Wave Structures on A Jet Entering the Bulk Liquid

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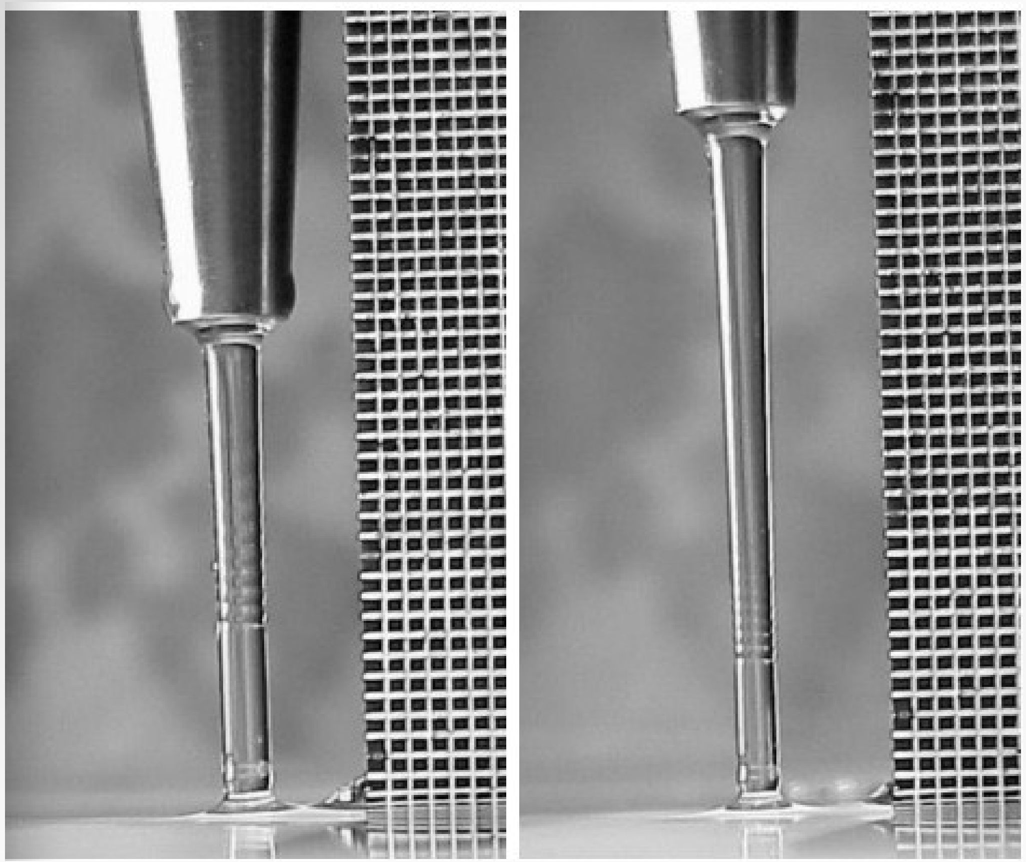
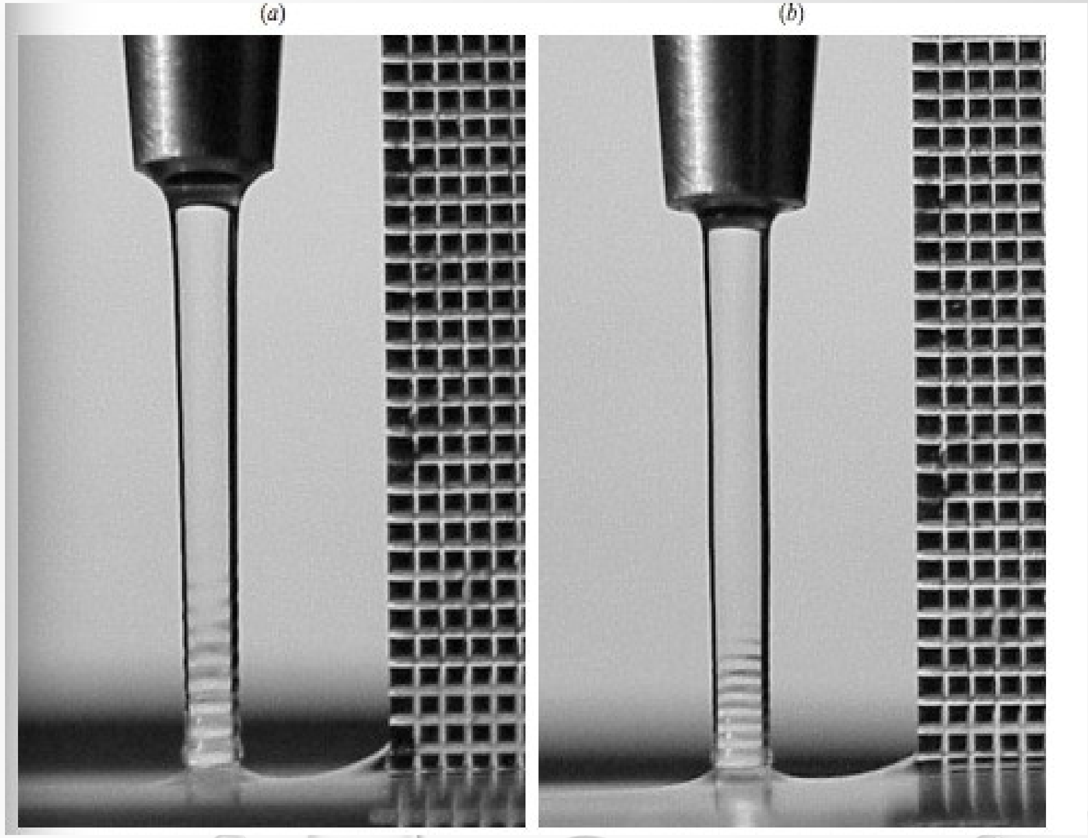
Introduction

