i = (D_i + {}^1\!/_{R_i})/2H_i$. Constants D and R come from the damping and droop equations respectively as seen in the first and second terms of the linearized equation. After finding this, we can then reduce it further to n-decoupled equations of 2 dimensions represented by:

$$\dot{\zeta}_j = \begin{pmatrix} 0 & 1 \\ -\alpha_j & -\beta \end{pmatrix}\zeta_j, \quad \zeta_j \equiv \begin{matrix} Z_{1j} \\ Z_{2j} \end{matrix}$$

Alpha is given by the eigenvalues of the matrix P and Beta is simply the common value of the diagonal elements of B.

From the matrix given earlier, we can find when the system is stable through the Lyapunov exponents of that matrix given by the equation:

$$\lambda_{j\pm}(\alpha_j, \beta) = -\frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4\alpha_j}$$

The entire power system is stable if and only if the real part of those exponents is negative. Because of this fact, we only need to consider the maximum real values of those exponents. If the maximum is less than zero, then every other value will be as well. From this, we want to find for which value of α_j can we minimize the maximum real part. As we analyze the behavior of the function $\lambda_{j\pm}$ with respect to α_j and fixed β, we find that the function is negative and decreases along the interval from $0 \le \alpha_j \le \frac{\beta^2}{4}$ and then remains constant past that interval. Past $\frac{\beta^2}{4}$, the square root portion of the equation turns complex, and no longer affects the real part any more than it has.

Now that we know how to minimize $\lambda_{j\pm}$, we want to know which α to choose. Again, a quick look at the behavior of the function reveals that it is smallest with the smallest non-negative value of α_j. That value is α_2. α_1 is a null eigenvalue, so we do not include that in this analysis of stability. Assuming a steady state and constant network system, α will be constant, so the only variable available to alter stability is β. Simple algebra gives the equation:

$$\beta = 2\sqrt{\alpha_2}$$

This is the ideal value of beta for system stability.

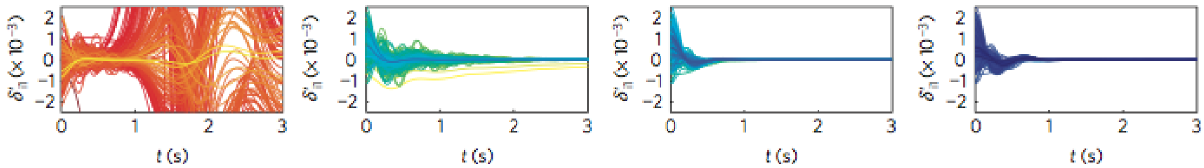
Enhancement of synchronization stability

Enhancement of the stability of the synchronous state happens by adjusting the parameters R_i (Drooping parameter), D_i (Damping parameter), H_i (Kinetic parameter) where $\beta = 2\sqrt{\alpha_2}$. The right side of the equation accounts for the network structure and depends on the generators, while β depends on the generator parameters R_i, D_i, H_i.

$$R_i = \frac{1}{4H_i\sqrt{\alpha_2} - D_i}, \quad i = 1, \dots, n$$

$$D_i = 4H_i\sqrt{\alpha_2} - \frac{1}{R_i}, \quad i = 1, \dots, n$$

The parameter R_i is altered for offline optimization of stability, when there is a slow change in demand. While, the parameter D_i is altered for online optimization of stability when there is rapid change in demand, usually caused by fluctuations in demand and faults.



The graphs above show the process of the enhancing the stability of the synchronous state. The first graph shows the network without β. The second graph is when β is first introduced to the network. The third graph shows the network with $\beta = 2\sqrt{\alpha_2}$. Finally the last graph is when perturbation is applied.

