Search for a car parked on a forest road

Math 485 Midterm Presentation

March 11, 2014

Mentor: Zhelezov, Gleb

Members: Rickel, Jayson Anthony
Nguyen, Alex
Braun, Rachel Anne
Alabkari, Mohammed
Gong, Yu

Outline

- Reference and link
- Abstract
- Introduction
- Mathematical Model
- Theoretical Calculations
- Monte Carlo Method
- Preliminary Simulation Results
- Conclusion
- Future work

Reference and link

Yu Baryshnikov and V Zharnitsky,
Search on the brink of chaos, Nonlinearity, 25 (2012), 3023–3047, doi:10.1088/0951-7715/25/11/3023

Abstract

Overall Goal:

The analysis of a linear search problem for an object placed from a given probability density function using analytical and numerical methods.

Our Current Project:

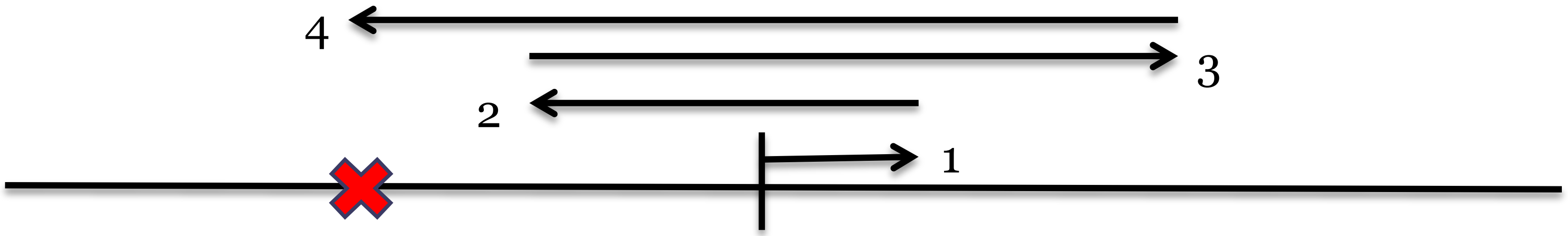
The development of numerical methods simulating the search for an object placed using an exponential distribution function and the minimization of the search for the object.

Search for a Parked Car on a Forest Road

Introduction

The traditional linear search problem:

- An object is placed on a line using a given probability density
- Using an initial step length, the searcher goes back and forth according to step lengths given by a specific formula



How to model the problem

- Why do we need a model?

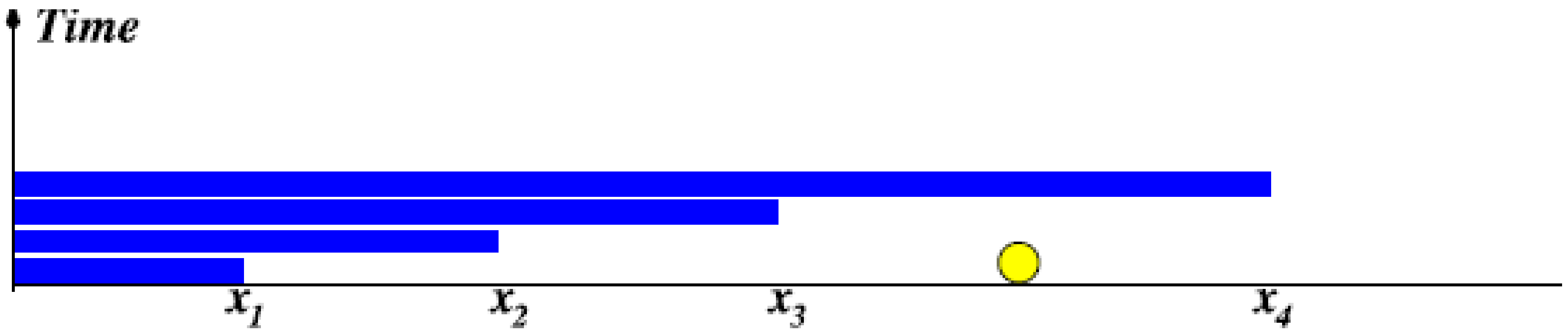
1. Can help to predict the position of a car (Probability distribution).
2. The prediction of mathematical model can help to save time and find the best cost for traveler (Minimizing problem).
3. Then lead the real world competition to better behavior (Modeling problem)

