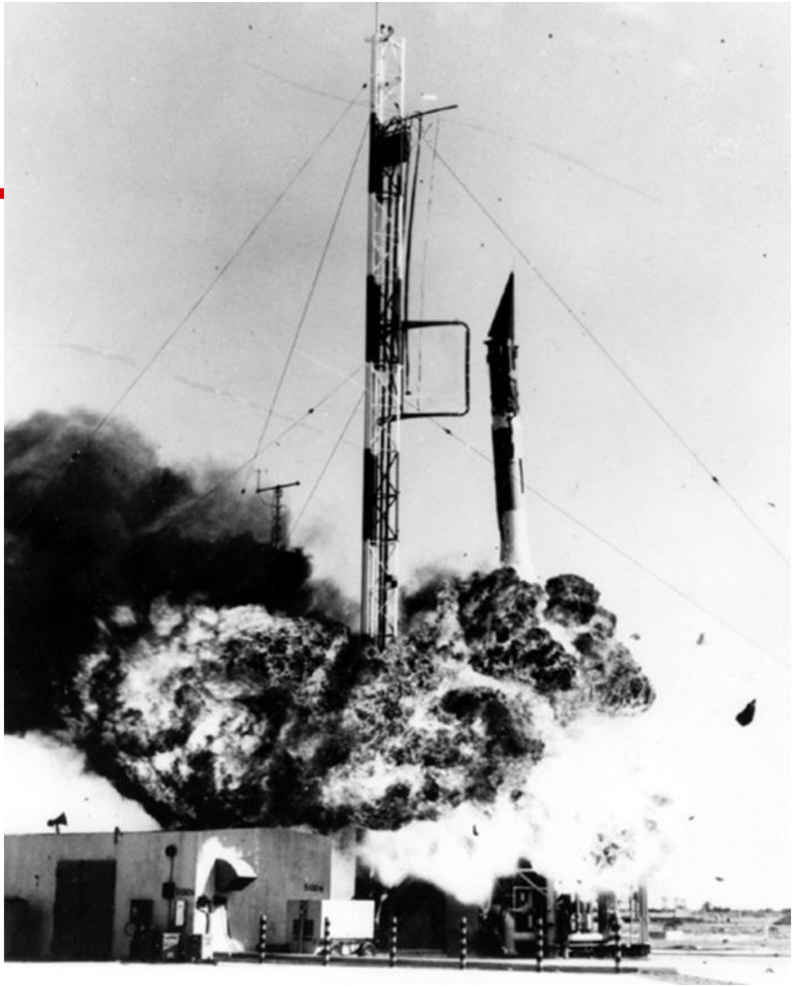
Bubble Dynamics in a Vibrating Liquid

By: James Wymer, Jaggar Henzerling, Aaron Kilgallon, Michael McIntire, Mohammed Ghallab