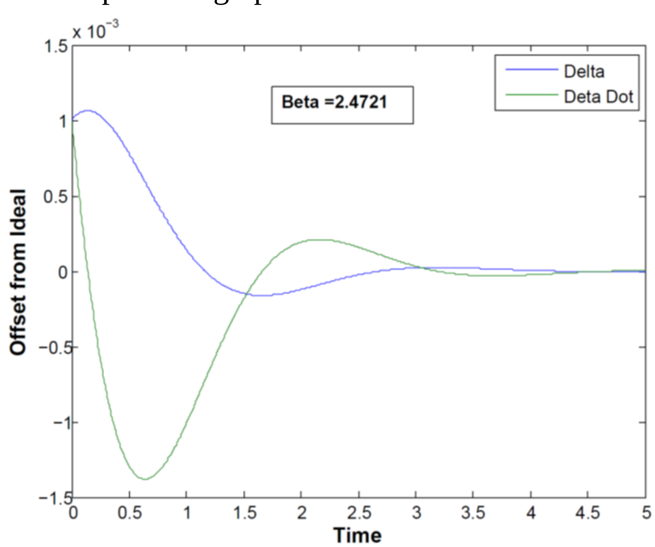
Numerical Simulation of System Stability

Our model is based around the simplified coupled set of 2n equations shown earlier of the form:

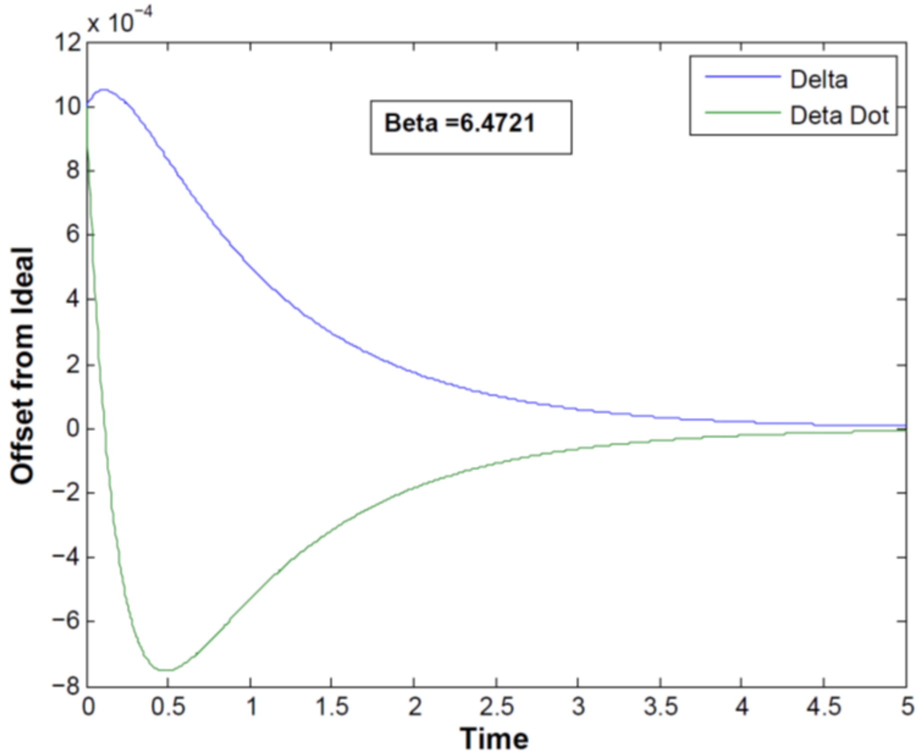
$$\dot{Z}_1 = Z_2$$
$$\dot{Z}_2 = -JZ_1 - \beta Z_2$$

To begin, we set up a "generator vector" in MATLAB using an arbitrary identical set of α variables. This vector represents the whole of every characteristic that affects stability. Afterwards, we add some randomness to the generator parameters with a simple random number generator with normal distribution and mean and standard deviation of 0 and 1 respectively. We then use this vector and use MATLAB's program "ode45" to solve for the unknown variable β contained within the B matrix. ODE45 is a versatile differential equation solver in MATLAB. The variable β in the model represents a common value for each generator. An assumption is made that the β value is similar for each generator which simplifies calculation greatly.

