Smallpox Epidemics

Graphical Sensitivity Analysis for an Epidemiological Model

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Epidemiology

Modeling spread of diseases

- Population Models
- Epidemic Aversion

Epidemiology

SIR Model

Basic Model for Population Epidemiology

$$\frac{dS}{dt} = -\beta IS$$

$$\frac{dI}{dt} = \beta IS - \nu I$$

$$\frac{dR}{dt} = \nu I$$

Epidemiology

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- Each variable S, I, R is a percentage of the total population, and thus are non-dimensional.

Epidemiology

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- Each variable S, I, R is a percentage of the total population, and thus are non-dimensional.
- The only stable solution is trivial (nobody is sick).

Sensitivity Analysis

Goal

To evaluate sensitivy of a model to the parameters describing it, i.e. to determine the amount that the entire model changes when each parameter is altered.

Used in models for which traditional analysis is impossible or inconclusive.

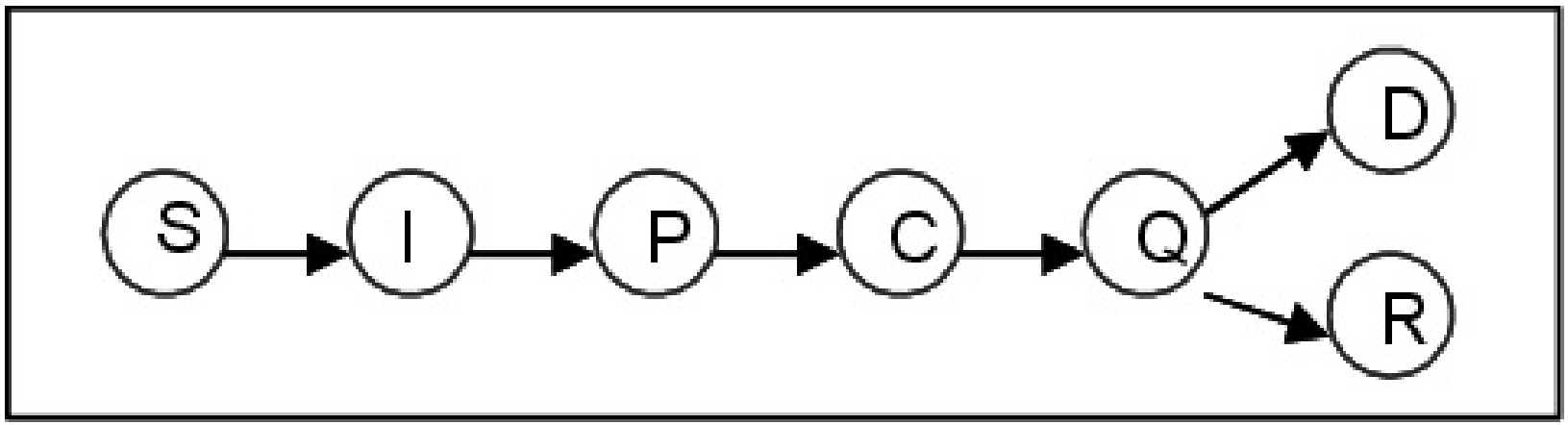
Smallpox Epidemic

Stages of Smallpox

1. Incubation (12-14 days after infection) - not contagious
2. Prodrome (2-4 days) - early symptoms and negligible infectivity
3. Fulminant- rashes appear and the patient is infectious within the next 7-10 days

Scabs fall off after 3 weeks but are still infectious.

Smallpox Epidemic



[1]

[1] Chen, Li-Chiou, et al. "Aligning simulation models of smallpox outbreaks." Intelligence and Security Informatics. Springer Berlin Heidelberg, 2004. 1-16.

Smallpox Epidemic

$$\frac{dS}{dt} = -\beta SC$$

$$\frac{dI}{dt} = \beta SC - \sigma I$$

$$\frac{dP}{dt} = \sigma I - \alpha P$$

$$\frac{dC}{dt} = \alpha P - \gamma C$$

$$\frac{dQ}{dt} = \gamma C - \nu Q$$

$$\frac{dD}{dt} = \lambda \nu Q$$

$$\frac{dR}{dt} = (1 - \lambda)\nu Q$$

Variables and Parameters

Eight Variables

S - Susceptible
I - Incubating
P - Prodrome
C - Contagious
Q - Quarantined
D - Dead
R - Recovered
t - Time

One Constant

N - Total Population

Note:

$$N = S + I + P + C + Q + D + R$$

Variables and Parameters

Six Parameters

β - Rate of Transmission

σ - Frequency of Incubation State

α - Frequency of Prodrome State

γ - Rate of Quarantine

ν - Frequency of the Course of the Disease

λ - Death Rate

Numeric Approximation of Solutions

System of ODEs numerically integrated using Matlab from $t = 0$ to $t = 100$.