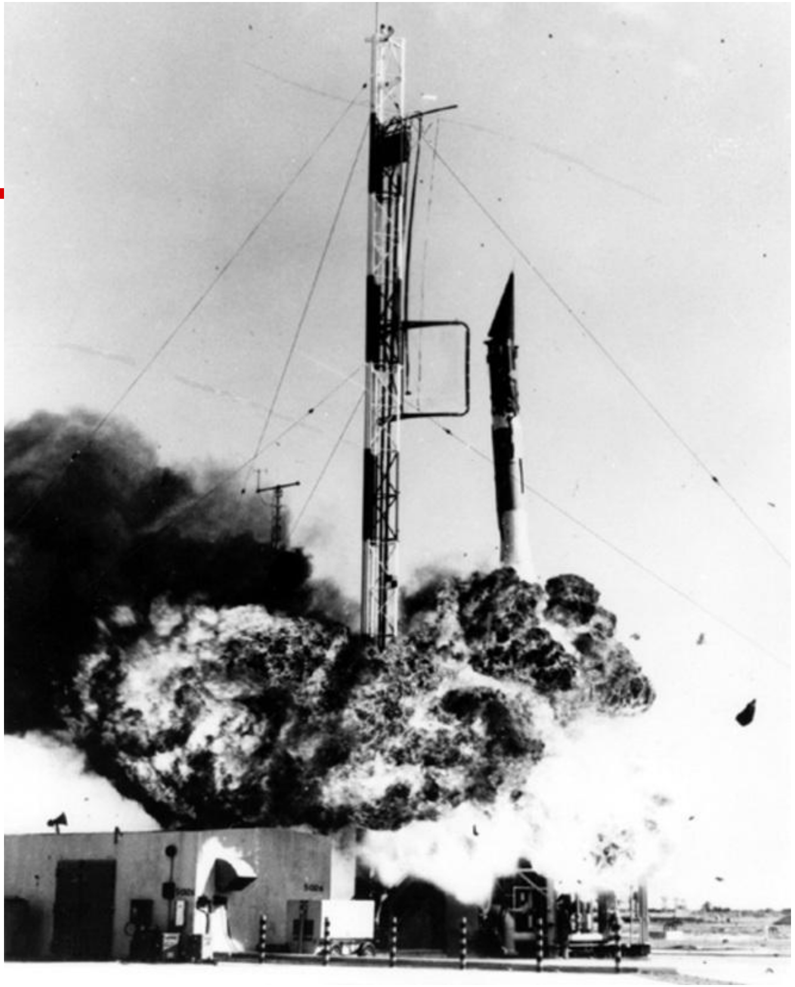
Background

In the early 1960's a series of rocket failures plagued leading space industries. The problems were caused when floating fuel indicators inexplicably sank and caused premature stage separation of the rockets.



Observation

- Bubbles can sink in a vibrating liquid
- The vibration can come from the liquid being directly agitated or agitation of its container
- Fluid density and pressure, bubble depth and amount of vibration are all key factors

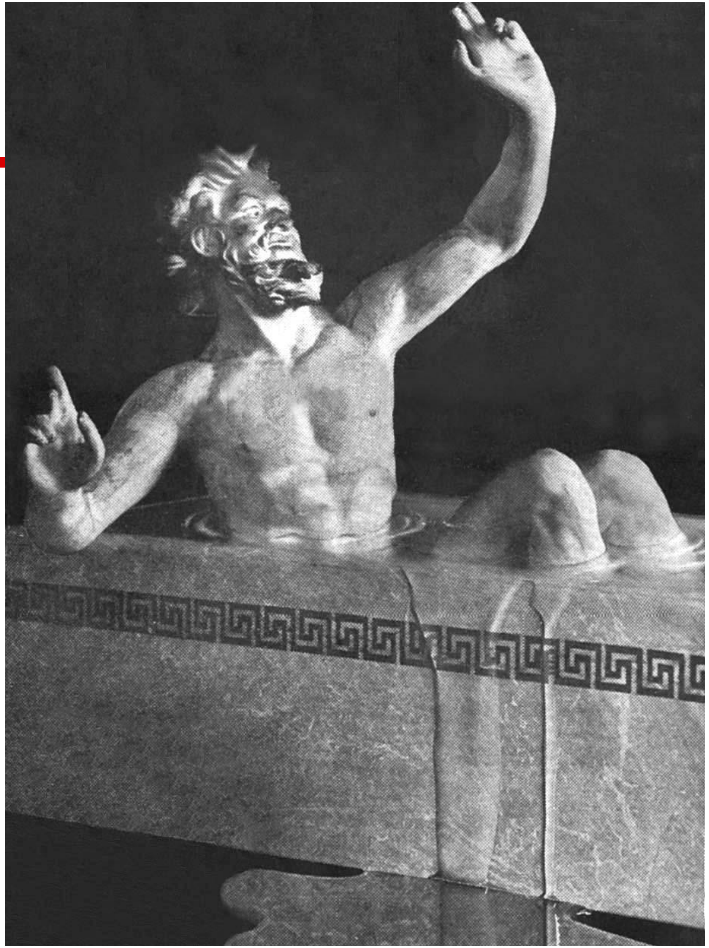


Problem

- Induced or Added mass was first proposed by Friedrich Bessel in 1828
- Vertical oscillations increase the induced mass that affects the bubble
- Phenomenon of induced mass creates a changing effective gravitational potential