The formation of capillary waves on fluid pipes—

Our goal is to describe the formation of stationary fields of capillary waves on a fluid jet entering a bulk liquid. The waves are affected by the velocity of the jet, the height of the jet ,the size of the stream, and the surface tensions of the fluids.

Picture of the jets with capillary waves





Cases-there are four primary cases to consider

- 1. A fluid jets enters a reservoir containing the same fluid.
- 2. A fluid jets enters a reservoir containing a fluid with a smaller surface tension.
- 3. A fluid jets enters a reservoir containing a fluid with a greater surface tension.
- 4. A fluid jet meets a fluid(air) with no surface tension.

Past research

There is a paper about fluid pipes by Maattew J. Hancock and John W. M.Bush. Show lots of information about capillary waves. So we will be comparing our results to those paper.

Derivation of formulas

- Simple formula
- Fourier transform formula

$$\omega^2 = \frac{\sigma}{\rho + \rho_1} |k^3|$$

ω is angular frequency
σ is surface tension number
 is density of the water
ρ_1 is density of the air, in this formula, it is nearly equal zero
K is wavenumber and it is equal

Simple formula

V is phase velocity, also said linear velocity, is $\frac{\omega}{k}$

$$v^2 = \frac{\omega^2}{k^2} = \frac{\sigma}{\rho}|k| = \frac{\sigma}{\rho} * \frac{2\pi}{\lambda}$$

$$\mathbf{V} = \sqrt{\frac{\sigma}{\rho} * \frac{2\pi}{\lambda}} = \frac{a}{\sqrt{\lambda}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

Fourier transform formula

$$-(i\omega)^2 = \sigma/\rho\ (ik)^3 i$$

$$\partial/\partial t = i\omega$$
$$\partial/\partial x = ik$$

$$-\partial/\partial t^2 = \sigma/\rho\ \partial/\partial x^3 i$$

$$-U_{tt} = \frac{\sigma}{\rho} i\, U_{xxx}$$

$$-\frac{\partial}{\partial t}^2\, \hat{U}(k,t) = \frac{\sigma}{\rho} i\, (-i\omega)^3\, \hat{U}(k,t)$$

$$\hat{U}_{tt}(k,t) = \frac{\sigma}{\rho} k^3\, \hat{U}(k,t)$$

$$\hat{U}_{tt}(k,t) - \frac{\sigma}{\rho} k^3\, \hat{U}(k,t) = 0$$

$$\hat{U}(k,t) = A_1\, e^{\sqrt{\frac{\sigma}{\rho} k^3} t} + A_2 e^{-\sqrt{\frac{\sigma}{\rho} k^3} t}$$

$$\mathbf{U(x,t)} = \frac{1}{\sqrt{2}\pi} \int_{-\infty}^{\infty} \hat{U}(k,t)\, e^{-ikx} dk$$

