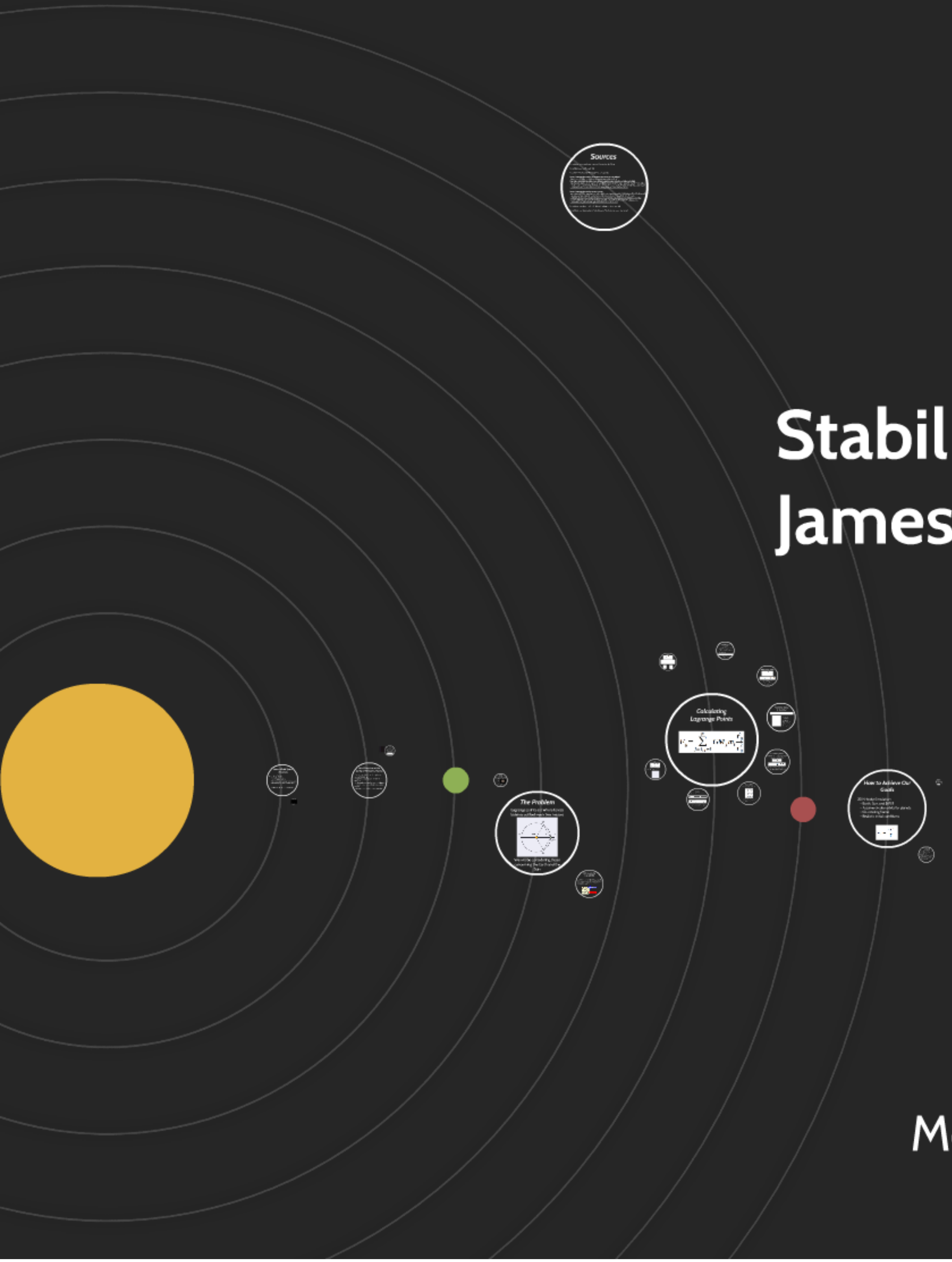