Initial Conditions

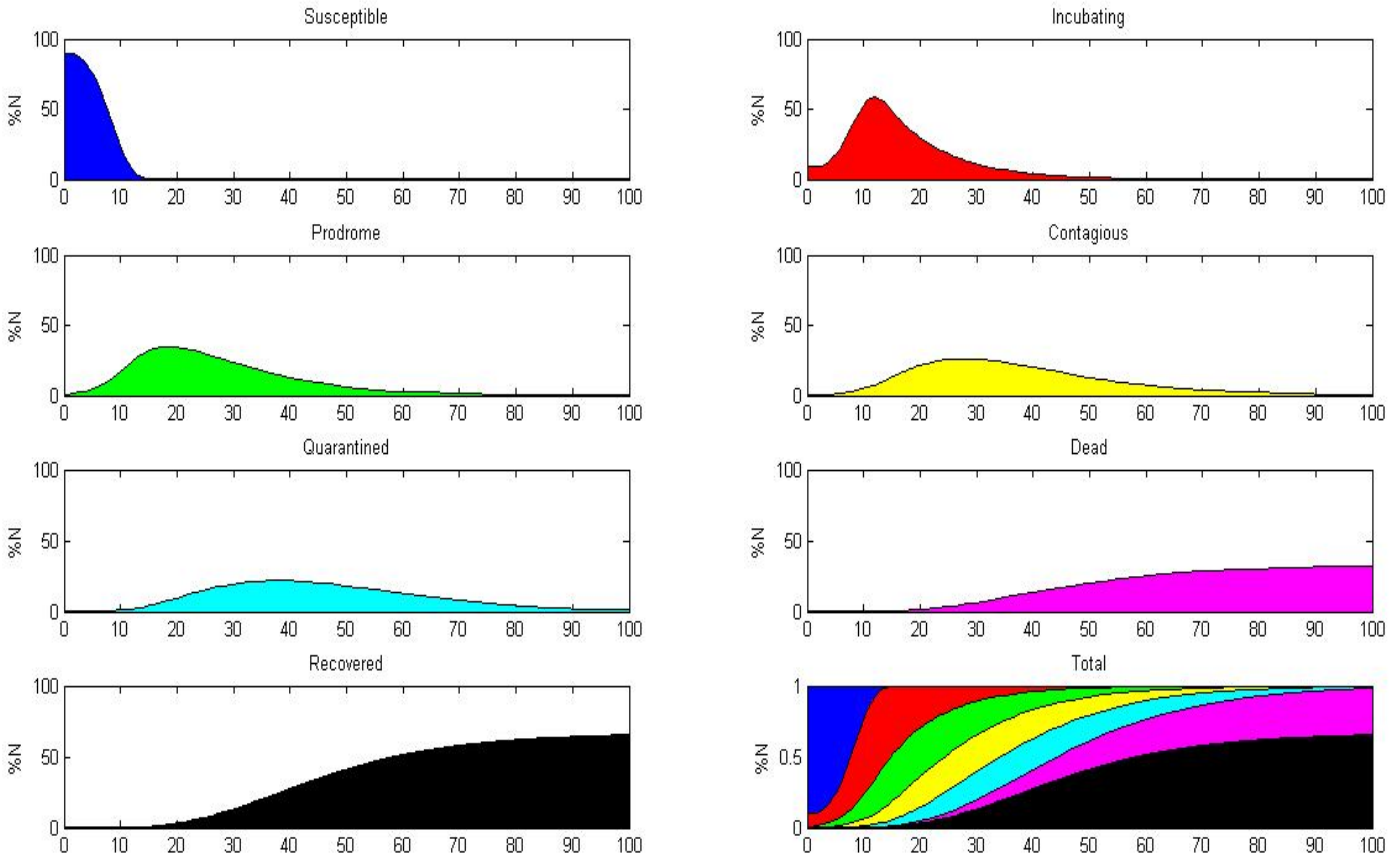
$S = 0.9$
$I = 0.1$
All other state vectors zero.

I.E., 10% of population gets infected.

$\lambda = .33$
All other parameters have value of 0.1

Numeric Approximation of Solutions

System of ODEs numerically integrated using Matlab from $t = 0$ to $t = 100$.



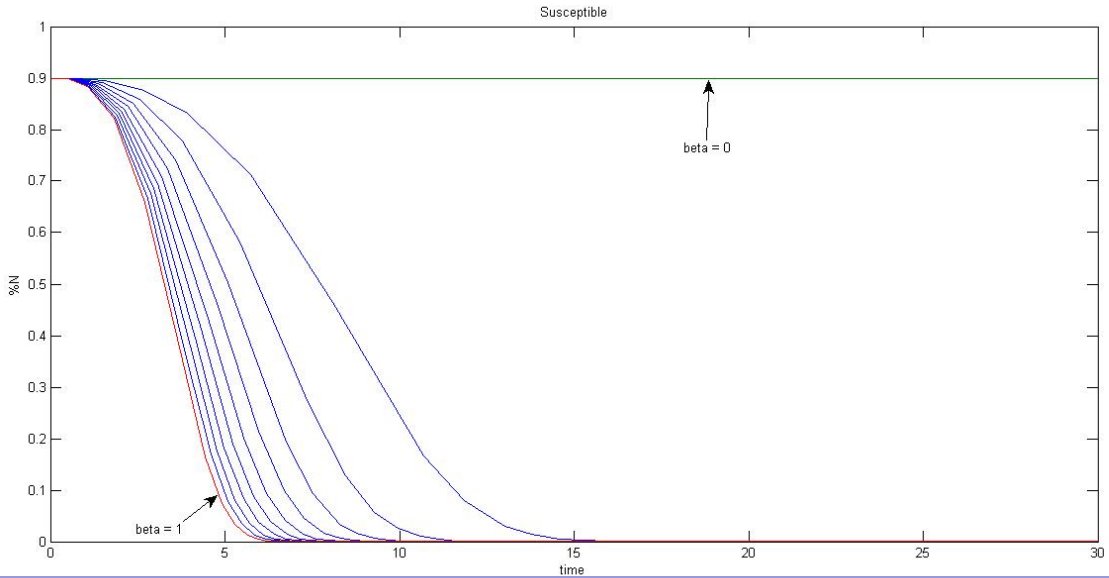
Introducing Graphical Analysis

Solutions of system as one parameter is changed, all others held constant.

Example

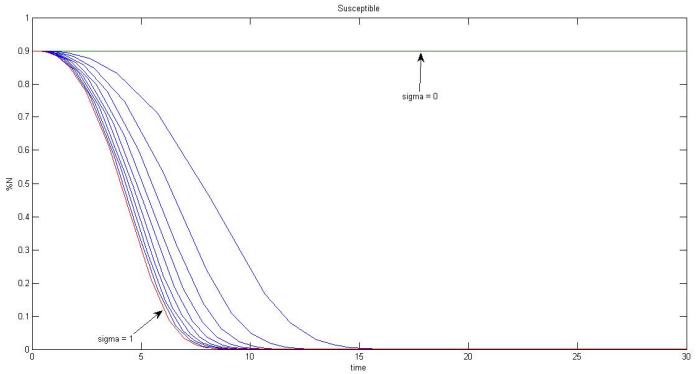
Plots of S, integrated from $t = 0$ to $t = 30$, as particular parameters are iterated from 0 to 1 by an interval of 0.1.

S as a function of time at different values of β

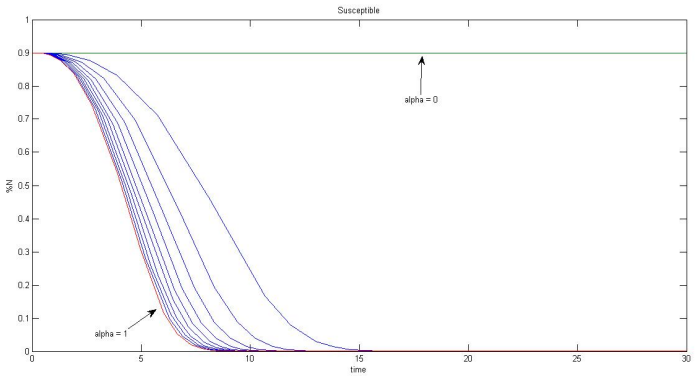


Introducing Graphical Analysis

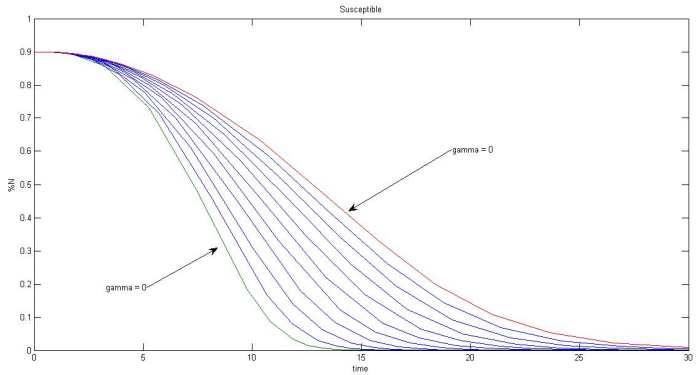
S as a function of time at different values of σ