After this solution is found, we repeated this process for many different β values. For each β value we solved for, the resulting plot was integrated and the mean-squared error was calculated. This was our determining factor in finding the optimal β system parameter. For example, with a beta value set too low, we end up with a graph like this:



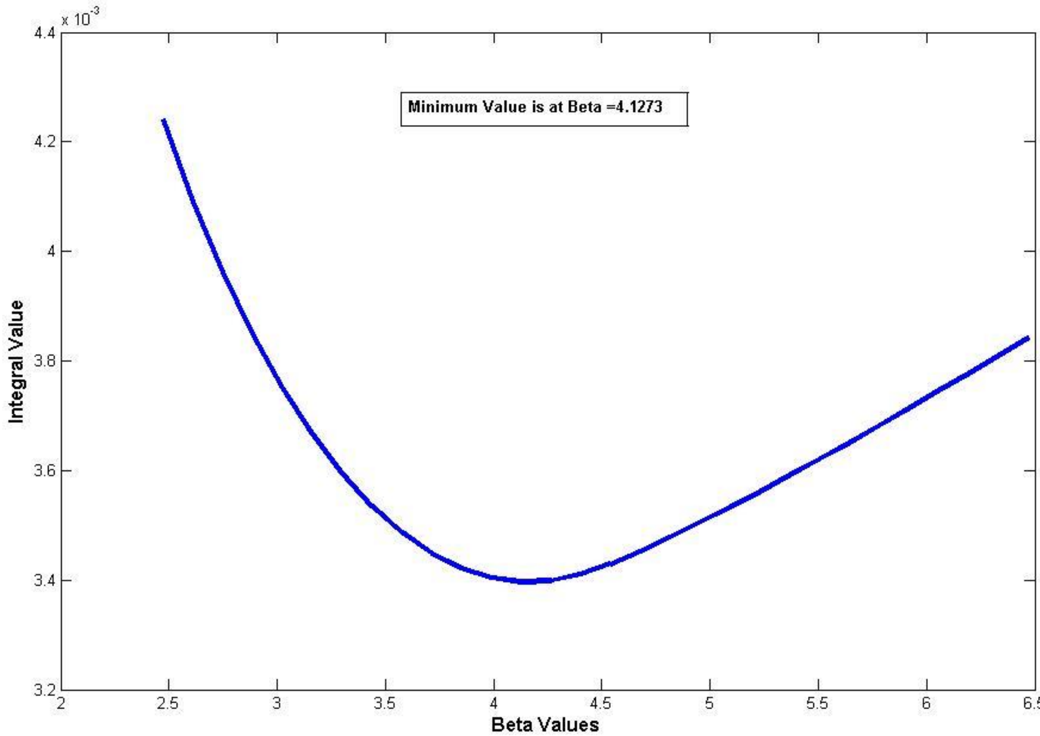
In this example, the system is not given a strong enough corrective force, governed by β, and it fluctuated around the stable point of zero. For a β that is too high, the plot is much too rigid and does not move towards stability in a fast enough manner. This can be seen in a plot like this:



As mentioned, we used the mean-squared error to find the ideal β value. The beta with the lowest mean-square error is the most efficient value for bringing the system back into stability. To find this error, we turned to calculations using a trapezoid integration function. This method was chosen by the help of our mentor, Joseph Dinius.

Results

After integrating the functions against each other, we plotted the integral values and calculated the minimum value to find the ideal β for our starting α value. These integral values were plotted against their beta values and it turned into a graph such as this for α=5:



This agrees with the findings of the research paper. The value for β we found coincides fairly closely with the value we found for ours. Accounting for variation due to randomness that was added, our calculated values for the ideal beta in the α=5 case fall within 10% error of an ideal $\beta = 2\sqrt{\alpha_2}$.