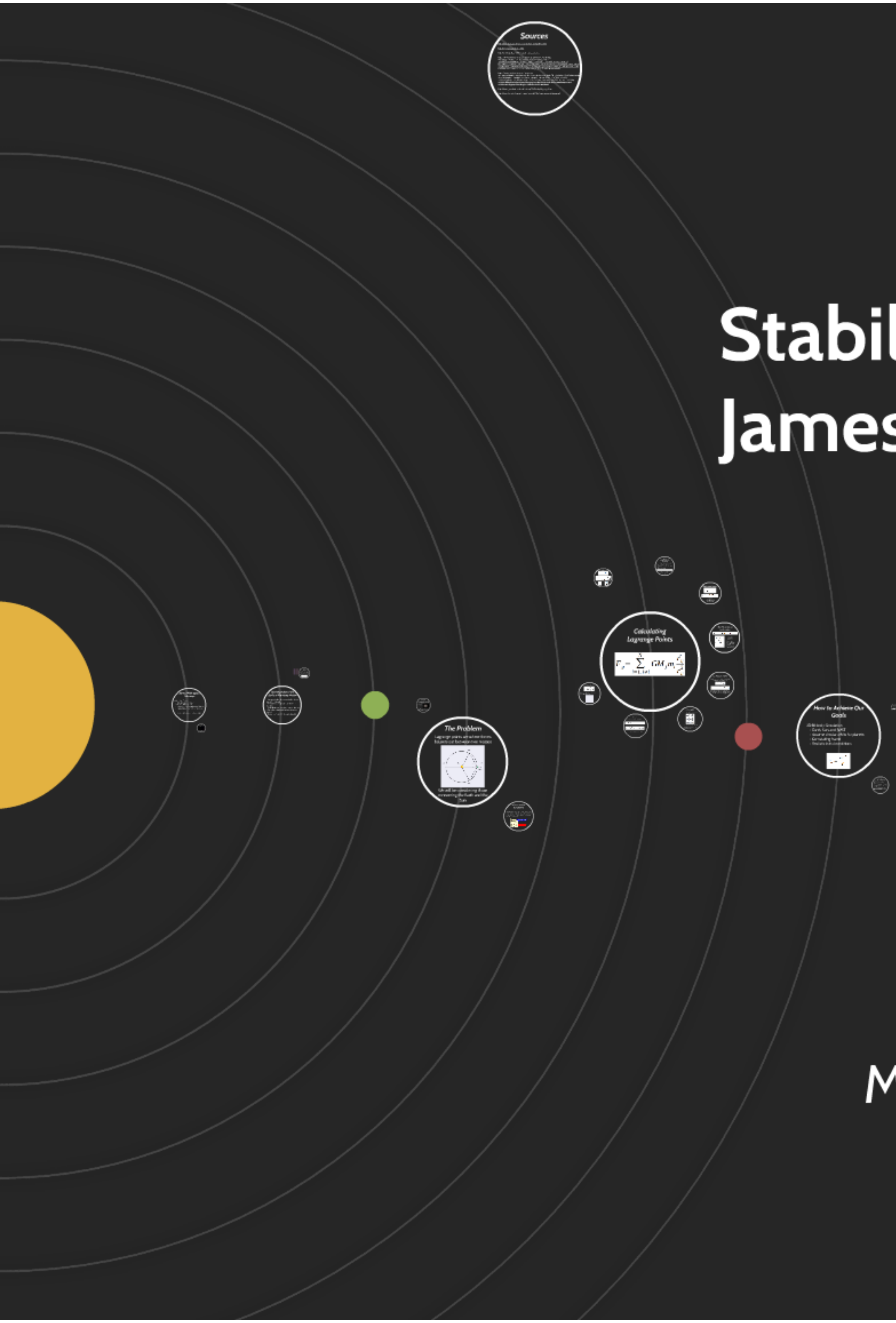


