Energy Flows in Electric Grids

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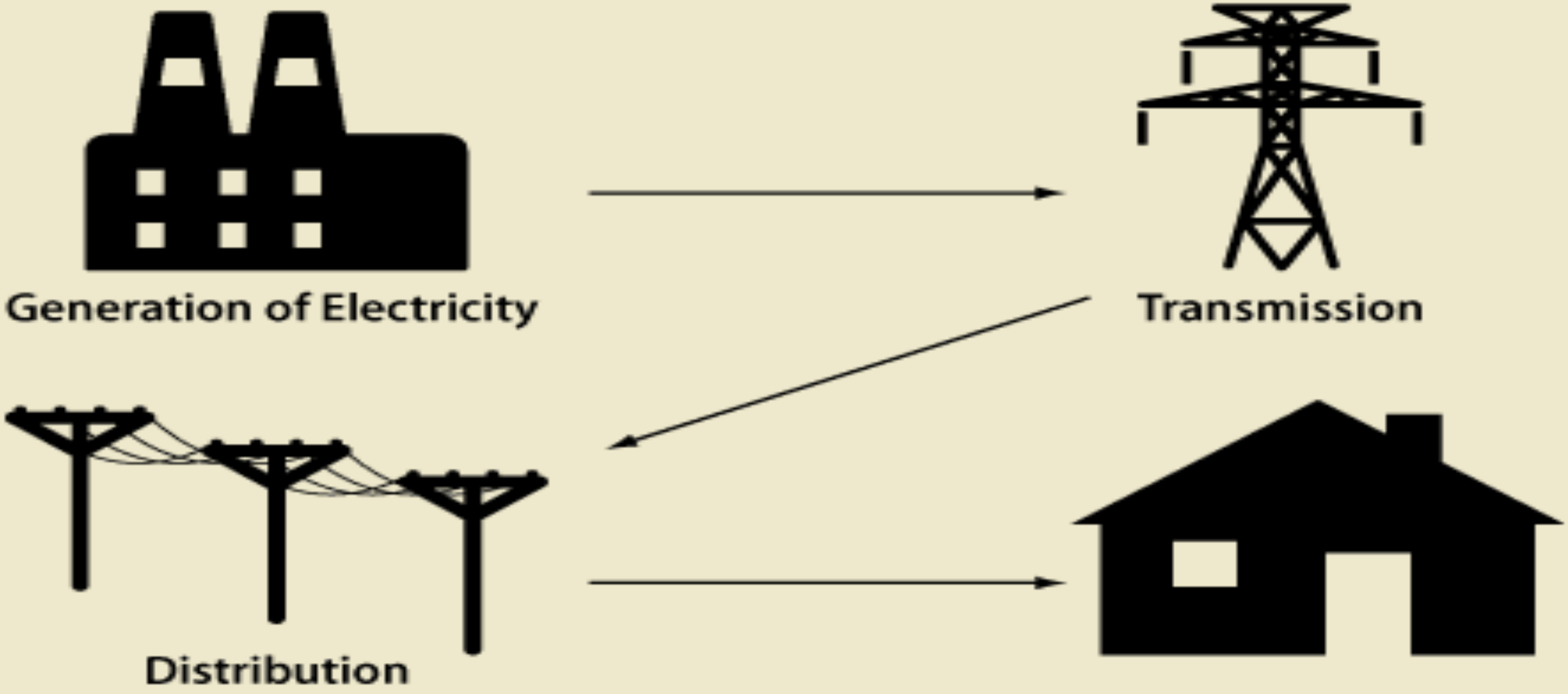
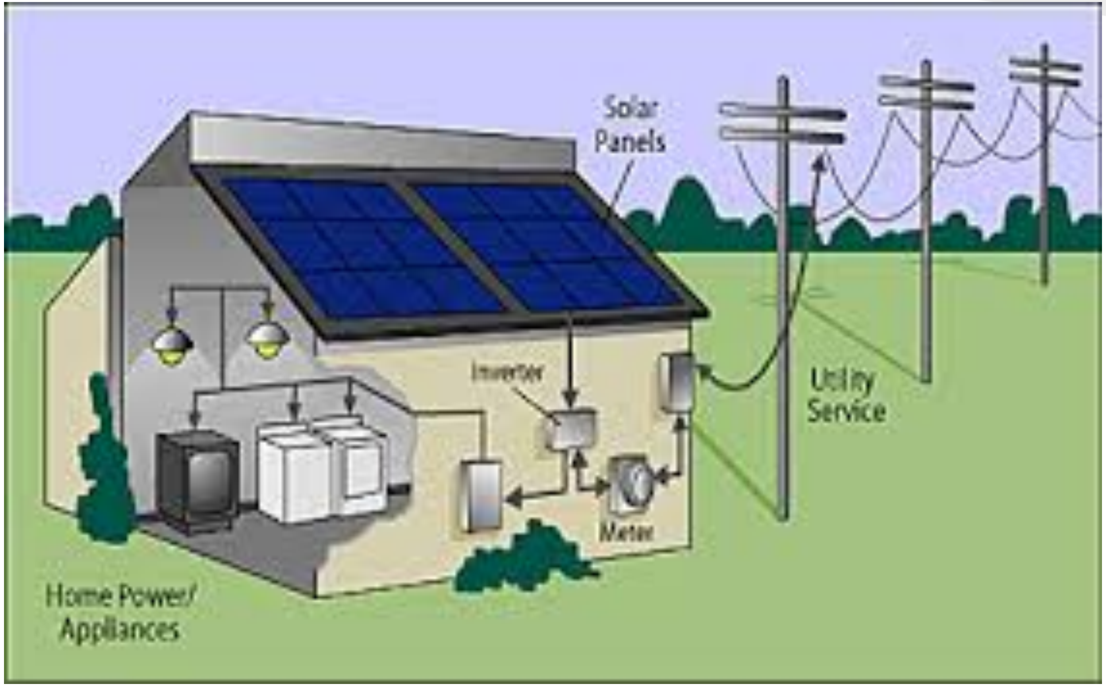
How Electricity Flows To Its Users