The procedure about this experiment

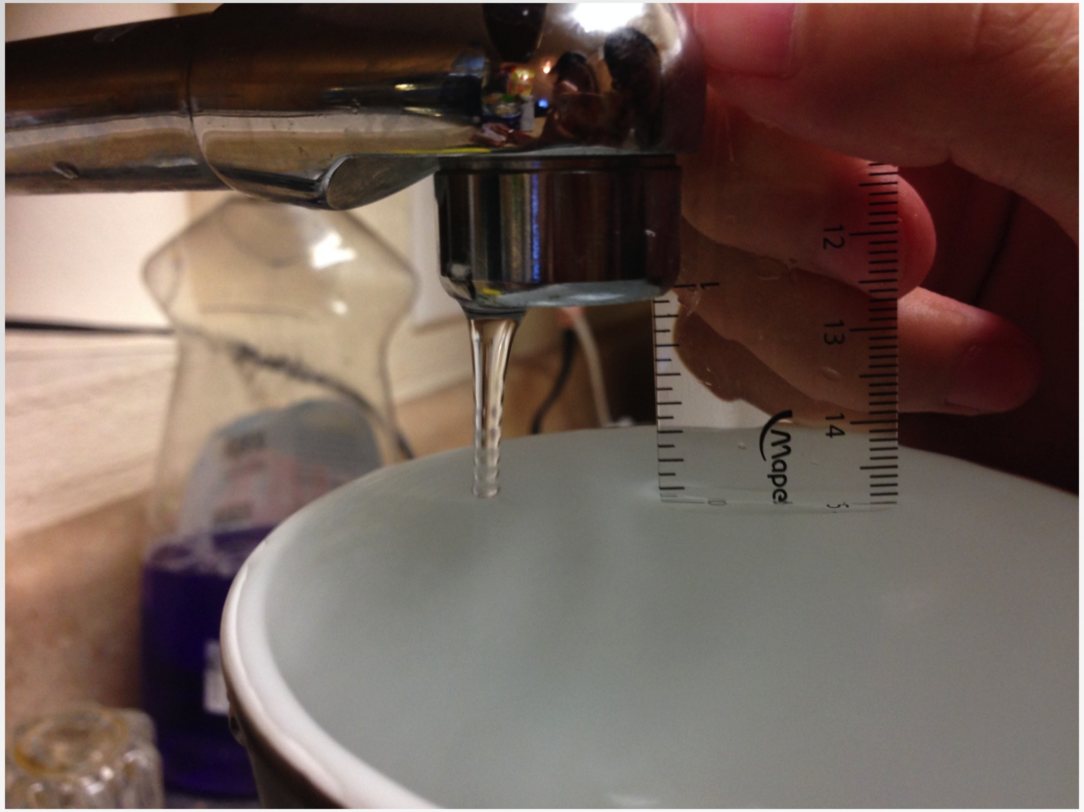
Equipment for this experiment

1. cup with volume
2. stopwatch to measure time
3. rule to measure the length of waves
4. camera to get pictures.

Procedures of the experiemnt:

- 1.we need to measure out the volume of the cup, so we can find out the capacity of the water.
- 2. use the stop-watch to measurement the time.
- 3. after we record the time about the flux, we need to move the cup until we can see the waves clearly.
- 4. put the ruler on the side, and the use camera to take pictures.
- 5. repeat those steps more times and take pictures for different length of waves.

Procedure to measure wave length and cross sectional area.