S as a function of time at different values of α

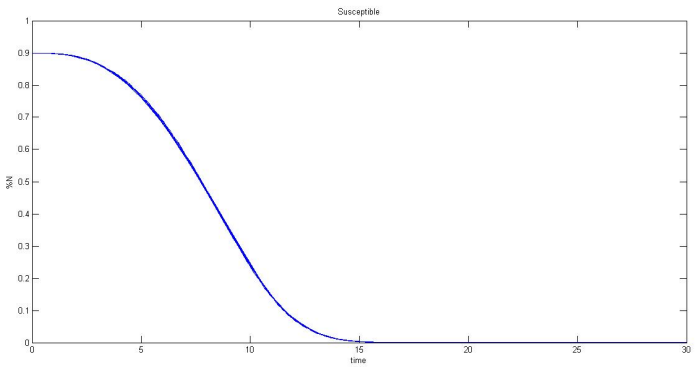


S as a function of time at different values of γ

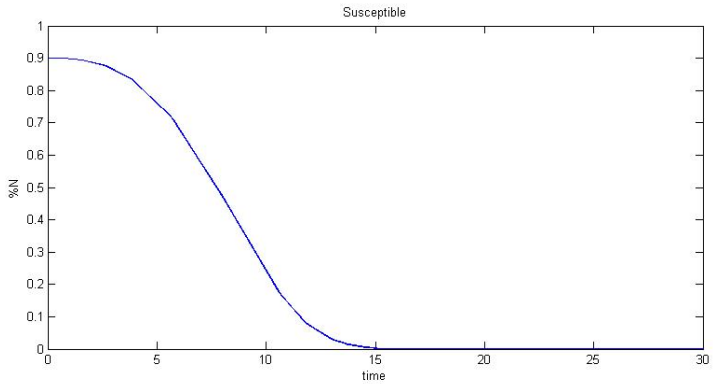


Introducing Graphical Analysis

S as a function of time at different values of ν

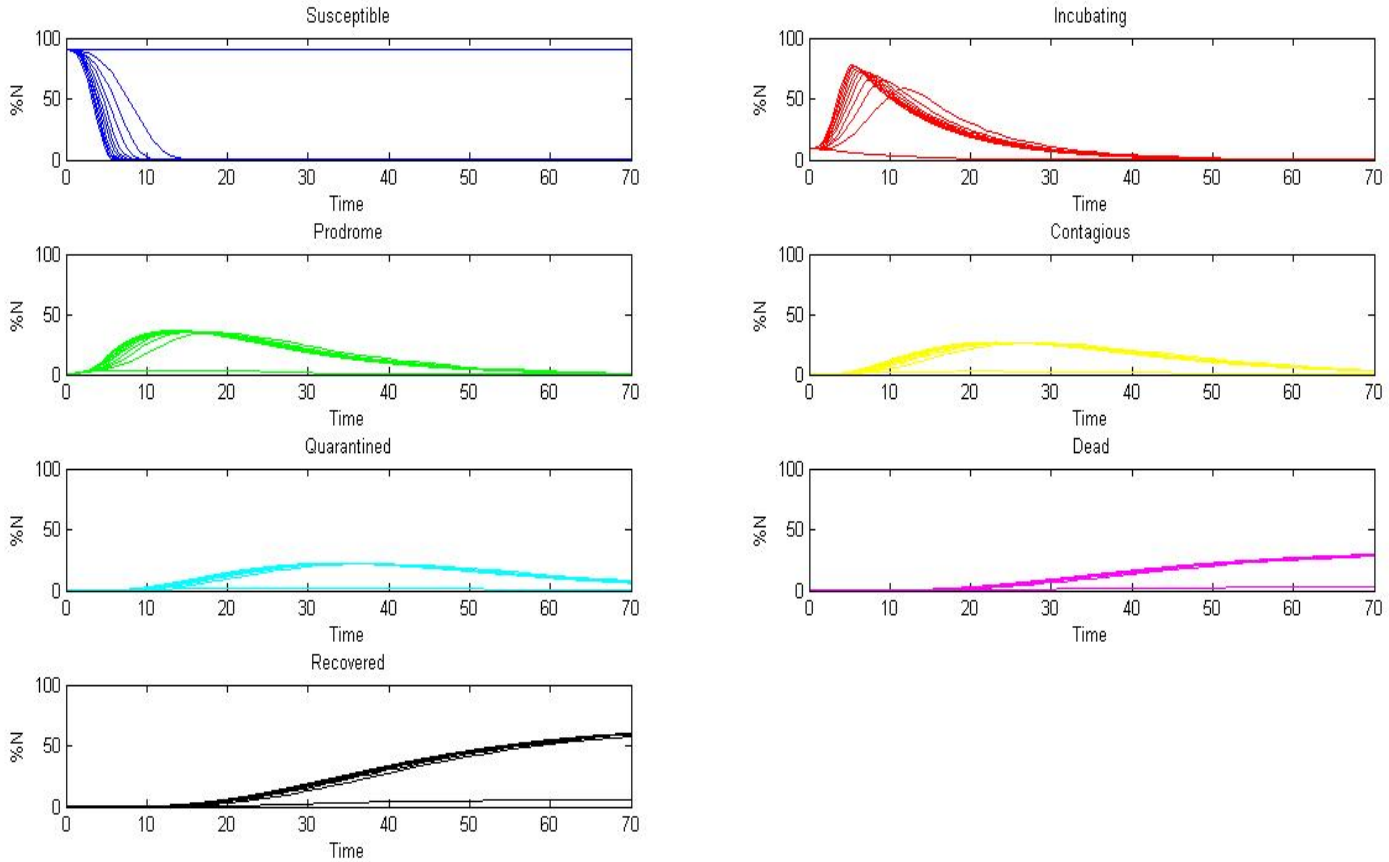


S as a function of time at different values of λ



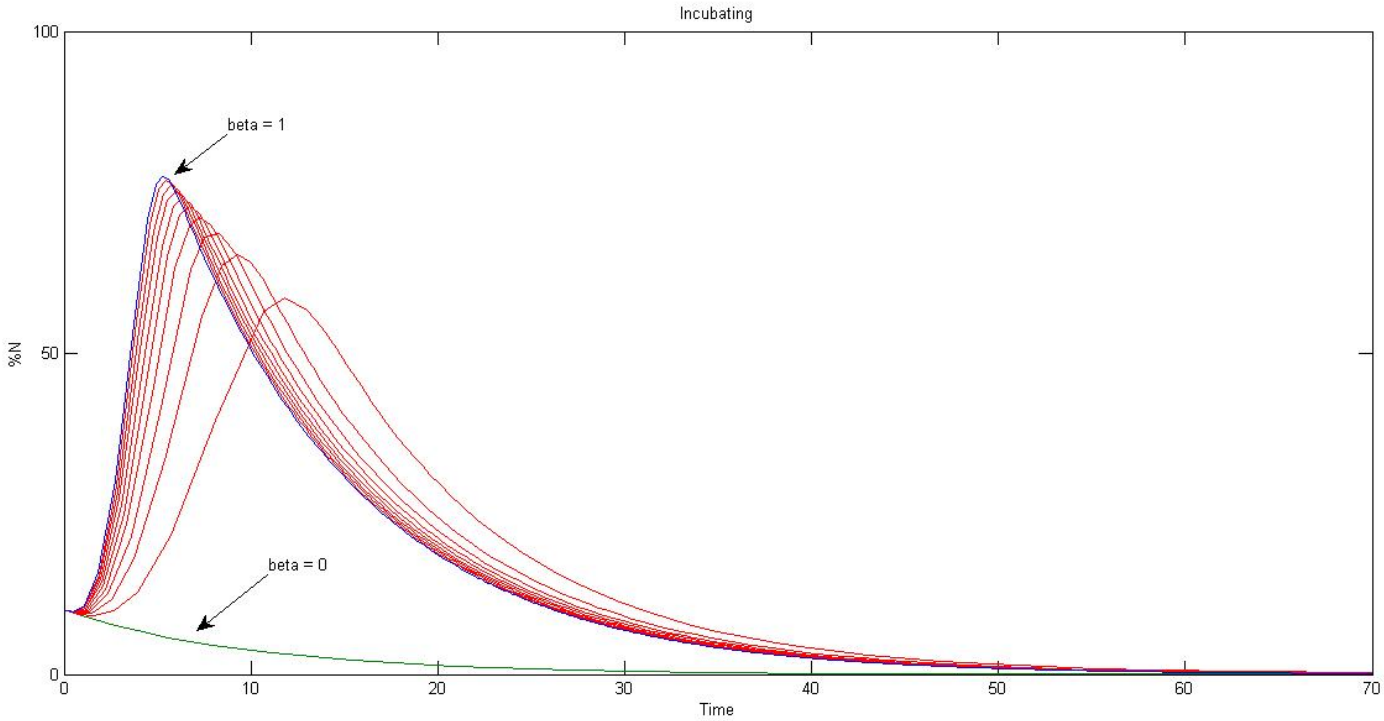
Graphical Analysis

Change in System as β Varies, integrated from $t = 0$ to $t = 70$.