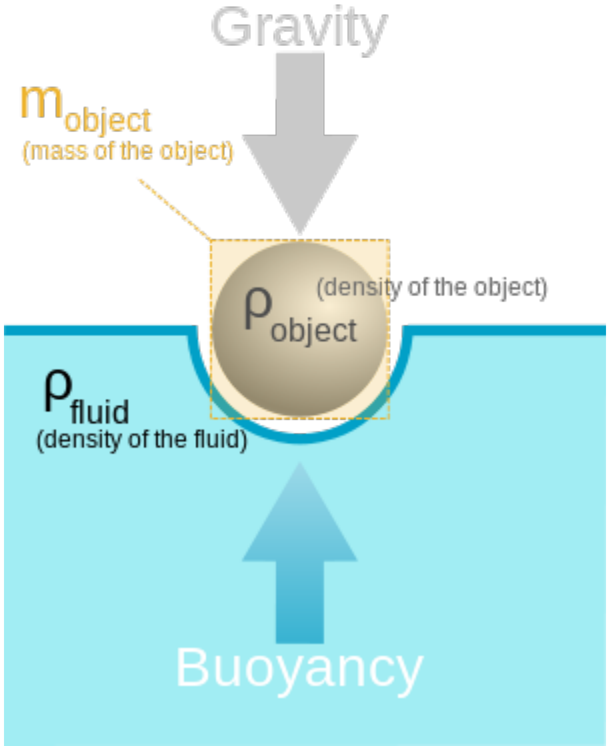
Model

- Concepts
  - Archimedes' principle
    - buoyancy
  - Laplace pressure
    - pressure difference between bubble inside and outside
  - Friction
    - drag on bubble's motion
  - Induced mass



Archimedes' Principle

- The buoyant force arises from a mass displacing fluid
- Depends on the density and volume displaced
- F = ρVg
- Archimedes postulated that the difference in pressure between an upper and lower face of an object caused this effect

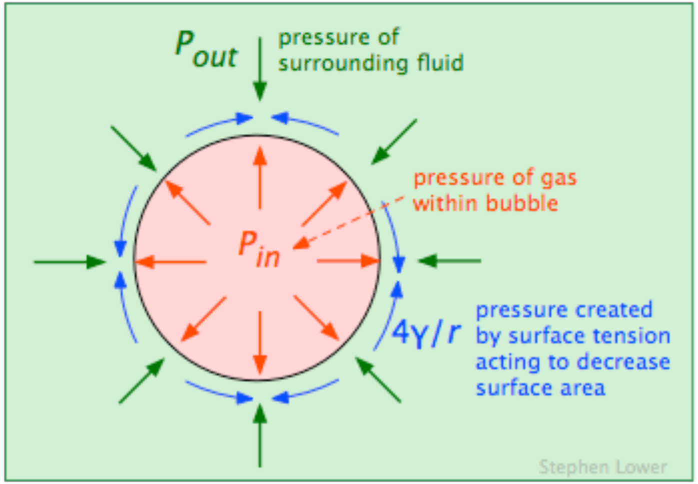


Laplace Pressure

- Is the pressure difference between two sides of a curved surface
- Arises from surface tension γ of the interface
- For spheres, Young-Laplace equation reduces to the second equation
- Smaller droplets have non-negligible extra pressure
- Commonly used for air bubbles in water or oil bubbles in water