Source: Energy Information Administration.

Application



Background

- Producers
  - power plants
  - self producers

- Consumers
  - Positive or negative
  - Amount varies

Background

- Real Power = I^2R
  - Where I = current & R = resistance
  - Energy that can be used to do work

- Reactive Power = I^2X
  - Where I = current & X = reactance
  - Reactive power flow is needed in an alternating-current transmission system to support the transfer of real power over the network.

Motivation

- Maximize production per length
  - Concerned about power flow
  - Situation: consumers and producers

- Create low-parametric ODE model of a feeder

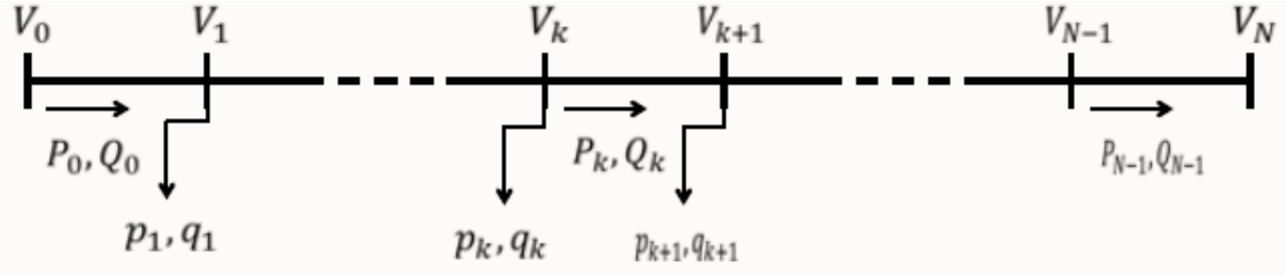
Motivation-the Problem

- Theoretical vs. Practical
  - power set to constant in the paper
  - power consumption varies

- Add randomness into the system
  - stochastic process
  - perturb both real & reactive power

Developing the Model

Diagram and notations below is the general idea for the energy flows in electrical grids.



DistFlow Equations

$$
\begin{aligned}
P_{k+1} - P_k &= p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2} \\
Q_{k+1} - Q_k &= q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2} \\
v_{k+1}^2 - v_k^2 &= -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{v_k^2}
\end{aligned}
$$

k = 0, 1, …, N -1 enumerates buses of the feeder (k = consumer)

Pk, real power flowing from bus k to bus k + 1. Qk, reactive power flowing from bus k to bus k + 1

pk, qk are net consumption for consumers

rk, xk represent the resistance and inductance of the line element connecting k and k + 1 buses

with Boundary Conditions: $P_{N+1} = Q_{N+1} = 0, \quad v_0 = 1$

Homogenization

1. Large number of consumers ($N >> 1$)
2. Continuous form with limit $N \to \infty$
3. To simplify, we reduce the parameter

   set rk / xk to constant. rk = r(lk) / L and xk = x(lk) / L,

   * where L is the total line of feeder line
   * lk length of line from bus k to bus k + 1

4. Relating differences to derivatives, $F_{k+1} - F_k \approx F'(z)\, l_k / L$

DistFlow ODEs (B.V.P.)

