Nonlinear Energy Harvesting

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Mathematical modeling is a significant addition to the analysis of energy harvesting systems. While experiments and modeling are able to obtain results on their own, the process of using experimentation to improve models and using models to guide experimentation quickens the discovery process and allows for more intelligent research. The primary benefit that modeling provides is not necessarily the quantitative data provided, but the qualitative information gleaned. One can analyze the model to predict the change in the physical behavior of the system without having to perform experiments, which would be far more costly. Further, by predicting the behavior of the system, it is known whether or not that change to the system is worth investigating further via experimentation or more modeling. In the following, we take a known equation for harvesting energy and analyze the energy harvested when the system is at various angles from the original position. The purpose of this is for application in hand held devices, which are often placed in pockets or purses sporadically, thus knowing how much energy is likely to be generated throughout a day rather than the maximum amount of energy generated.

There is no such thing as a perfect transfer of energy. This theoretical situation is called the "Carnot cycle", though such a process cannot be replicated. All processes are subject to energy loss to the environment, such as heating or cooling the ambient air, or performing work on the walls of the system. When work is performed on the environment through methods such as walking or driving on a surface, vibrations are released into the surface that eventually dissipate due to friction. One form of energy harvesting is to recapture these vibrations and recapture as much of the lost energy as possible. This can be done with a capacitor or inductor, but also with a piezoelectric material (PM). In the later situation, the vibrations are used to place compressive and tensile forces on the PM. When this is done below the materials Curie temperature, the atoms deform in such a way that a voltage drop forms across the PM, the direction depending on the direction of the deformation. ***(CITE)*** This voltage is normally used to reform the material after the stress is no longer applied, but in an energy harvesting device it is siphoned. ***(CITE)*** A rectifier must be used in such a circuit because the voltage formed by the PM will essentially be AC.

One design is to use an inverted pendulum to harvest the energy. Layers of a PM are placed at the base around a rod, the latter being attached to an inertial mass in order to more easily recapture the vibrations. The rod in this system is clamped, but the elasticity of the rod and PM allow for the inertial mass to oscillate linearly. It has been found, however, that non-linear oscillation was able to better harvest oscillations. This can be achieved by adding magnets with repelling polarities to the system. One is attached to the end of the inertial mass opposite of the PM and one is placed some distance λ from the pendulum. At some distance λ_c the magnetic repulsion between the magnets causes the energy potential of the oscillator to turn from having a single potential well to two potential wells. As λ is decreased further from λ_c, the energy barrier between the two wells increases. The creation of the energy barrier causes an increase in the deformation of the PM, resulting in more voltage generated by the system and more energy harvested. The exact distance λ at which the system most efficiently harvests energy varies depending on the applied oscillating force. ***(CITE)*** More specifically, as the oscillating force applied increases, λ_{ideal} decreases. This is because the increase in harvested energy is due to the oscillator transitions between the two stable domains. If the energy barrier is too small then it transitions too frequently; if the energy barrier is too large, it does not transition often enough if at all.

The model for this system is:

$$m_{eff}\ddot{x} = \frac{dU(x)}{dx} - \gamma\dot{x} - K_v V(t) + \sigma\varepsilon(t)$$

$m_{eff}\ddot{x}$ is the kinetic force of the oscillator. U(x) is the potential energy of the oscillator, given by:

$$U(x) = Kx^2 + (ax^2 + b\lambda^2)^{-3/2} + c\lambda^2$$

$\gamma\dot{x}$ is the energy dissipated due to the bending of the rod and PM. $K_v V(t)$ is the energy transferred from the PM, where V(t) is given by:

$$\dot{V}(t) = K_c\dot{x} - \frac{V(t)}{R_L C}$$

$\sigma\varepsilon(t)$ is the driving force of the oscillator, which is represented as a stochastic process. The value of each variable is listed in Table 1.