Mathematical Model

Variables

$L(x, H)$: the total distance travelled until the point H is found in function of x

$E(x) = \mathbb{E}[L(x, H)]$: the cost of the search plan x



F(x) : chosen probability density function

Mathematical Model

Let us consider:

- Hamiltonian dynamics model on the half-line $\mathbb{R}_+$
- Homogeneous tail distribution function or Pareto distribution is differentiable
- If the plan x is optimal, then the term $\{x_k\}$ satisfy the variational recursion, where f(x) is the cumulative density function of the distribution

$$x_n = -\frac{f(x_{n-2})}{f'(x_{n-1})}$$

If we find x_1 and x_2, we can optimally define the rest of the sequence

- For example, the optimal cost of a plan, where $F(x) = x^{-b}$, is given by

$$E(x) = \sum_{k=1}^{\infty} x_k f(x_{k-1})$$

Theory

We are attempting to minimize the expectation for the cost function for the total length of the search (EL).

$$EL = \int_0^\infty F(x)L(x)dx$$

Where $$L(x) = \begin{cases} 2x_1 & 0 < x \leq x_1 \\ 2(x_1 + x_2) & x_1 < x \leq x_2 \\ \ldots & \\ 2\sum_{i=1}^{N} x_i & x_{N-1} < x \leq x_N \end{cases}$$

Theory

$$EL = \int_0^{x_1} 2x_1F(x)dx + \int_{x_1}^{x_2} 2(x_1+x_2)F(x)dx + \ldots + \int_{x_{N\_1}}^{x_N} 2\sum_{i=1}^{N} x_i \; F(x)dx$$

CDF: $f(x) = \int_0^x F(x)dx$

$$EL = 2x_1[f(x_1) - f(x_0)] + 2x_1[f(x_2) - f(x_1)] + 2x_2[f(x_2) - f(x_1)] + \ldots.$$

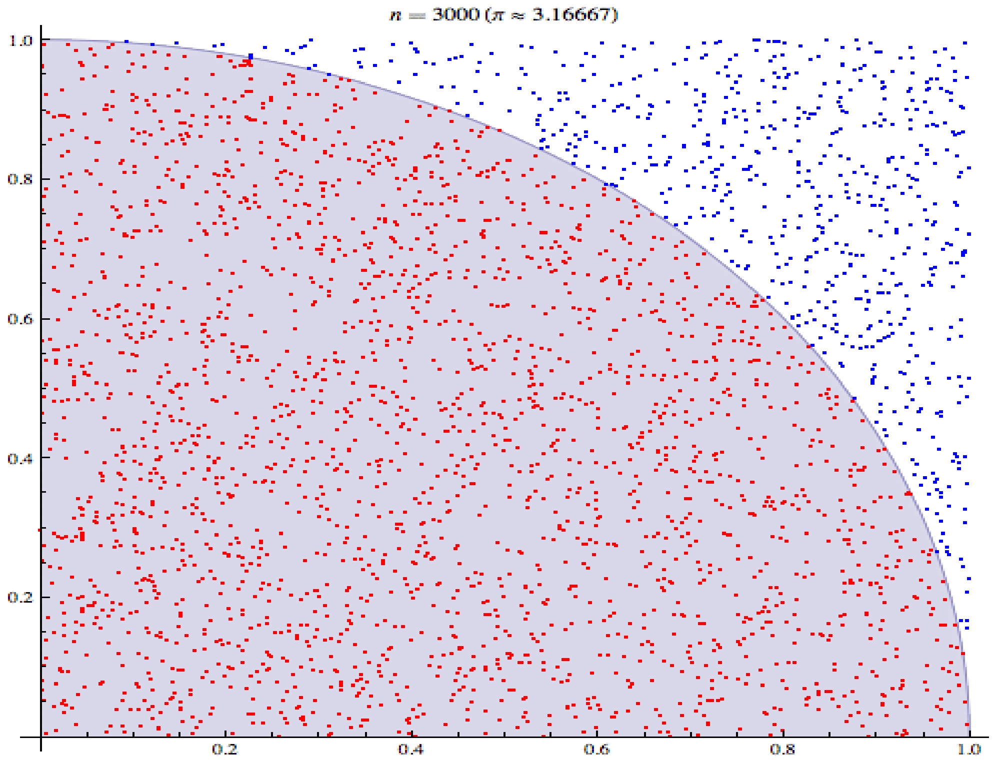
$$EL = 2[\sum_{n=1}^{\infty} x_n - \sum_{m=2}^{\infty} x_m f(x_{m-1})]$$