Stability of Lagrangian Points: James Webb Space Telescope

Gianna Cacolici
Jake Hanson
Cassandra Lejoly
Kyle Pearson
Katie Reynolds

Mentor: Alexander Young

Stability of Lagrangian Points: James Webb Space Telescope



Gianna Cacolici
Jake Hanson
Cassandra Lejoly
Kyle Pearson
Katie Reynolds

Mentor: Alexander Young

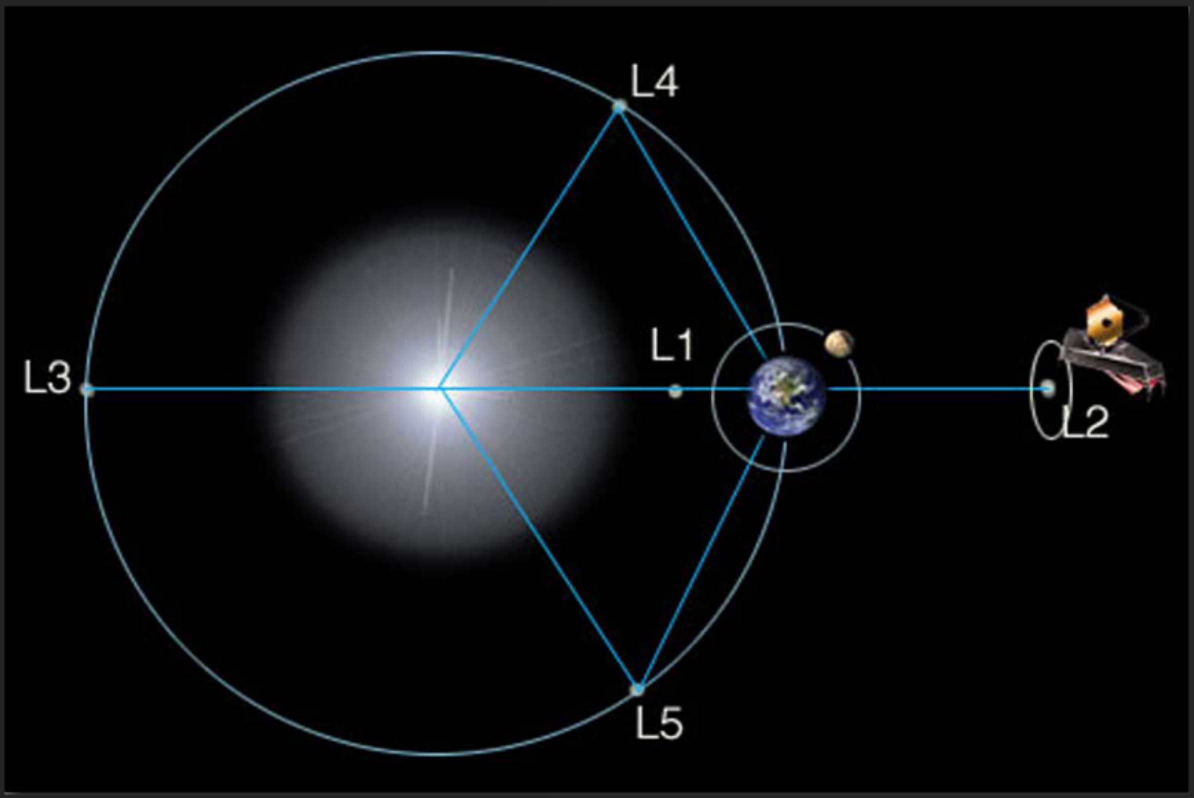
James Webb Space Telescope

Main Science Themes:

- First Light and Reionization
- The Assembly of Galaxies
- Birth of Stars and Protoplanetary Systems
- Planetary Systems and the Origin of Life

Studies the Universe in infrared light

Introduction to Lagrange Points



Where in space would it be ideal to place a telescope guaranteeing stability with minimal energy?