Table 1

Variable	Meaning
m_{eff}	Effective mass of the oscillator
x	Cartesian displacement of the inertial mass from the point of measurement
K	$K_{eff}/2$
K_{eff}	Effective elastic constant of the pendulum
a	$d^2\left(\frac{\mu_0 M^2}{2\pi d}\right)^{-\frac{2}{3}}$
d	Geometrical parameter related to the distance between the measurement point and the pendulum length
μ_0	The permeability constant
M	Effective magnetic moment
b	a/d^2
λ	The distance between the magnets
c	$K_{eff}/2d^2$
γ	Dampening coefficient
K_v	Efficiency of the voltage transferred
C	Capacitance of the piezoelectric
R_L	Resistive load of the circuit
K_C	Coupling constant of the piezoelectric material
σ	Standard Deviation

For the purposes of analyzing the efficiency of the system at various angles, the exact value of the constants is irrelevant, so long as they remain constant. When analyzing this system, one has to look at the three degrees of freedom involved in rotating this system: rotation around a line running through the center of the system (z-axis), rotation around a line running perpendicular to the direction of oscillation, through the base of the system (x-axis), and rotation around a line running parallel to the direction of oscillation, through the base of the system (y-axis). The system as designed makes the assumption that the oscillations are small, and thus the effects of gravity have thus far been ignored in terms of modeling the oscillation of the pendulum. However, as the pendulum rotates around the x and y axes, the effect of gravity becomes non-negligible and must be taken into account. However, the force applied by gravity on the oscillator does not change with rotation around the z-axis, and thus only two degrees of freedom need to be considered. As the system is rotated around the x-axis, the force applied by gravity increases until the system is effectively rotated $\frac{\pi}{2}$ radians. After this point, the gravity begins to apply less force until an effective rotation of π radians, after which the force applied by gravity is 0. When rotated around only the y axis gravity applies force in the direction perpendicular to the direction of oscillation. The rod is stiff enough in this direction that no bending is expected in this direction. However, when there is rotation about both the x and y axes, the force applied by gravity differs than if there was solely rotation about the x or y axis. For clearer representation, reference Figures 1, 2, and 3.

The force applied by gravity must be scaled based on the angle at which the system is titled about the x and y axes. The angle of rotation around the x axis will be labeled β and the angle of rotation around the y axis will be labeled ψ. To start, the case will be considered where ψ=0. The force on the oscillator applied by gravity can be seen in Image 1. The force applied by gravity is decomposed into two components, one being parallel to the direction of oscillation and the other being perpendicular. Only

Figure 1

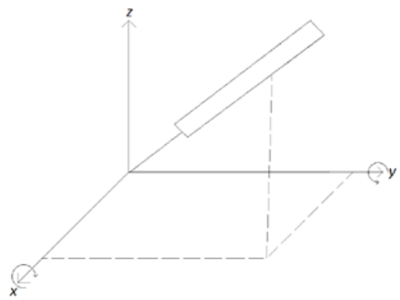


Figure 2

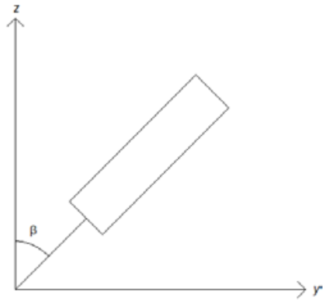
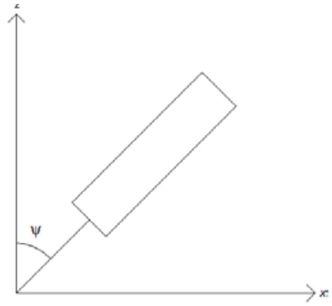