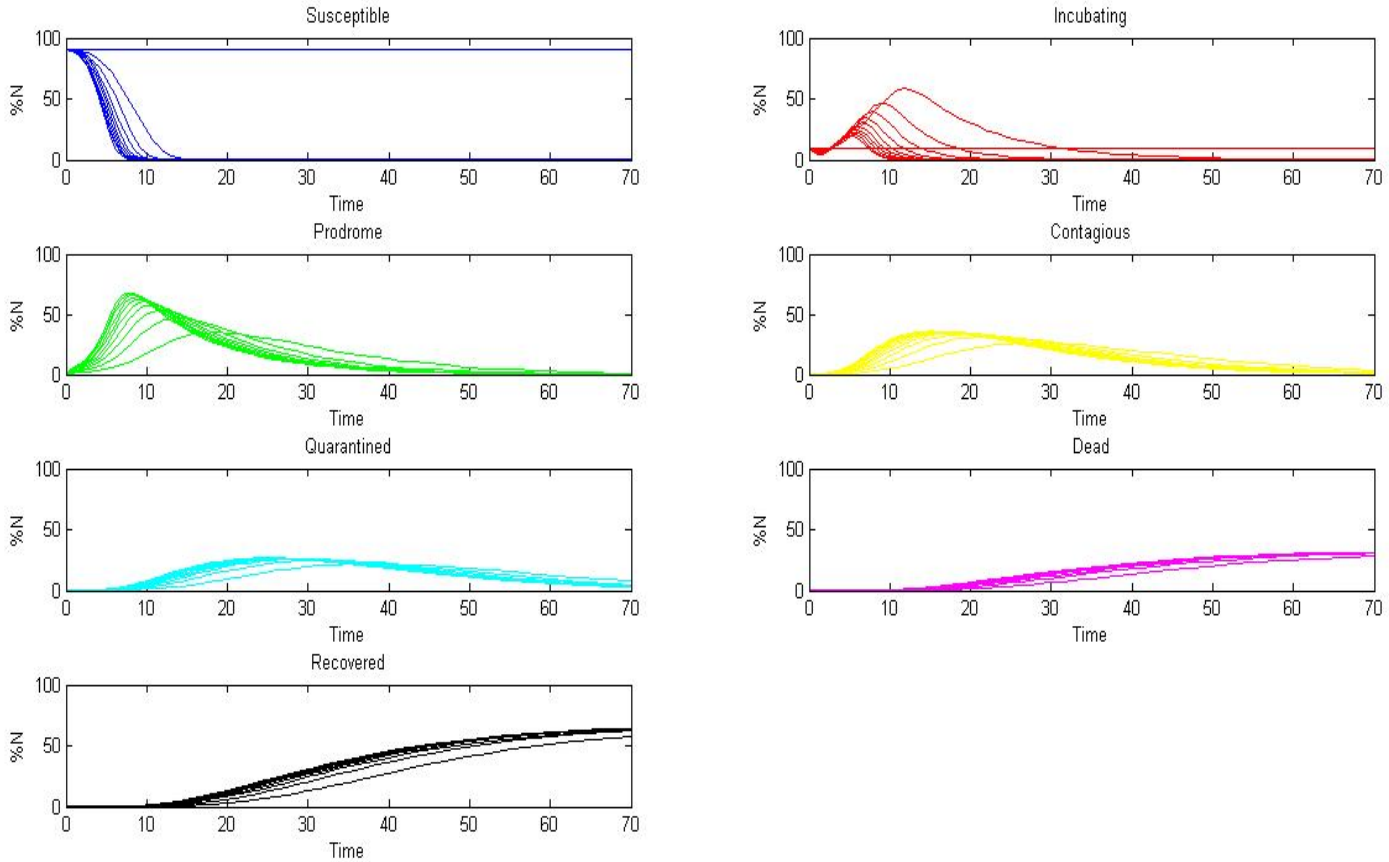
Graphical Analysis

I as a function of time at different values of β



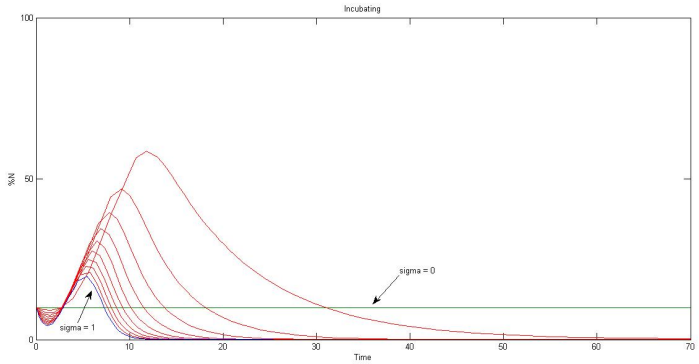
Graphical Analysis

Change in System as σ Varies, integrated from $t = 0$ to $t = 70$.