1. use ruler to find out the waves length in pictures.
2. use ruler to get the proportion about the ruler in pictures.
3. use proportion to find out the real length about the waves.
4. repeat those steps and find out the radius about the cross sectional in different high.
5. After all those steps,we will use the measurements to do following test.

result of experiment

- 1. time
- 2. cup volume: 497 cm^3
- v = ((diameter/2)^2) * pi * height of cup
- 3.wave

4.k: water wave cross section area

get the value of flow

time(sec)	flow(cm^3/s)	k(cm^2)
39	12.74	0.32
42.1	11.8	0.31
45.18	11	0.3
47.3	10.5	0.29
49.7	10	0.277
50	9.94	0.276
52	9.55	0.265
52.7	9.43	0.262
54	9.14	0.254
55.2	9.03	0.252

flow = volume of cup / time

get the value of velocity

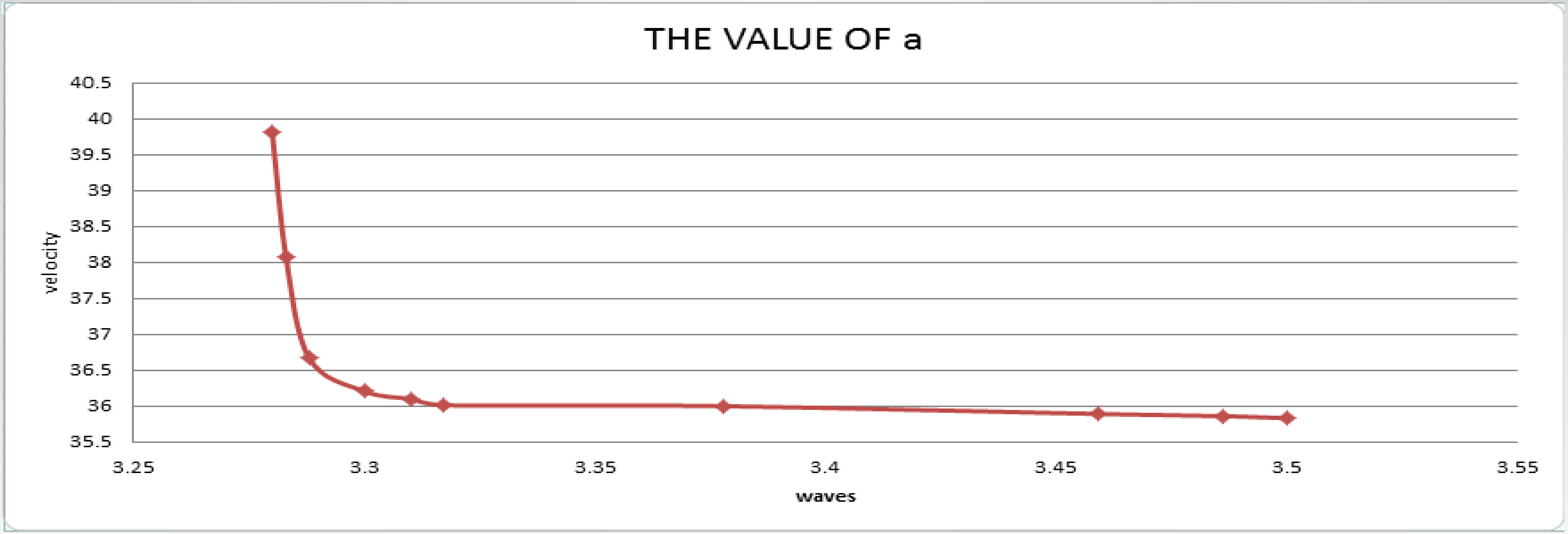
k(cm^2)	velocity(cm/s)
0.32	39.8125
0.31	38.06451613
0.3	36.66666667
0.29	36.20689655
0.277	36.10108303
0.276	36.01449275
0.265	36.03773585
0.262	35.99236641
0.254	35.98425197
0.252	35.83333333