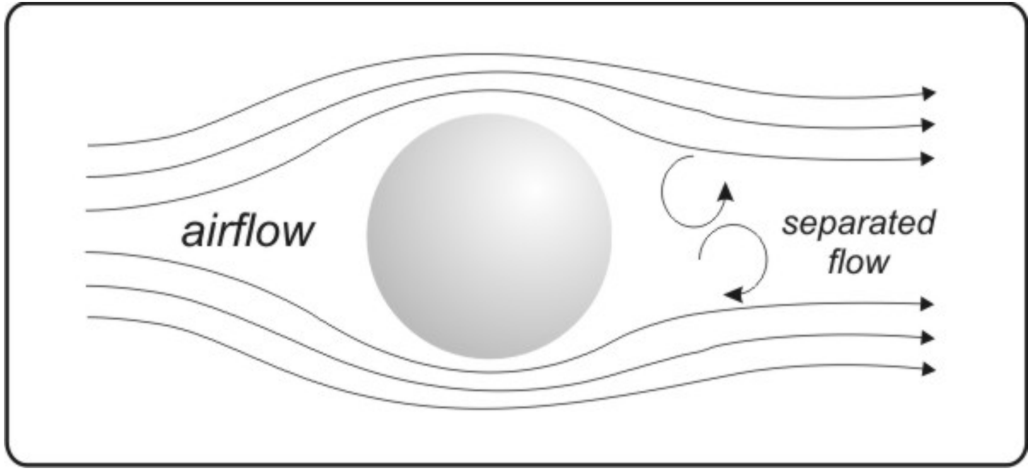
$$\Delta P \equiv P_{\text{inside}} - P_{\text{outside}} = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right),$$

$$\Delta P = \frac{2\gamma}{R},$$



Friction

- Also known as drag or fluid resistance
- Described mathematically by the drag equation
- Depends on the velocity of the object and fluid properties
- Drag coefficient C is dimensionless and describes drag in fluids



$$F_D = \frac{1}{2} \rho v^2 C_D A$$

Induced Mass

- Various changes in velocity affect the Kinetic energy
- The attached mass determines the work done to change the kinetic energy