Brief Overview of the Study of Planetary Motion

-Ptolemy studied the motion of planets in a geocentric ideology
-Copernicus came up with a heliocentric ideology
-Tycho Brahe observed the motion of planets
-Kepler then developed his laws planetary motion
-Newton came up with his laws of gravity

Lagrange

1736-1813

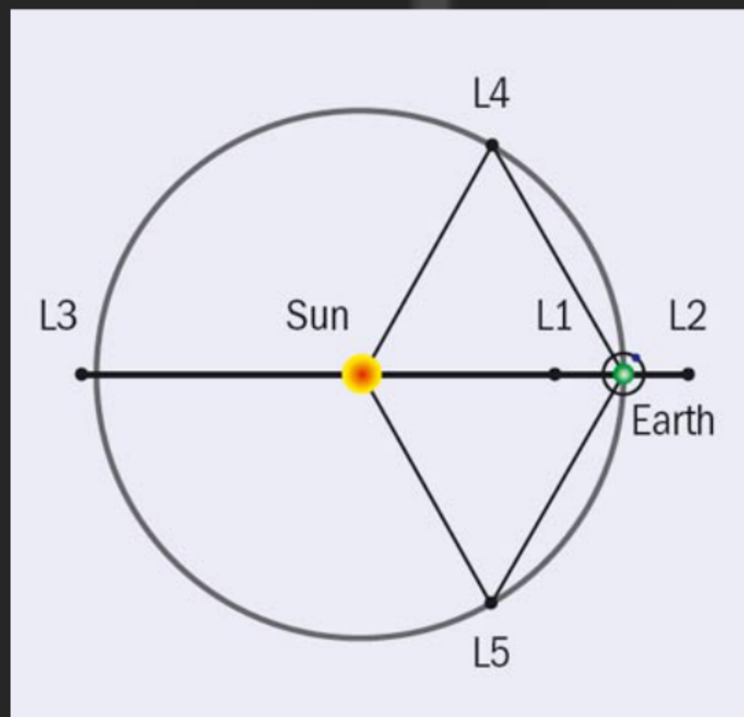
Wrote "Le Problème Des Trois Corps" in 1772 in which he attempts to solve the three body problem, which is not solvable analytically

"If the 'Académie' wishes to honor my work, it would be a great motivation to perfect my work; I don't lose hope to be able to form from my method a theory of the Moon as complete as we could ask from the still imperfect state of the analysis" – p230

$$i = \frac{\text{parall. } \odot}{\text{parall. moy. } ☾};$$

The Problem

Lagrange points are where forces balance out between two masses



We will be considering those concerning the Earth and the Sun

Why we want to test stability

Forces

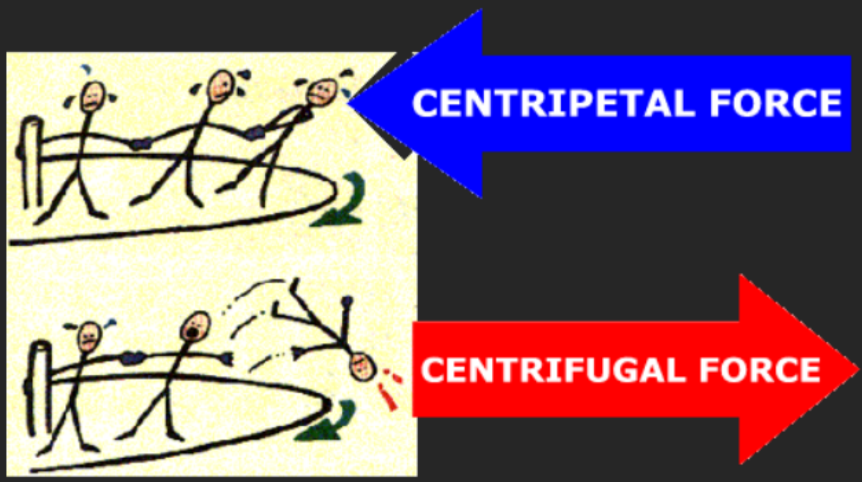
Gravity – "Let A, B, C be the masses of three bodies that attract each other proportionally in mass and inversely in the square of the distances" – p231