- velocity = flow / k

the realtionship between velocity and wave

waves(mm)	velocity(cm/s)
3.28	39.8125
3.283	38.06451613
3.288	36.66666667
3.3	36.20689655
3.31	36.10108303
3.317	36.01449275
3.378	35.99993
3.459	35.89236641
3.486	35.86214414
3.5	35.833333

the graph of wave v.s velocity



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calculate the value of a

waves(mm)	velocity(cm/s)	a
3.28	39.8125	22.8
3.283	38.06451613	21.8
3.288	36.66666667	22.1
3.3	36.20689655	20.8
3.31	36.10108303	20.76
3.317	36.01449275	20.74
3.378	35.99993	20.92
3.459	35.89236641	21.1
3.486	35.86214414	21.17
3.5	35.833333	21.19

use the function of experiment

$$V = \sqrt{\frac{\sigma}{\rho} * \frac{2\pi}{\lambda}} = \frac{a}{\sqrt{\lambda}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

the result of a

- from the function of experiment, we can get the result of a have a range.
- from 20.74 to 22.8

use the function of experiment

- velocity = a / (wave)^0.5
- get the average value of a, a = 21.338

$$V = \sqrt{\frac{\sigma}{\rho} * \frac{2\pi}{\lambda}} = \frac{a}{\sqrt{\lambda}}$$

$$a = \sqrt{\frac{2\sigma\pi}{\rho}}$$

So:
surface tension = density of water * a^2 / 2 pi
And the surface tension is about 72 mN/m.

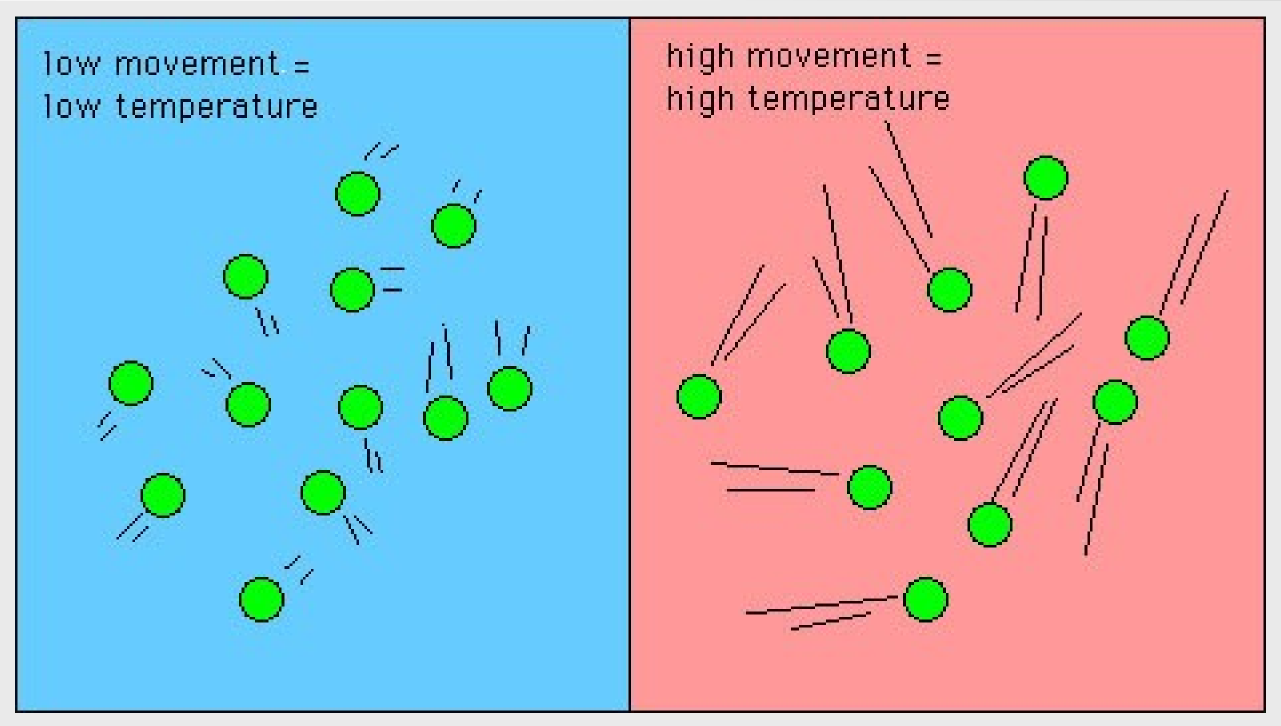
experiment error

- personal error(photo, feeling and reflect, calculation)
- temperature
- lighting

More about Surface Tension

The Relationship between Temperature and Surface Tension

- **Surface tension** - Contraction force caused by the imbalance of **molecular attraction** in liquid surface layer.
- When the liquid **temperature** **increases**, the **molecular motion** will be **enhanced**.