$$T = \frac{\rho}{2}\int_V (u_1^2 + u_2^2 + u_3^2)dV$$

$$T = \rho \frac{1}{2} U^2$$

Equations from CR82.010

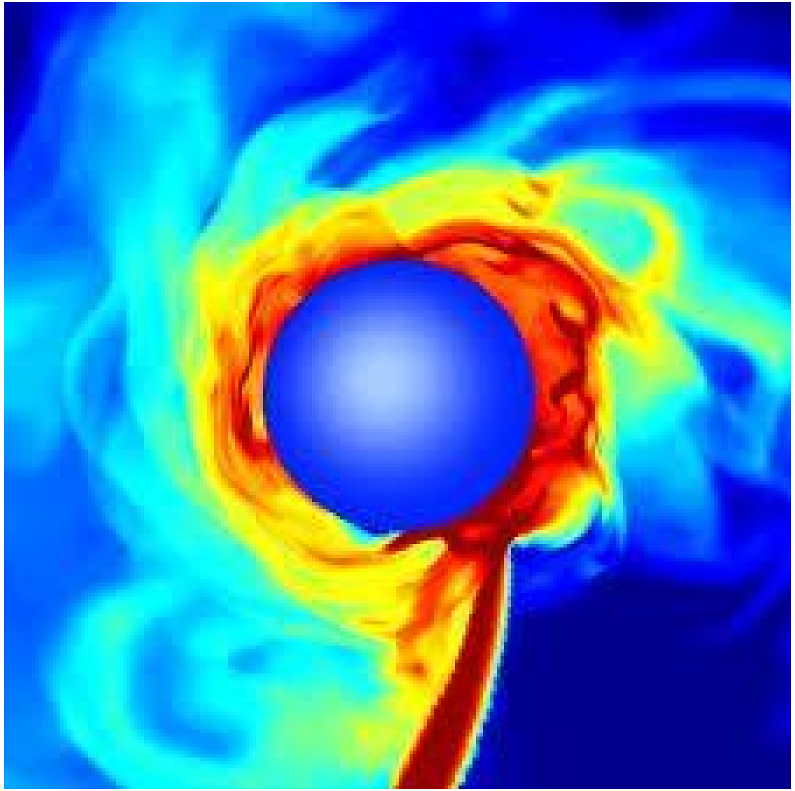
Induced Mass

- Introduce I which represent how the volume of the sphere changes with velocity
- Attached mass is equal to I*density, so attached mass is 1/2m (displaced fluid)

$$I = \frac{2}{3}\pi R^3$$

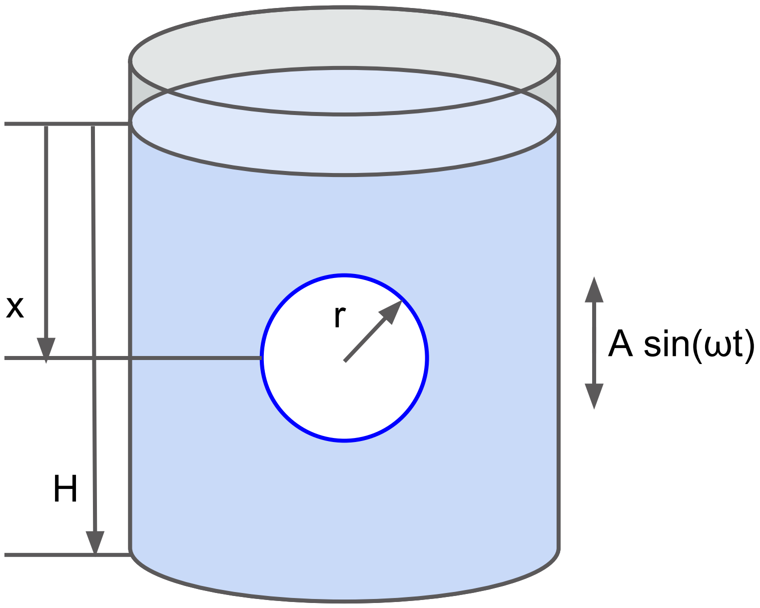
Equation from CR82.010

Induced Mass Concept



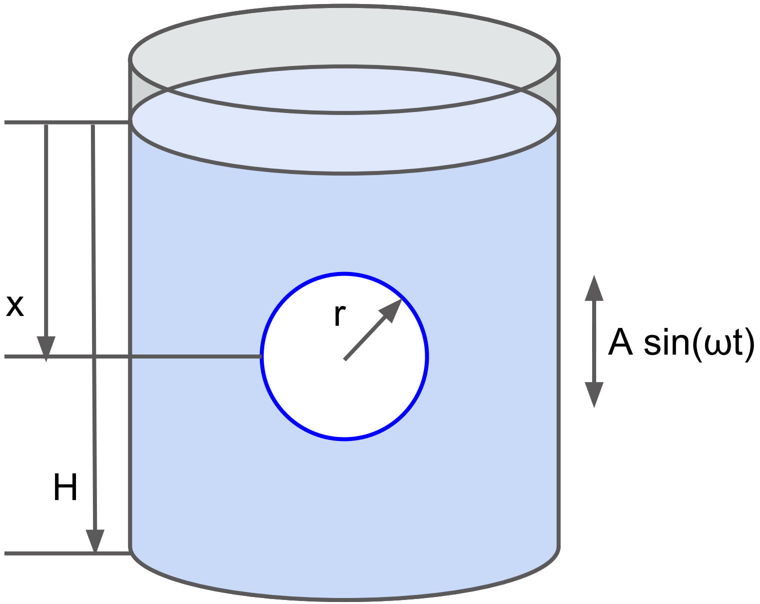
Model

- Assumptions
  - Spherical bubbles
  - Incompressible liquid (div of velocity is zero)
  - Container is open on top
  - Bubble volume changes are insignificant (quasistatic)
  - Ideal pressure conditions (too much and no oscillations, too little and cavitation occurs)