Figure 3



the parallel component will alter the potential energy of the pendulum. β increases and decreases as the system oscillates, thus one must account for the oscillations. Thus, the applied force that is relevant to the analysis is $\vec{G} * \sin(\beta + \theta)$. The next case is when β=0, but as has been mentioned before gravity will have no effect on the system as none of gravity applies in the direction of oscillation. The third case is when $\beta \neq 0 \neq \psi$. In order to consider this case, one must consider a unit vector perpendicular to the direction of oscillation. At this time, it is also perpendicular to the force applied by gravity. However, as ψ increases, the angle between the vectors decreases. By using geometric relations, more easily observed in figure 2, one finds that when $\psi \neq 0$, the force applied in the plane of the oscillator is no longer mg, where $m = m_{eff}$ and g is the gravitational constant. Rather, it is $mg * \cos(\psi)$. Thus, the force of gravity applied by gravity is written as:

$$\vec{G} = -m_{eff}g * \cos(\psi) * \sin(\beta + \theta)\,\hat{z}$$

It is known that $\vec{F} = -\vec{\nabla}U$, where $\vec{\nabla}$ is the gradient in cylindrical coordinates. ***(CITE)*** Thus, $-m_{eff}g * \cos(\psi) * \sin(\beta + \theta) = \frac{1}{L}\frac{dU}{d\theta}\hat{\theta}$ and $-\mathrm{L} * m_{eff}g * \cos(\psi) * \int_0^\theta \sin(\beta + \theta)d\theta \sin(\beta + \theta) = U$. From this point, we see that the potential energy due to gravity is:

$$U = m_{eff}gL * \cos(\psi) * [\cos(\beta + \theta) - \cos(\beta)]$$

Figure 4

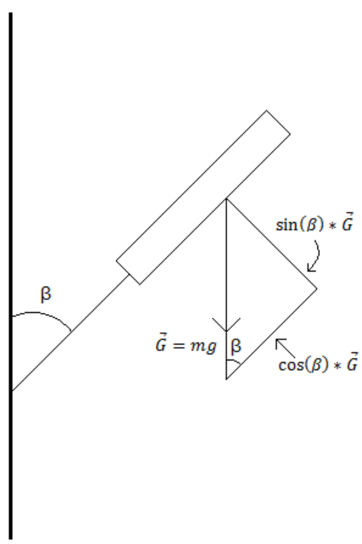
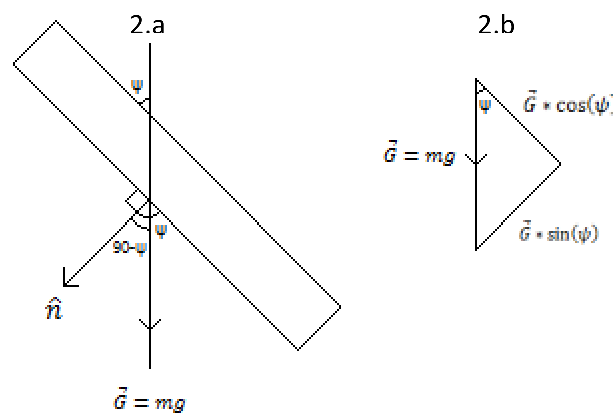


Figure 5



The effect of gravity will change as the system oscillates and as a result the model is more easily analyzed in terms of θ rather than x. Again, since the system is being analyzed qualitatively rather than quantitatively, the point along the rod at which x is measured is irrelevant so long as it is taken consistently. So, the point will be the end of the rod. The transition to a model in terms of θ is achieved by converting the system to cylindrical coordinates, which results in the following equivalencies:

$$x = \mathrm{L} * \sin(\theta)$$

$$\dot{x} = \mathrm{L} * \cos(\theta)\dot{\theta}$$
$$\ddot{x} = -\mathrm{L} * \sin(\theta)\dot{\theta} + L * \cos(\theta)\ddot{\theta}$$
$$dx = L * \cos(\theta)\, d\theta$$
$$\frac{dU}{dx} = \frac{1}{L * \cos(\theta)}\frac{dU}{d\theta}$$

