Spontaneous Synchronization in Power Grids

Oren Lee, Thomas Taylor, Tianye Chi, Austin Gubler, Maha Alsairafi

Motivation

- Complex Power Grids
- Optimization
- Reliability

What is a Network?

- Generators
  - Nodes
- Transmission lines
  - Matrix

- What is synchronization

$$\dot{\delta}_1 = \dot{\delta}_2 = \cdots = \dot{\delta}_n$$

Building the Grid

- Nodes
- Conservation of angular momentum

$$J\frac{d^2\delta_i}{dt^2} = T_{mi} - T_{ei},$$

- Conservation of Energy

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei}$$

Network Structure

- Assumptions?
- Physical network vs Effective network
- Admittance (Y v Y_0)

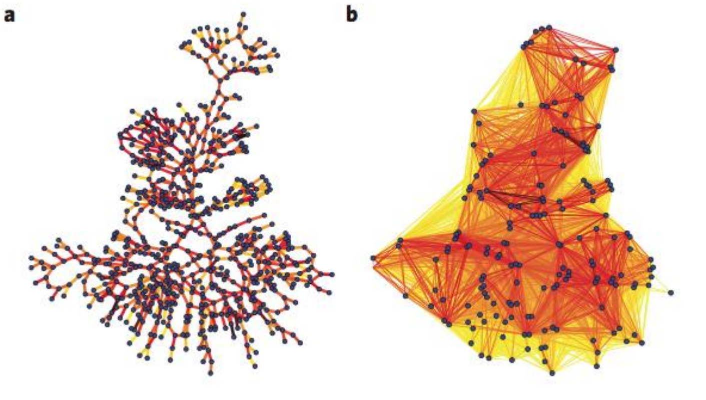
  $Y = \underline{P} / |V|^2$

  $Y_0=(Y_{0ij})$: Y_{0ij} is the negative of the admittance between nodes i and

  j /= i, and Y_{0ii} is the sum of all

  admittances connected to

  node i.



Stability condition from the swing equation

assume $\delta_i = \delta_i^* + \delta_i'; P_{ei} = P_{ei}^* + P_{ei}'; P_{mi} = P_{mi}^* + P_{mi}'$

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i'}{dt^2} = \frac{\partial P_{mi}}{\partial \omega_i}\omega_i' - \frac{\partial P_{ei}}{\partial \omega_i}\omega_i' - \sum_{j=1}^{n}\frac{\partial P_{ei}}{\partial \delta_j}\delta_j'$$

Term 1: $\partial P_{mi}/\partial \omega_i = -1/(\omega_R R_i)$

Term 2: $\partial P_{ei}/\partial \omega_i = D_i/\omega_R$

Term 3: $P_{ei}' = D_i\omega_i'/\omega_R + \sum_{j=1}^{n} E_i E_j (B_{ij}\cos\delta_{ij}^* - G_{ij}\sin\delta_{ij}^*)\delta_{ij}'$

coupled set of 2n first-order equation &
n decoupled two-dimensional systems

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \vec{\delta} \\ \dot{\vec{\delta}} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \vec{\omega} \\ \dot{\vec{\omega}} \end{bmatrix}$$

$$\dot{X}_1 = X_2 \tag{4}$$

$$\dot{X}_2 = -PX_1 - BX_2 \tag{5}$$

$$B = \frac{1}{2H_i} + \frac{D_i}{2H_i}$$

B is the diagonal matrix of elementsβ

$$P_{ij} = \begin{cases} \frac{\omega_R E_i E_j}{2H_i}(G_{ij}\sin\delta_{ij}^* - B_{ij}\cos\delta_{ij}^*), & i \neq j \\ -\sum_{k \neq i} P_{ik}, & i = j \end{cases} \quad (6)$$

Simplifying coupled equations

$$\dot{\zeta}_j = \begin{pmatrix} 0 & 1 \\ -\alpha_j & -\beta \end{pmatrix} \zeta_j, \quad \zeta_j \equiv \begin{pmatrix} Z_{1j} \\ Z_{2j} \end{pmatrix} \qquad (7)$$

From the different equation, this has the solution

$$(\vec{X}_1)e^{\lambda_1 t} + (\vec{X}_2)e^{\lambda_2 t}$$

How we guarantee stabilities?

By ensuring the REAL part is negative in both terms

Finding Stability from Matrix

- From ODE's, we can find requirements for stability
- Lyapunov Exponents:
- $\lambda_{j\pm}(\alpha_j, \beta) = -\frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4\alpha_j}$

Keeping λ Negative

- Need to keep all Re(λ) < 0
- Alter function to $\Lambda = \max(\mathrm{Re}(\lambda)) \le 0,\ j \ge 2$
- J=1 is a null eigenvalue
  - Related to shift in all phases

- How can we tune variable α to keep λ negative?

Function Behavior

- For α<0
  - Λ>0; not usable
- For α>0
  - Λ<0; usable
- For 0≤α≤$^{\beta^2}/_4$
  - $\Lambda < 0$; reaches min at $\alpha = {}^{\beta^2}/_4$
  - Past this function forms imaginary part

What value for α?

- A is based on the eigenvalues of the earlier matrix P
- Are dependent on generator in system network
- Want smallest non-zero value
    - Reduces any reduction in stability
    - Choose α_2

Parameters for changes in demand

- Droop Parameter: $\beta = \beta_{\text{opt}} \equiv 2\sqrt{\alpha_2}$

$$R_i = \frac{1}{4H_i\sqrt{\alpha_2} - D_i}, \quad i = 1, \ldots, n$$

- Slow changes in demand (Off-line optimization)

- Damping Coefficient:

$$D_i = 4H_i\sqrt{\alpha_2} - \frac{1}{R_i}, \quad i = 1, \ldots, n$$

- Rapid changes in demand (Online optimization)

Enhancement of the stability of synchronous states