Coriolis -

Centrifugal – a fictitious force that draws a rotating body away from the center of rotation

Why we want to test stability

- To minimize the work of the JWST to stay in orbit
- If the telescope would orbit the point if perturbed
- How long the telescope would remain in orbit at L2
- For stable observations



Calculating Lagrange Points

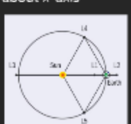
$$F_g = \sum_{j=1, j \neq i}^{N} GM_j m_i \frac{\hat{r}_{ij}}{r_{ij}^2}$$

L4, L5

$$L4: \quad \left(\frac{R}{2}\left(\frac{M_1 - M_2}{M_1 + M_2}\right), \frac{\sqrt{3}}{2}R\right),$$

$$L5: \quad \left(\frac{R}{2}\left(\frac{M_1 - M_2}{M_1 + M_2}\right), -\frac{\sqrt{3}}{2}R\right)$$

quidistant from sun and Earth, and ymmetric about x-axis



Co-Rotat

Origin at the center

velocity given

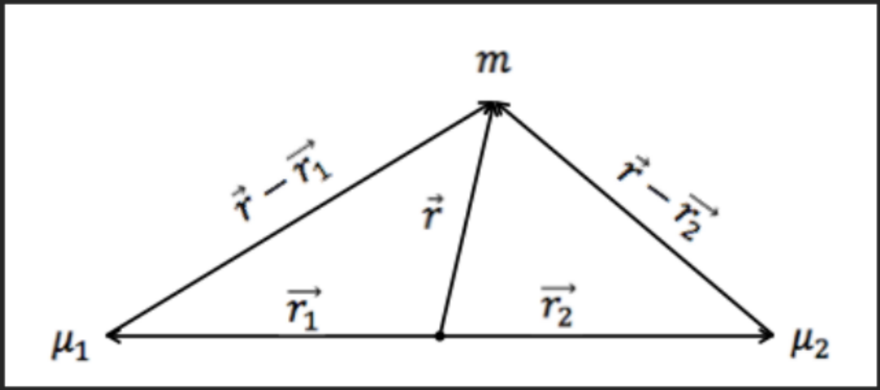
Effective force in

- Coriolis and

L1, L2, and L3

- To lowest order of alpha we find:

Stationary Frame



$$\vec{F} = \frac{-GM_1 m(\vec{r}-\overrightarrow{r_1})}{|\vec{r}-\overrightarrow{r_1}|^3} - \frac{-GM_2 m(\vec{r}-\overrightarrow{r_2})}{|\vec{r}-\overrightarrow{r_2}|^3}$$

- We seek solutions to F=0 that keep the relative locations of the three masses fixed in time
    - These stationary solutions are our Lagrange points

Co-Rotating Frame

Origin at the center of mass and an angular velocity given by Kepler's law:

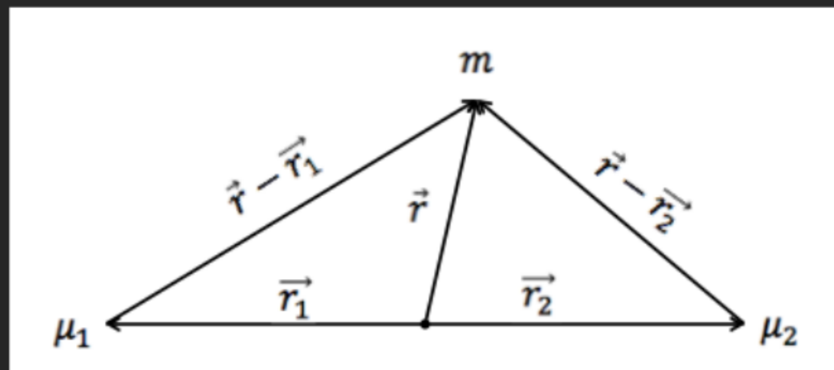
$$\Omega = \sqrt{\frac{G(m_1 + m_2)}{R^3}} = constant$$

$$\overrightarrow{F_\Omega} = \vec{F} - 2m\left(\Omega \times \frac{d\vec{r}}{dt}\right) - m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