Model

- Parameters
  - Total water depth - H
  - Bubble depth - x
  - Bubble radius - r
  - Oscillation amplitude - A
  - Oscillation frequency - ω
  - Time duration - t



Bubble Volume

Assume the bubble to be isothermal:

The Ideal Gas Law implies that $P(t)V(t) = P(0)V(0)$

Fluid oscillations implies the pressure is:

$$P(t) = P(0) + \rho x(g + A\omega^2 \sin \omega t)$$

The final result for the volume of the bubble is:

$$V_b = \frac{P_e V_{b0}}{P_e + \rho x(g + A\omega^2 \sin \omega t)}$$

Model

- Combining these varying functions into a general equation results in this differential equation of motion for the bubble:

$$(m + m_0)\ddot{x} + \dot{m}_0\dot{x} = -F(\dot{x}) + (m - \rho V_b)(A\omega^2 \sin \omega t + g)$$

- Induced mass m_0, Mass Variation Friction,

Model

- Combining these varying functions into a general equation results in this differential equation of motion for the bubble:

$$(m + m_0)\ddot{x} + \dot{m}_0\dot{x} = -F(\dot{x}) + (m - \rho V_b)(A\omega^2 \sin \omega t + g)$$

Drag force, buoyancy term, oscillating fluid term.

Drag Force:

$$F(\dot{x}) = 4\rho R^2 \Psi(\mathrm{Re})\dot{x}^2 \mathrm{sgn}\dot{x}$$

$$\mathrm{Re} = 2\rho R V/\mu \rightarrow$$ Reynolds Number

Model - Separation of Variables

- Method of Separation of Variables
- Harmonics of these types of oscillations imply that one can assume that the solutions are of the form: $x(t,\tau) = X(t) + \Psi(\tau)$
- $X(t)$ is the ‘slow’ solution
- $\Psi(\tau)$ is the ‘fast’ solution

Time Average Position of the Bubble

Average Position of Bubble:

$$\langle x(t,\tau) \rangle = \langle X(t) \rangle + \langle \Psi(\tau) \rangle$$

Slow Fast

Since $\Psi(\tau)$ is periodic its average is zero

Therefore the average position of the bubble is described by the changes that take place slowly in time

Slow Solution

Condition for bubble sinking is:

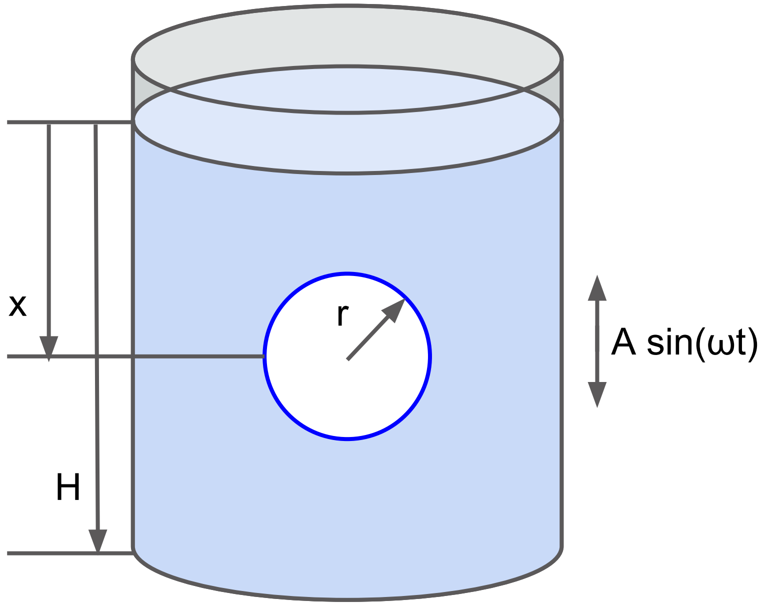
$$\gamma \cdot \omega^2 \frac{X}{H_0} \frac{1}{2} \left(1 - \frac{2}{3} \frac{\theta \frac{A^2}{R_0^2}}{2\left(1 + \sqrt{1 + \theta \frac{A^2}{R_0^2}}\right) + \theta \frac{A^2}{R_0^2}} \right) > 1$$