Monte Carlo Method

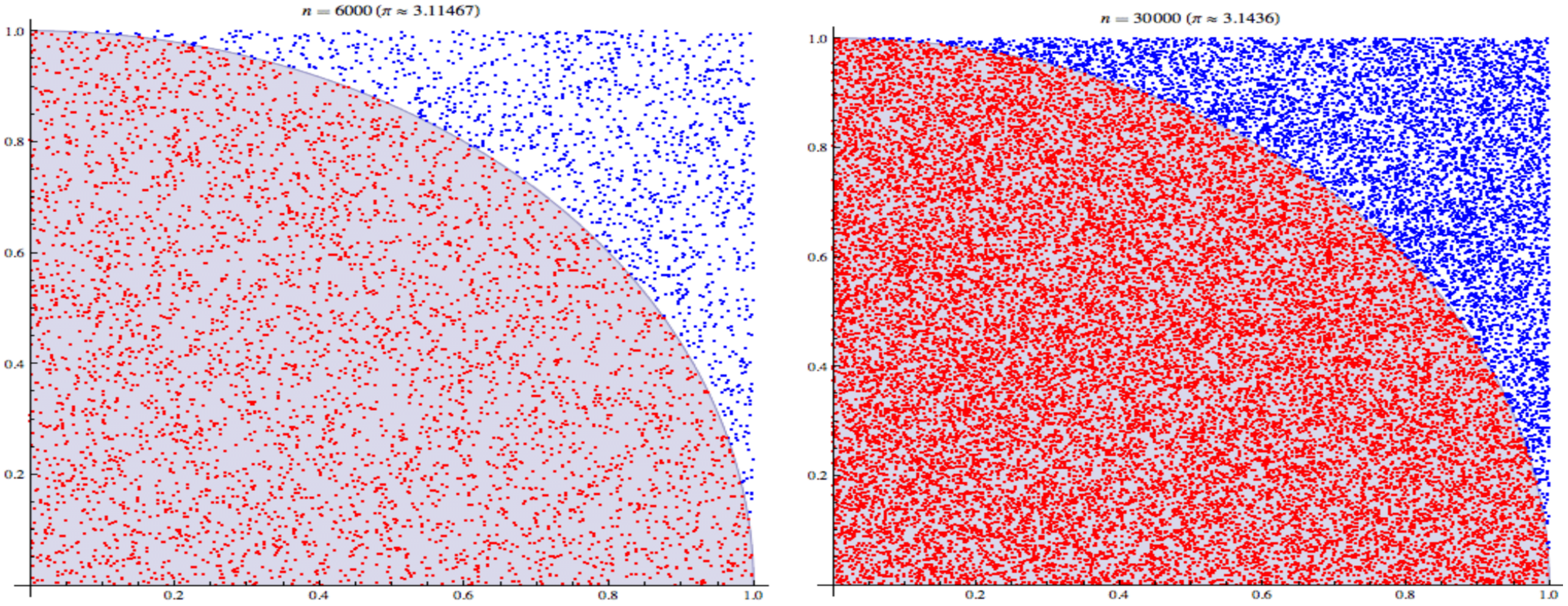
Many random samplings to obtain numerical results

For our purposes:

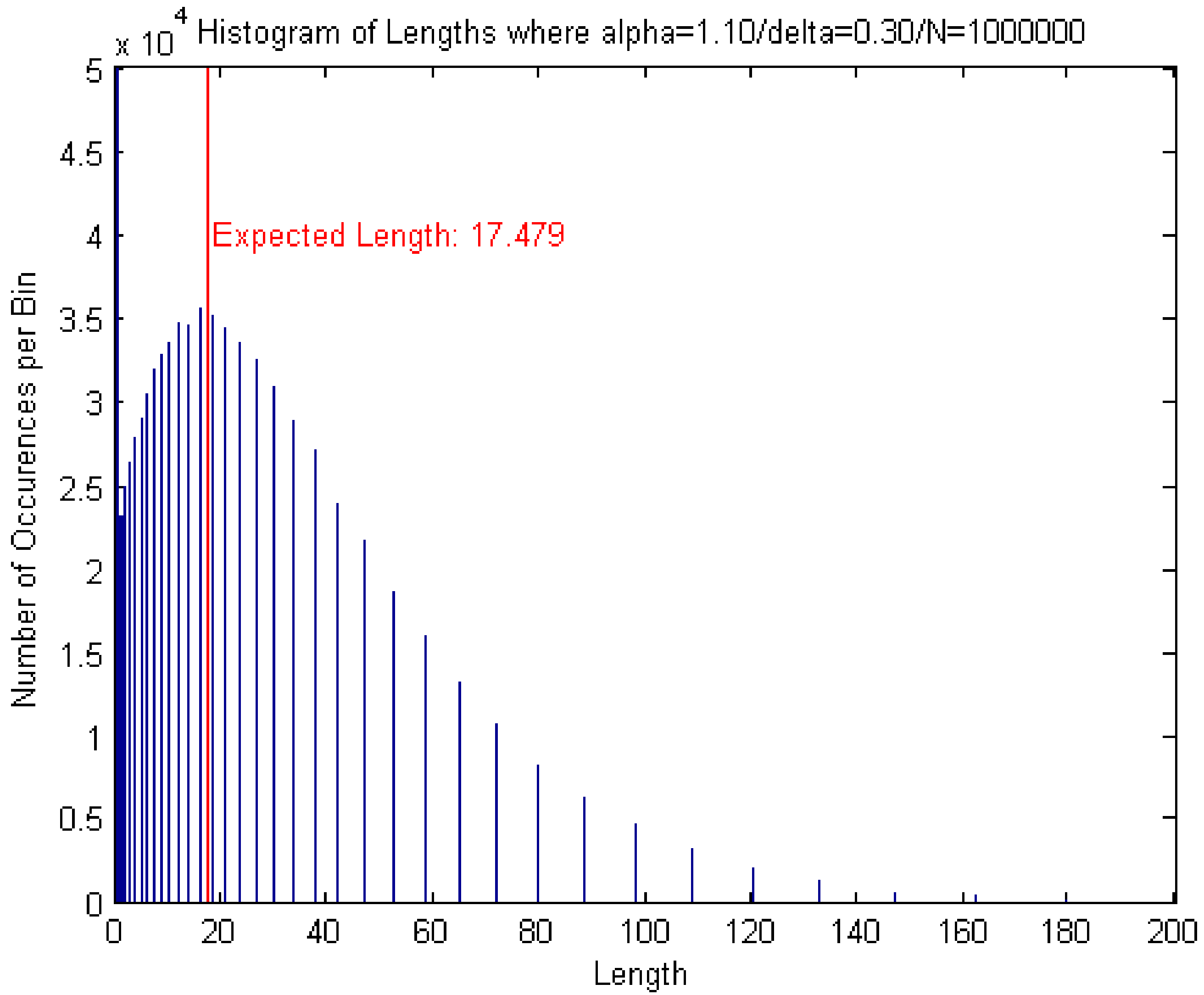
- Select placement of the car based on the probability density
- Determine the total cost to find car
- Average all lengths to find the expected value
- Find ideal pivots based on the comparison of many different expected values



Monte Carlo Method Example



Preliminary Simulations



Steps- Geometric Series:

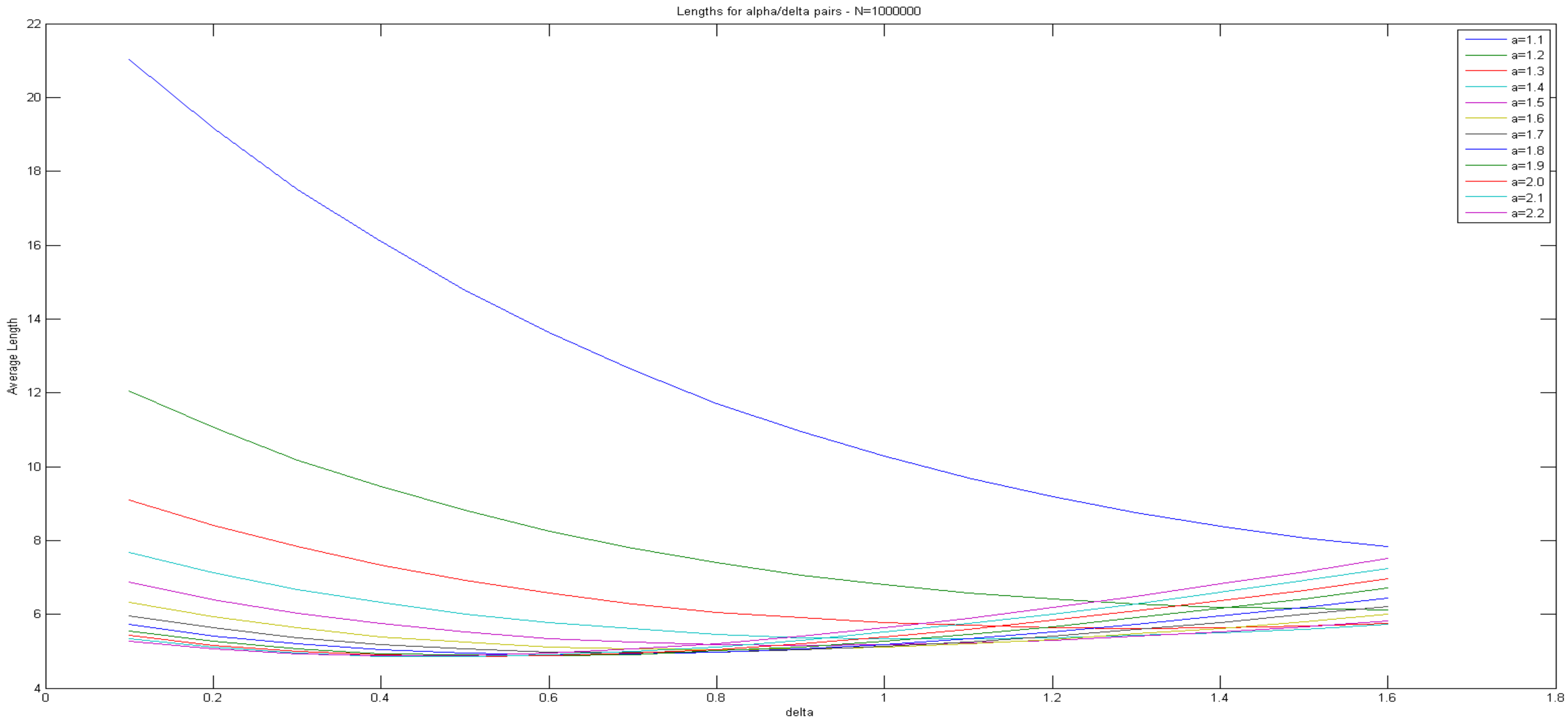
$x_n = \Delta\alpha^n$

Probability Density:

$F(x) = e^{-x}$

Preliminary Simulations

Optimal -> $\alpha = 2.1$; $\Delta = 0.5$; L = 4.7932



Conclusion

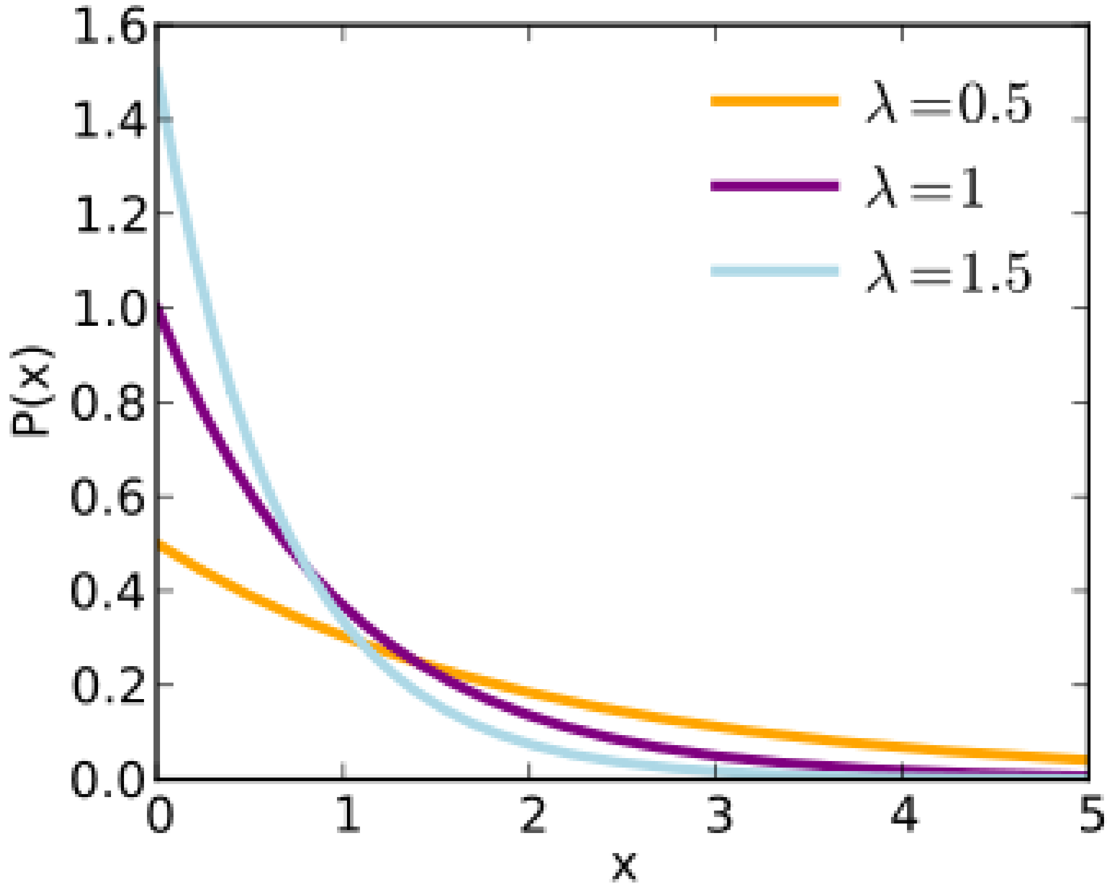
- We know that the expectation of the length converges
- We cannot find the value analytically
- Use numerical methods and Monte Carlo simulations to find expectation length

Future work

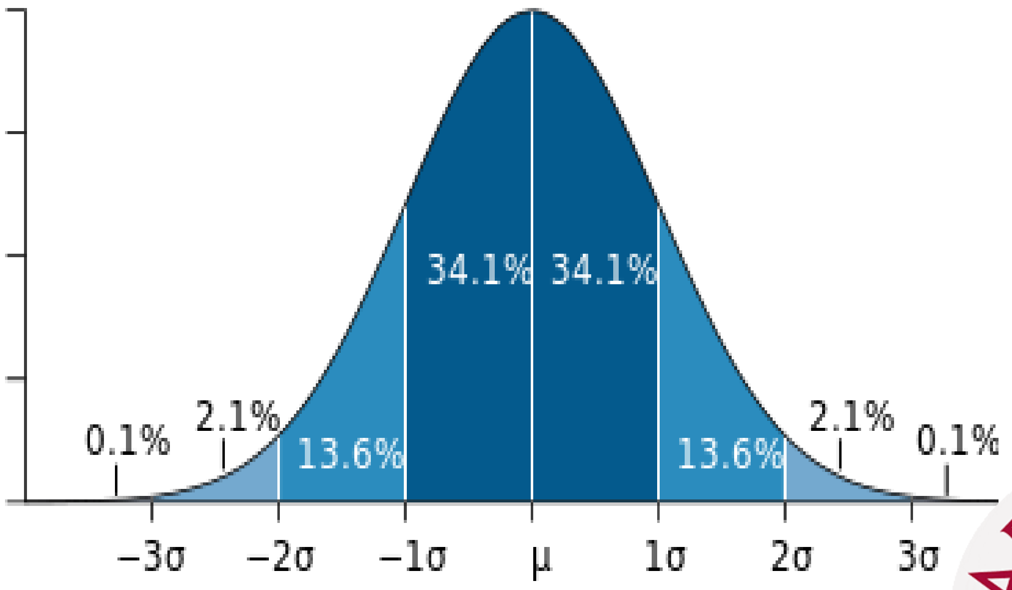
- Repeat problem for different distributions
- Find E(x)
- Find optimal x_1 and x_2 via numerical simulations
- Monte Carlo Simulations
- Random Steps
- Road has a known, finite length

Distributions to Consider

Exponential Distribution



Normal/Gaussian Distribution



Distribution Equations

Exponential Distribution

$$f(x{:}\,\lambda) = \begin{cases} \lambda e^{-\lambda x} \,;\, x \geq 0 \\ 0 \,;\, x < 0 \end{cases}$$

Normal/Gaussian Distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Questions?