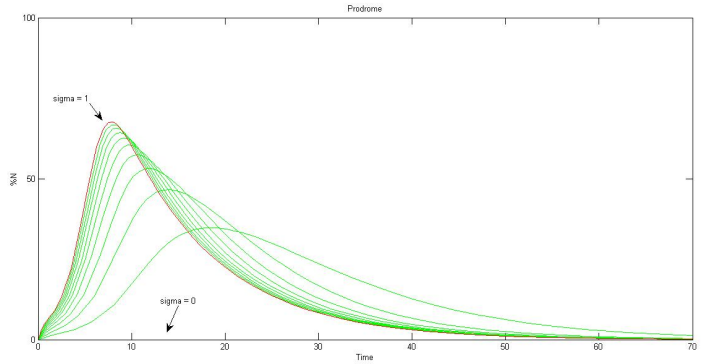


Graphical Analysis

I as a function of time at different values of σ

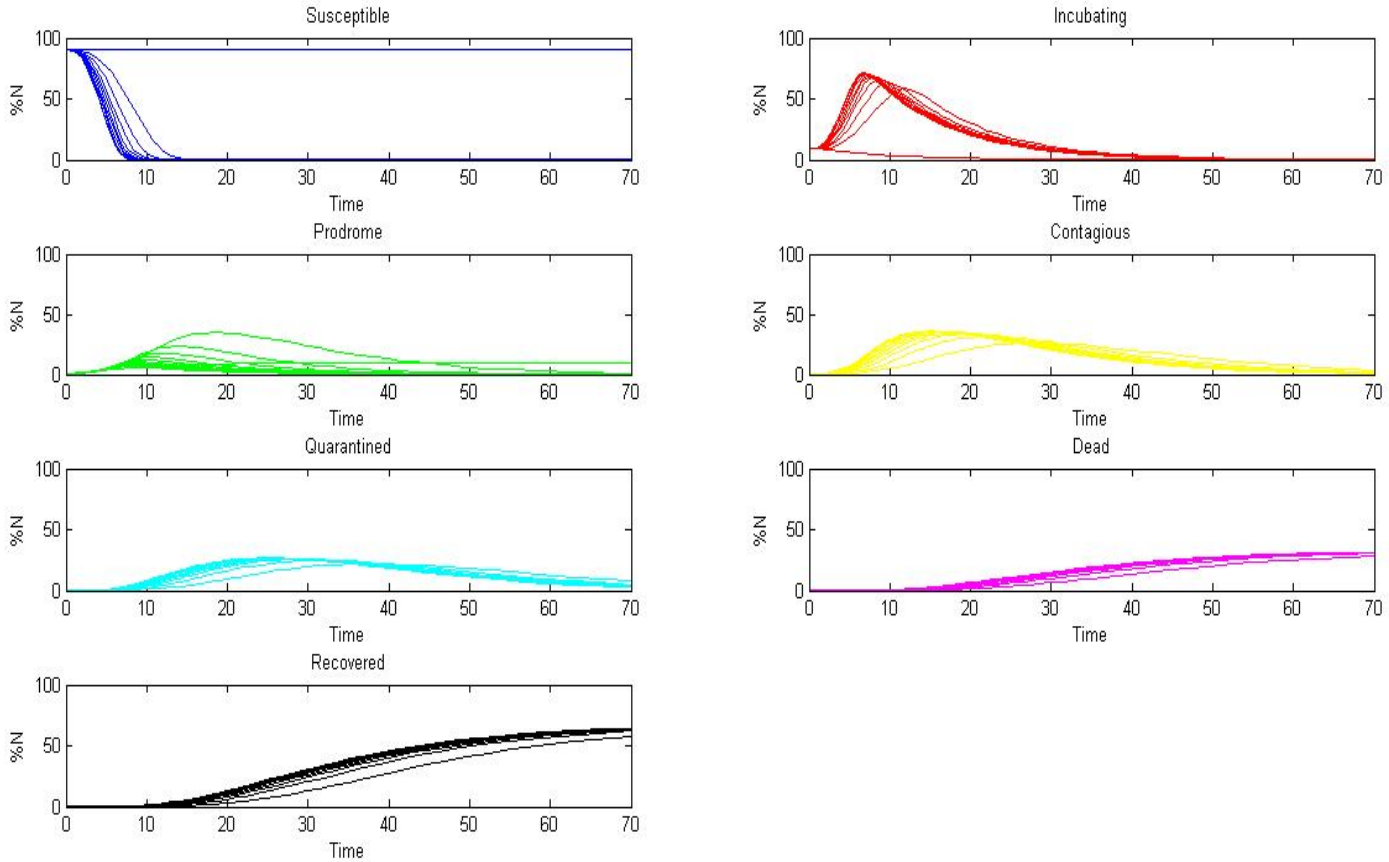


P as a function of time at different values of σ



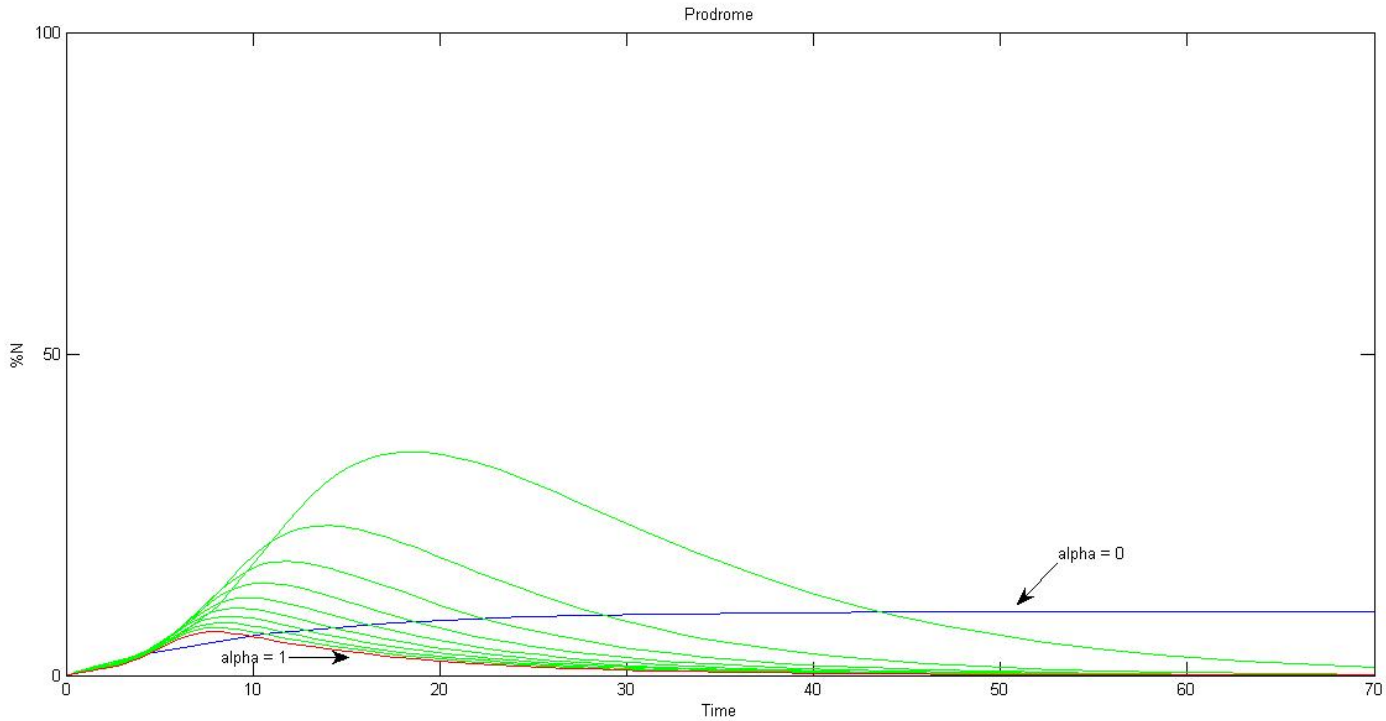
Graphical Analysis