The Relationship between Temperature and Surface Tension

- Then the **average distance** between liquid molecules will be **increased**
- The **attraction** between liquid molecules **decreases**.

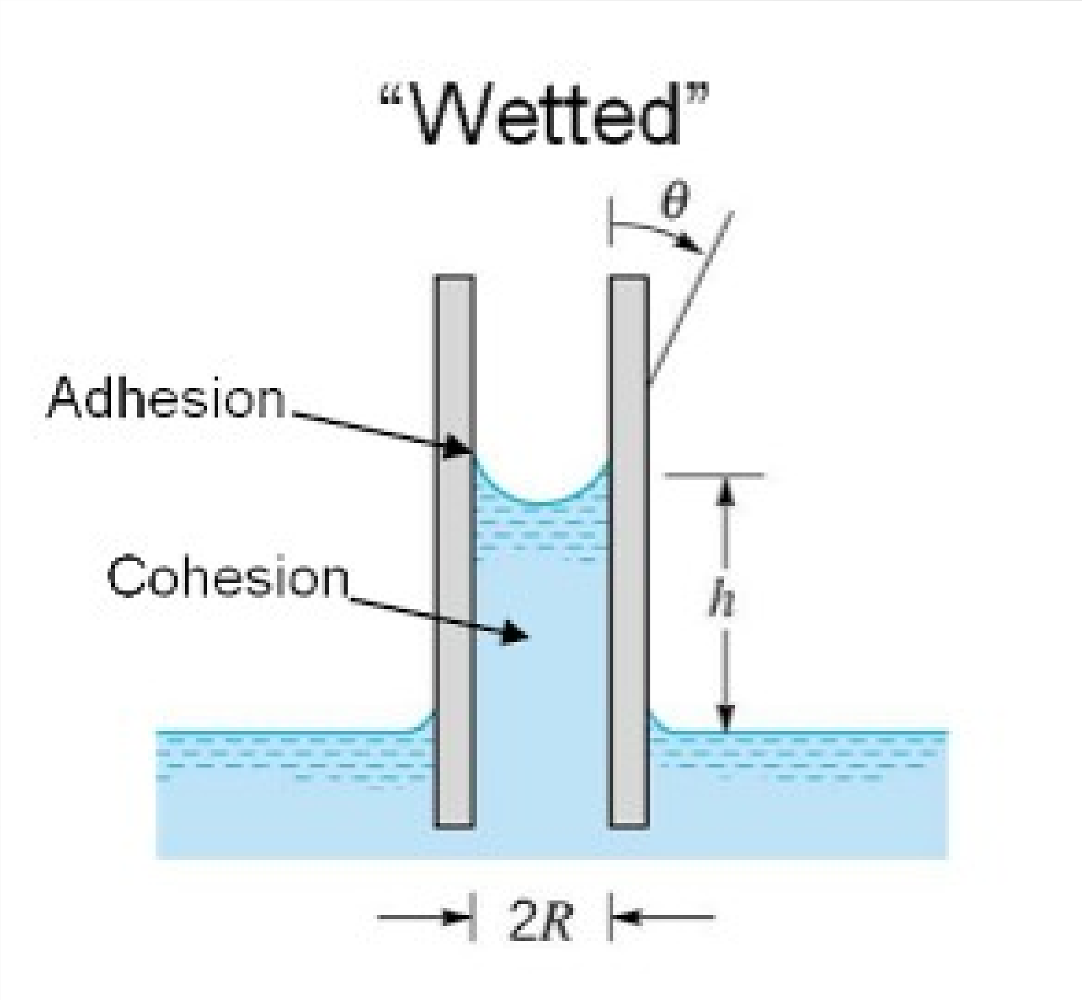
Major Assumption:

Surface tension is **inverse proportional** to temperature.

Future Work

Capillary Rise Method:

- Using a circular capillary to dip into water
- water wets the walls and rises in the capillary to get a balance level
- The surface is a hemispherical concave.



Future Work

- On the surface, there is an equilibrium with gravity force and surface tension
- To derive the equation of surface tension from these balance forces
- Compare the result of experiment to verify the relationship between temperature and surface tension