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2+Q^2}{v^2} \\ q - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix}$$

The DistFlow ODEs with the following mixed conditions

$$v_0 = 1, P(L) = Q(L) = 0$$

By Solving this ODE’s for known length of feeder line , one can evaluate real and reactive power and voltage along the given line.

Re-Scaling and Simplification of The B.V.P.

Assuming that p is constant, and changing from the flow densities P, Q, voltage, v, and position along the feeder line, z, to the new variables

A new variable s, this rescaling change the position of the line and the end of line in terms of s

it will then show the power at the beginning.

$$s = \frac{\sqrt{|p|r}}{v(L)}(L - z)$$

$$\varrho(s) = \sqrt{\frac{r}{|p|}}\frac{P(z)}{v(L)}$$

$$\tau(s) = \sqrt{\frac{r}{|p|}}\frac{Q(z)}{v(L)}$$

$$\upsilon(s) = \frac{v(z)}{v(L)}$$

DistFlow ODEs v.s. Rescale DistFlow ODEs (unit)

Rescale DistFlow ODEs

→ Calculate the initial value problems (dimensionless)

DistFlow ODEs

→ Calculate the boundary value problems (not dimensionless)

Dimensionless

$$\rho(s) = \sqrt{r/|p|} * P(z)/v(L)$$

Units of r, p, P, and v:

$$[r] = \Omega$$

$$[p] = v^2/\Omega$$

$$[P] = v^2/\Omega$$

$$[v] = V$$

$$[\rho] = \sqrt{[r]/[p]} * [P]/[v]$$

$$\sqrt{\Omega/|v^2/\Omega|} * (v^2/\Omega)/V = \Omega/V * V/\Omega = 1$$

τ & υ are also equal to 1.

They are unitless → dimensionless variable.

After Rescaling

we get the initial value problem equations as below:

$$-\frac{d}{ds}\begin{pmatrix} \rho \\ \tau \\ \upsilon \end{pmatrix} = \begin{pmatrix} \mathrm{sign}(p) - \frac{\rho^2+\tau^2}{\upsilon^2} \\ A - B\frac{\rho^2+\tau^2}{\upsilon^2} \\ -\frac{\rho+B\tau}{\upsilon} \end{pmatrix}$$

initial conditions

$$\upsilon(0) = 1, \varrho(0) = \tau(0) = 0$$

Rescaled Formulation

When q is constant, the rescaled DistFlow ODEs defines initial value problem stated in only one end of the s interval.

We solve it forward in the rescaled time, s : 0 → s_*, (stopping point) then arriving at

$\varrho(s_*)$, $\tau(s_*)$ and $v(s_*)$ n recomputes L, v, P and Q.

$$L = \frac{s_*}{v(s_*)\sqrt{|p|r}} \qquad v(L) = \frac{1}{v(s_*)}$$

$$P(0) = \frac{\varrho(s_*)\sqrt{|p|/r}}{v(s_*)} \qquad Q(0) = \frac{\tau(s_*)\sqrt{|p|/r}}{v(s_*)}$$

Compare DistFlow ODEs & Rescale DistFlow ODEs

DistFlow ODEs

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2+Q^2}{v^2} \\ q - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix} \qquad v_0 = 1, P(L) = Q(L) = 0$$

Rescale DistFlow ODEs

Eqs. (1) (2) allow efficient computation of the original mixed problem(DistFlow ODEs) for different value of the feeder length L by simply scanning s_*

$$-\frac{d}{ds}\begin{pmatrix} \rho \\ \tau \\ \upsilon \end{pmatrix} = \begin{pmatrix} \mathrm{sign}(p) - \frac{\rho^2+\tau^2}{\upsilon^2} \\ A - B\frac{\rho^2+\tau^2}{\upsilon^2} \\ -\frac{\rho+B\tau}{\upsilon} \end{pmatrix} \qquad \upsilon(0) = 1, \varrho(0) = \tau(0) = 0 \tag{1}$$

$$L = \frac{s_*}{\upsilon(s_*)\sqrt{|p|r}} \qquad v(L) = \frac{1}{\upsilon(s_*)}$$

$$P(0) = \frac{\varrho(s_*)\sqrt{|p|/r}}{\upsilon(s_*)} \qquad Q(0) = \frac{\tau(s_*)\sqrt{|p|/r}}{\upsilon(s_*)} \tag{2}$$