Change in System as α Varies, integrated from $t = 0$ to $t = 70$.



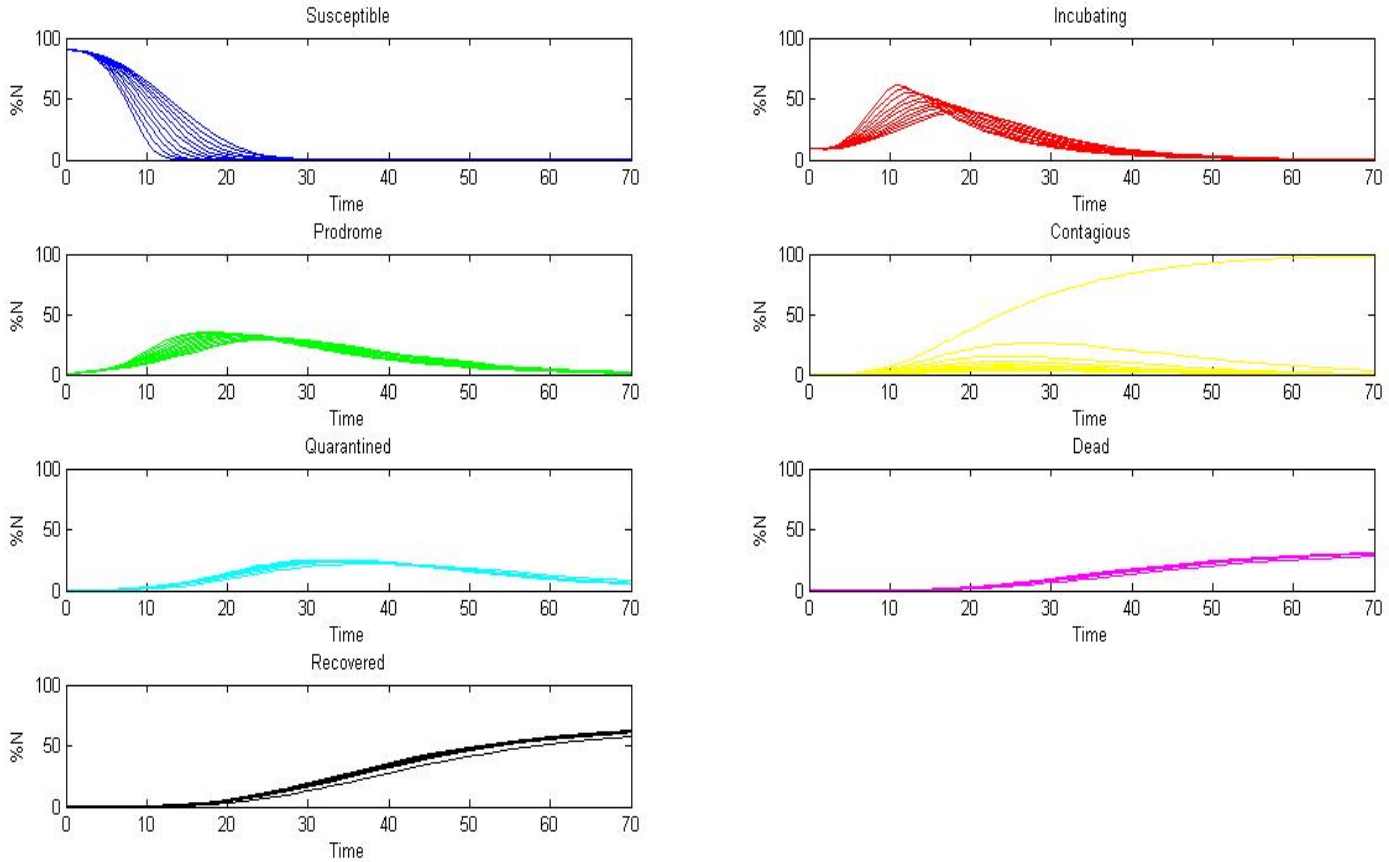
Graphical Analysis

P as a function of time at different values of α



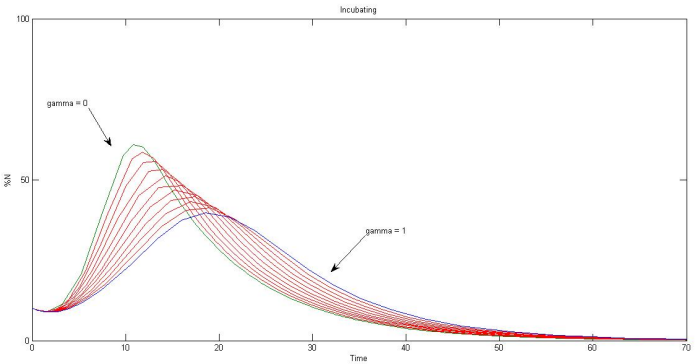
Graphical Analysis

Change in System as γ Varies, integrated from $t = 0$ to $t = 70$.