Velocity of Bubble

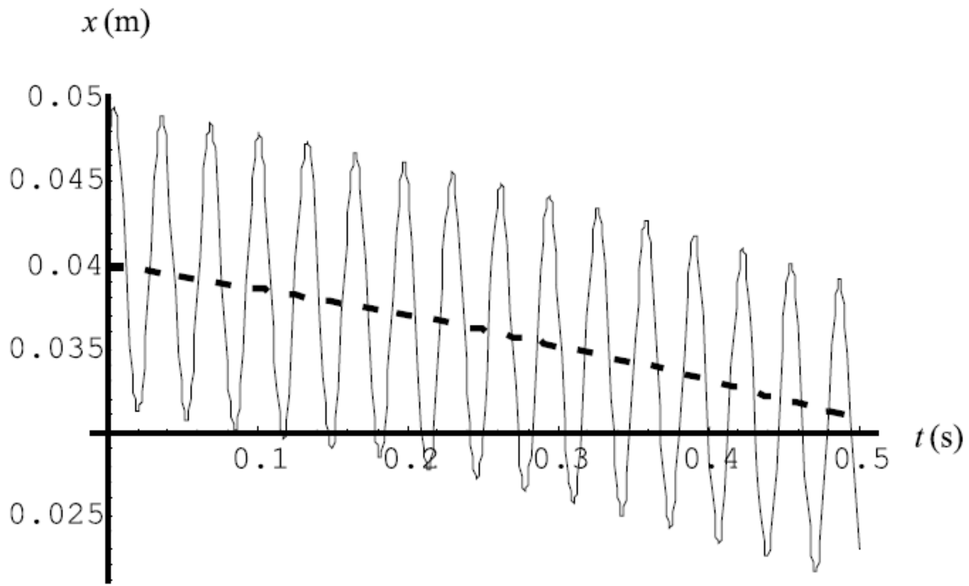
- Acceleration of bubble is relatively small

$$\dot{x} \approx v\left[\frac{x}{x_0} - 1\right]$$

- Results in 3 cases
  - Bubble sinks $x > x_0$
  - Bubble remains motionless $x = x_0$
  - Bubble floats $x < x_0$