Case Studies

Set r =x=1, q=p/2

Case 1: standard case, p and q are negative along the feeder -- consuming

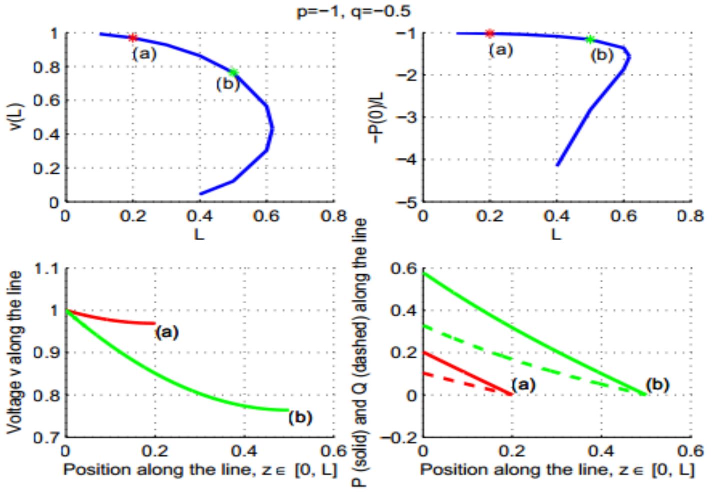


Fig. 1. The case of uniform distributed consumption of real and reactive powers.

Case Studies

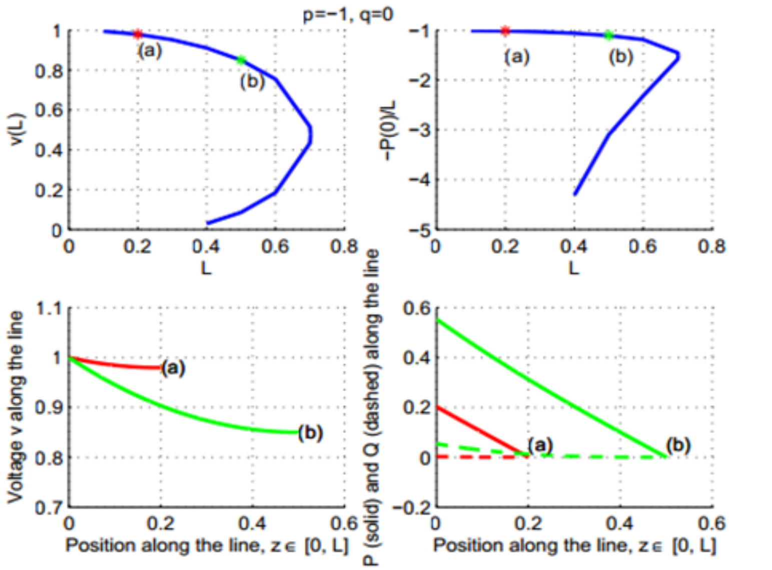


Fig. 2. The case of uniform consumption and zero-power factor reactive control.

Distributed Generation Graph

Case 2: when p and q are positive along the feeder -producing

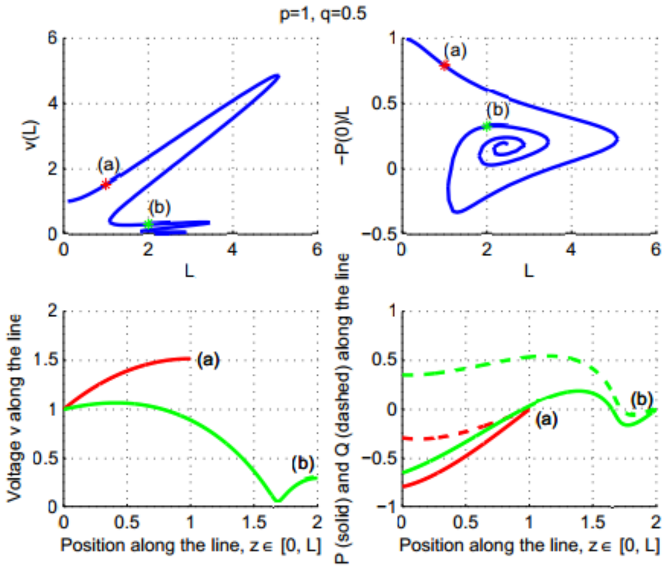


Fig. 4. The case of uniformly distributed and comparable generation of real and reactive powers

Previous Results

- A desirable system, in which the voltage along the line is satisfactory

- Undesirable scenario, in which the system experiences a power failure

The Plan From Here

Goal: telling the story of what happens when all consumers are all producers but also not everyone being the same - adding randomness only perturbing the real component

-Can do from here:

1. Testing the results from previous studies.
2. Cases of a large amount of individual producers and their effect on the grid
3. Test real and reactive power independently and perturb them independently
   -Add randomness in the system (stochastic process)

Any questions?

Thank you!