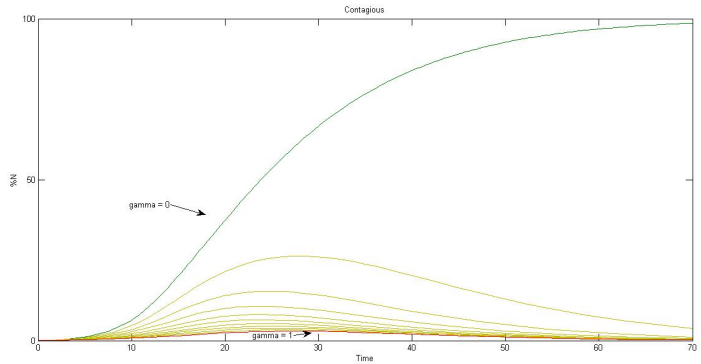


Graphical Analysis

I as a function of time at different values of γ

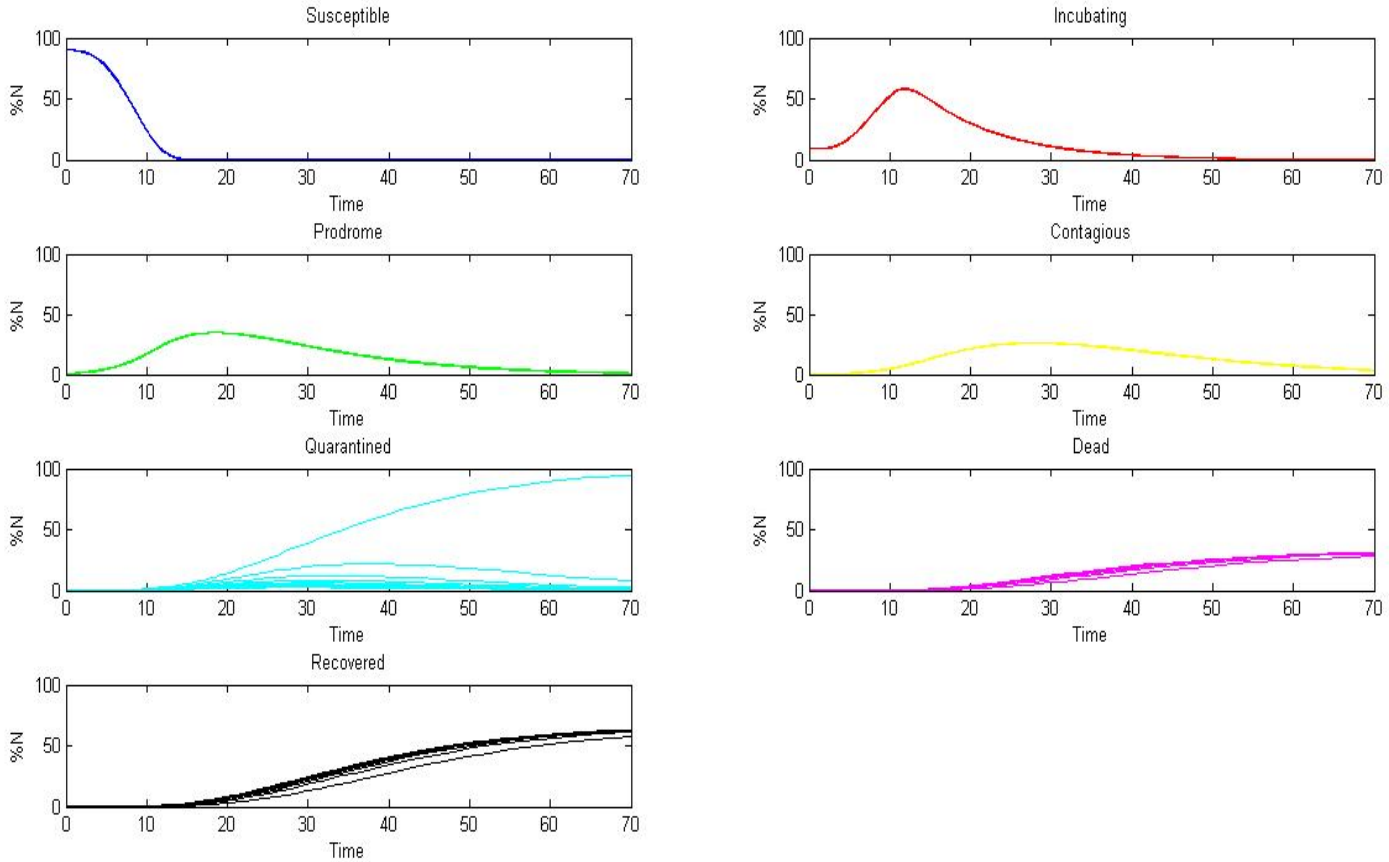


C as a function of time at different values of γ



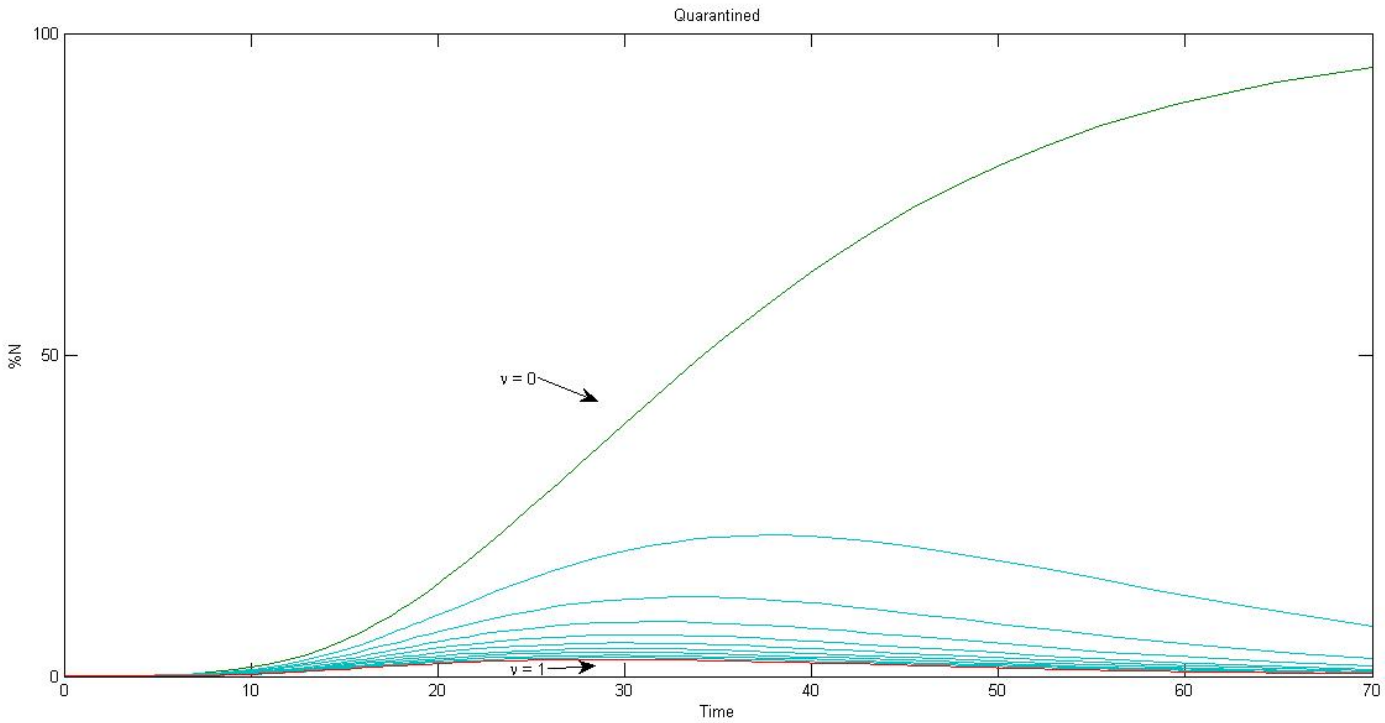
Graphical Analysis

Change in System as ν Varies, integrated from $t = 0$ to $t = 70$.



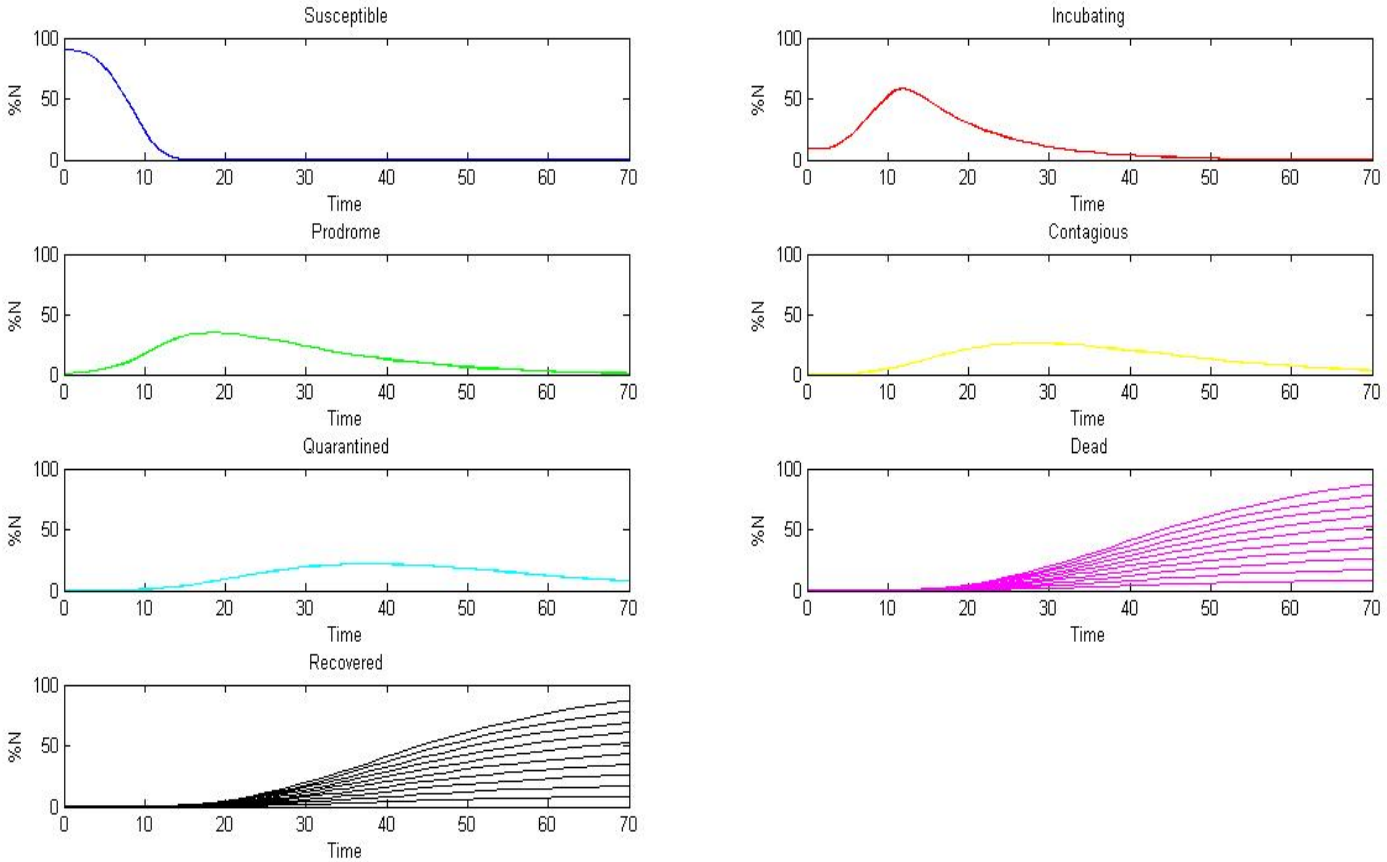
Graphical Analysis

Q as a function of time at different values of ν



Graphical Analysis

Change in System as λ Varies, integrated from $t = 0$ to $t = 70$.



Summary

- Picked smallpox model and determined factors affecting transmission.
- Developed a code to simulate model and solve numerically.
- All variables sensitive to changes in α, γ, σ, and β.
- γ has an effect opposite those of β, σ, and α.
- Only Q, D, R sensitive to ν.
- Only D and R are sensitive to λ.

Summary

What does it mean?

- The disease spreads slowest when rates of transmission are low, and when periods of illness are quick.
- Lots of quarantine can significantly slow the spread of infection.
- Progress of the illness after quarantine is inconsequential to the spread of the disease.

Future Work

- Non-dimensionalization of ODEs: removing time parameter.
- Forward Sensitivity Analysis: Analytical bounds on state vectors with respect to parameter perturbations.

$$\eta = \frac{\partial y}{\partial p}\frac{p}{y}$$