Effective force in co-rotating frame

- Coriolis and Centrifugal force

Solving



$$\vec{F_\Omega} = \Omega^2\left(x - \frac{\beta(x+\alpha R)R^3}{((x+\alpha R)^2+y^2)^{\frac{3}{2}}} - \frac{\alpha(x-\beta R)R^3}{((x-\beta R)^2+y^2)^{\frac{3}{2}}}\right)\hat{\imath} + \Omega^2\left(y - \frac{\beta y R^3}{((x+\alpha R)^2+y^2)^{\frac{3}{2}}} - \frac{\alpha y R^3}{((x+\beta R)^2+y^2)^{\frac{3}{2}}}\right)\hat{\jmath}$$

$$\vec{\Omega} = \Omega\hat{k}$$

$$\vec{r} = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

$$\vec{r_1} = -\alpha R\hat{\imath}$$

$$\vec{r_2} = \beta R\hat{\imath}$$

$$\alpha = \frac{\mu_2}{\mu_1+\mu_2}$$

$$\beta = \frac{\mu_1}{\mu_1+\mu_2}$$

Simplifications: L1, L2, L3

- Set m=1 without loss of generality
- Let x=R(u+B) so that u is the distance from the Earth in units of R
- Set y=0 and solve:

$$u^2\left((1-s_1)+3u+3u^2+u^3\right)=\alpha(s_0+2s_0u+(1+s_0-s_1)u^2+2u^3+u^4)$$

- Alpha << 1 which is a realistic approximation

The Three Cases to Consider

$$u^2\left((1-s_1)+3u+3u^2+u^3\right)=\alpha(s_0+2s_0u+(1+s_0-s_1)u^2+2u^3+u^4)$$

3 cases are :

$(s_0, s_1) = (-1,1)$

$(s_0, s_1) = (1,1)$

$(s_0, s_1) = (-1,-1)$

s_o=sign(u)
s_1=sign(u+1)

In each case there is ***one*** real root thus giving us three Lagrange Points

L1, L2, and L3

- To lowest order of alpha we find:

$$L1: \left(R\left[1-\left(\frac{\alpha}{3}\right)^{\frac{1}{3}}\right], 0\right)$$

$$L2: \left(R\left[1+\left(\frac{\alpha}{3}\right)^{\frac{1}{3}}\right], 0\right)$$

$$L3: \left(-R\left[1+\frac{5\alpha}{12}\right], 0\right)$$

- L1 and L2 = 1.5 million kilometers from earth
- L3: Slightly farther out than Earth's orbit, directly opposite the sun

L4, L5

- Resolve Force into Components:

$$F_\Omega^\perp = \alpha\beta y\Omega^2 R^3 \left(\frac{1}{((x-R\beta)^2+y^2)^{3/2}} - \frac{1}{((x+R\alpha)^2+y^2)^{3/2}}\right)$$

$$F_\Omega^\parallel = \Omega^2 \frac{x^2+y^2}{R}\left(\frac{1}{R^3} - \frac{1}{((x-R\beta)^2+y^2)^{3/2}}\right)$$