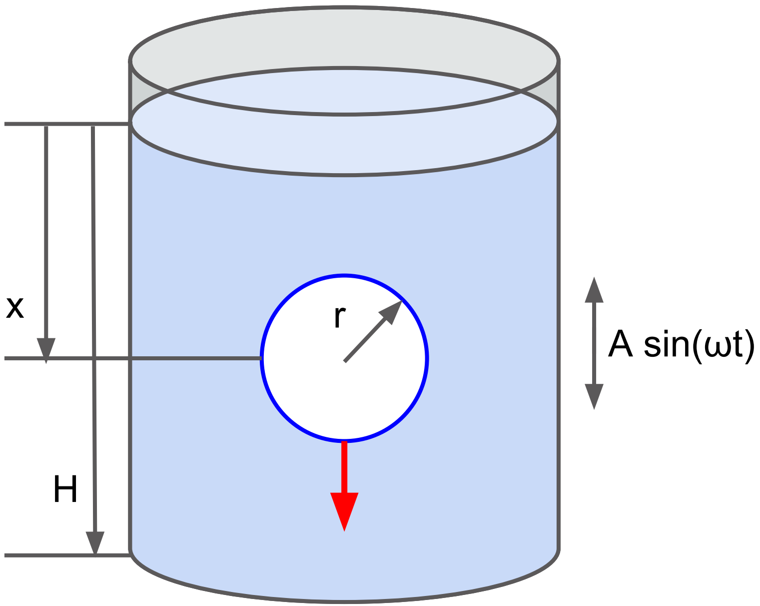


Velocity of Bubble



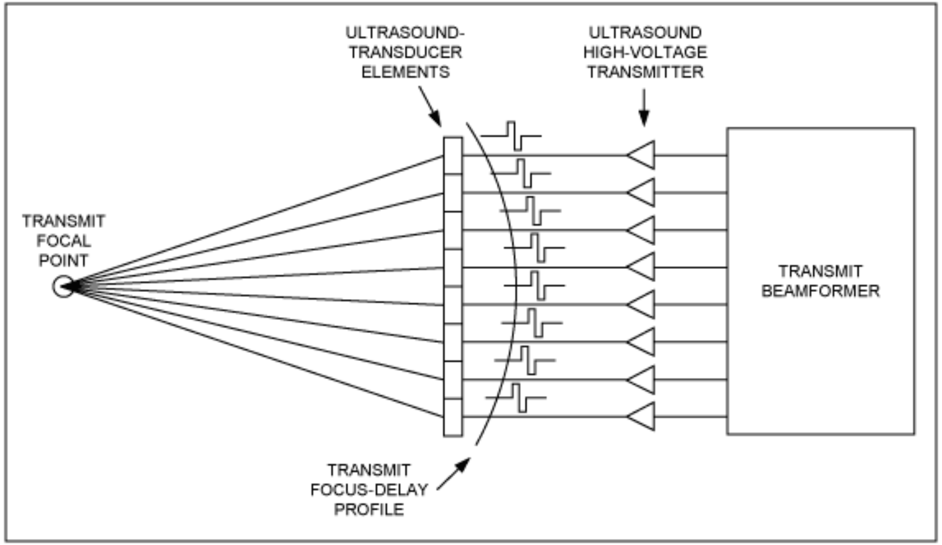
Conclusion

- Bubbles in vibrating fluids will sink given certain circumstances
    - Dependent factors
        - Bubble depth
        - Vibration amplitude
- Cause
    - Induced mass overcomes buoyancy



Future Steps

- Create computer model simulating bubble effects
- Determine quantifiable difference between liquid being agitated by vibrating container and pressure waves
- Attempt to determine a method to prevent sinking of bubbles (thermal excitation, focused ultrasound)



Sources

- Brennen, C. E., comp. "A Review of Added Mass and Fluid Inertial Forces." (1982): Naval Civil Engineering Laboratory. Web. 8 Mar. 2014. http://authors.library.caltech.edu/233/1/BRE052.pdf
- Falkovich, G. Fluid Mechanics: A Short Course for Physicists. Cambridge: Cambridge UP, 2011. Print.
- Techet, A. "Object Impact on the Free Surface and Added Mass Effect." (2005): Massachusetts Institute of Technology. Web. 8 Mar. 2014. http://web.mit.edu/2.016/www/labs/L01_Added_Mass_050915.pdf
- V.S. Sorokin, “Motion of a Gas Bubble in Fluid Under Vibration,” (201): Springer. Web.

Model

- Derivations

$$X_0 = \frac{2H_0}{\gamma \cdot \omega^2} \cdot \frac{2\left(1 + \sqrt{1 + \theta \frac{A^2}{R_0^2}}\right) + \theta \frac{A^2}{R_0^2}}{2\left(1 + \sqrt{1 + \theta \frac{A^2}{R_0^2}}\right) + \frac{\theta}{3} \frac{A^2}{R_0^2}}$$