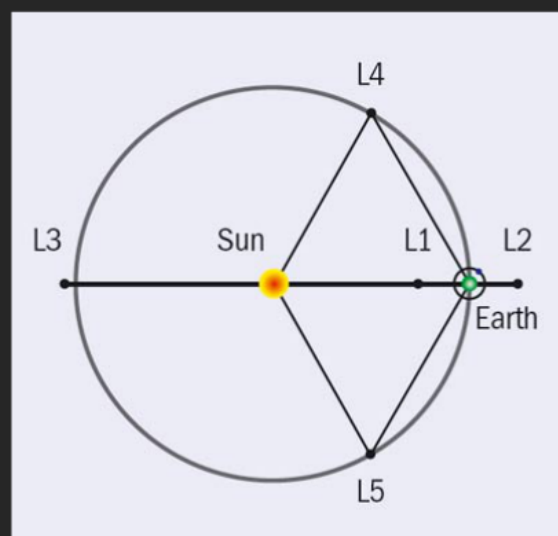
- Set components equal to zero and solve

L4, L5

$$L4: \quad \left(\frac{R}{2}\left(\frac{M_1 - M_2}{M_1 + M_2}\right), \frac{\sqrt{3}}{2}R\right),$$

$$L5: \quad \left(\frac{R}{2}\left(\frac{M_1 - M_2}{M_1 + M_2}\right), -\frac{\sqrt{3}}{2}R\right).$$

- Equidistant from sun and Earth, and symmetric about x-axis



How to Achieve Our Goals

2D N-body Simulation

- Earth, Sun, and JWST
- Assume circular orbits for planets
- Co-rotating frame
- Realistic initial conditions

$$v_o = \frac{\pi r^2}{T}$$

Simulation

-Python and C/C++
-Fourth Order Hermite Integrator (Predictor-Corrector)
-Higher order terms for accuracy
-Jerk to update Acceleration

$$\mathbf{j}_i = G \sum_{\substack{j=1 \\ j \neq i}}^{N} M_j \left[\frac{\mathbf{v}_{ji}}{r_{ji}^3} - 3 \frac{(\mathbf{r}_{ji} \cdot \mathbf{v}_{ji}) \mathbf{r}_{ji}}{r_{ji}^5} \right]$$

Testing Stability

-Validate physics by testing that the Earth orbits the Sun (assess energy budget)
-Get our co-rotating frame set up
-Test our analytic solutions
-Place the "space telescope" at those points in our code and let it run
-Slightly perturb initial conditions to see what happens to telescope
-Search for stability regions, classify fixed points and investigate what would happen after a longer period of time

Sources

http://www.nasa.gov/topics/universe/features/webb-l2.html

http://jwst.nasa.gov/launch.html

http://en.wikipedia.org/wiki/Joseph-Louis_Lagrange

https://www.google.com/search?q=centripetal+force+vs+centrifugal+force&espv=210&es_sm=93&source=lnms&tbm=isch&sa=X&ei=-BoZU5bDO8W42wXNroCwDQ&ved=0CAkQ_AUoAQ&biw=1366&bih=624#q=centrifugal+force&tbm=isch&facrc=_&imgdii=_&imgrc=ID9CnWzEAyQMQM%253A%3B5FqR-Ap6sRrPwM%3Bhttp%253A%252F%252Fwww.powermasters.com%252Fimages%252FCAROSEL.gif%3Bhttp%253A%252F%252Fwww.powermasters.com%252FCentrifugal_Force.html%3B488%3B268

https://www.google.com/search?q=lagrange+point&espv=210&es_sm=93&source=lnms&tbm=isch&sa=X&ei=9hsZU_HSMaWg2AXW2IDIAQ&ved=0AoQ_AUoAg&biw=1366&bih=624#facrc=_&imgdii=_&imgrc=XTNA_sHWTKGRuM%253A%3BfHX7iJq5mf6FuM%3Bhttp%253A%252F%252Fwww.labspaces.net%252Fpictures%252Fblog%252F4cd8b19d3cc591289269661_blog.jpg%3Bhttp%253A%252F%252Fwww.labspaces.net%252Fview_blog.php%253FblogID%253D895%3B400%3B386

http://www-gap.dcs.st-and.ac.uk/history/PictDisplay/Lagrange.html

http://www.ltas-vis.ulg.ac.be/cmsms/uploads/File/Lagrange_essai_3corps.pdf