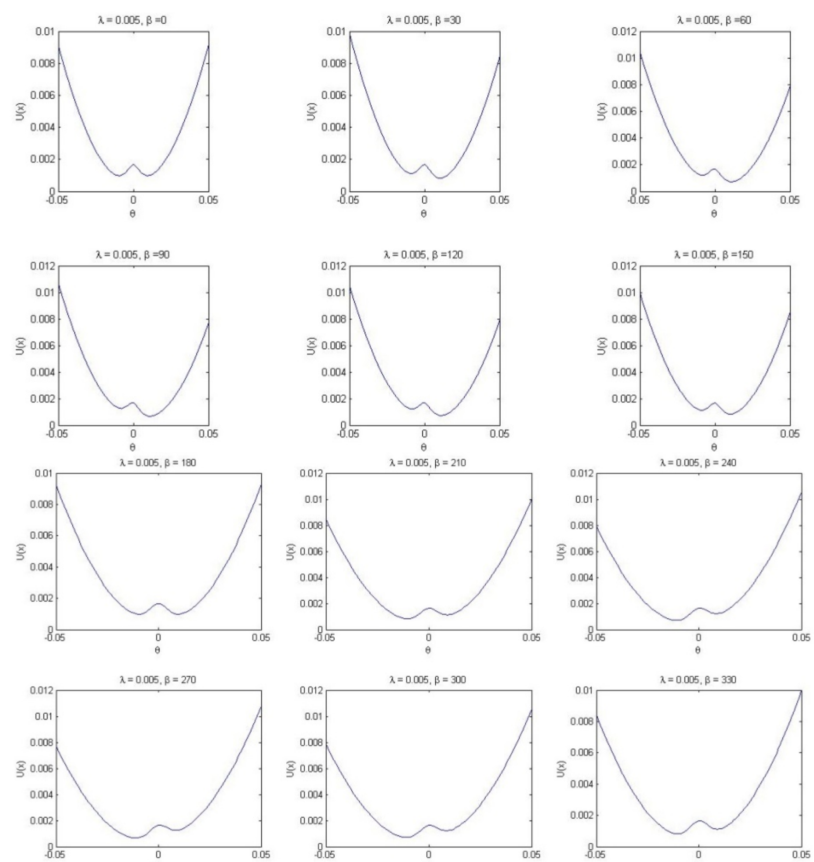
Thus, the model and potential energy in terms of θ are written as:

$$m_{eff}\left(-\mathrm{L} * \sin(\theta)\dot{\theta} + L * \cos(\theta)\ddot{\theta}\right) = \frac{1}{L * \cos(\theta)}\frac{dU}{d\theta} - \gamma\, \mathrm{L} * \cos(\theta)\dot{\theta} - K_v V(t) + \sigma\varepsilon(t)$$
$$U(\theta) = K[\mathrm{L} * \sin(\theta)]^2 + (a[\mathrm{L} * \sin(\theta)]^2 + b\lambda^2)^{-3/2} + c\lambda^2 + m_{eff} g L * \cos(\psi) * [\cos(\beta + \theta) - \cos(\beta)]$$

As noted before, it is assumed that θ is small, so the above equation simplifies to:

$$\ddot{\theta} = \frac{1}{m_{eff} * (\mathrm{L})^2}\frac{dU}{d\theta} + \theta\dot{\theta}^2 - \frac{\gamma}{m_{eff}}\dot{\theta} - \frac{K_v}{m_{eff} * L}V(t) - \frac{\sigma}{m_{eff} * L}\varepsilon(t)$$
$$U(\theta) = K\,\mathrm{L}^2\,\theta^2 + (a\,L^2\,\theta^2 + b\lambda^2)^{-3/2} + c\lambda^2 + m_{eff} g L * \cos(\psi) * [\cos(\beta + \theta) - \cos(\beta)]$$

Figure 6



They potential energy was first analyzed at varying values of λ and β, mapping the potential U as a function of θ while ψ=0. The results are shown in Figure 6, where λ is in meters, β is in degrees, and θ is in radians. It was found that, as beta increases, the stability of each domain changes. The potential energy of the system is decreased in the direction which gravity is applying to the oscillator, leading to a deeper potential well. Simultaneously, oscillation in the opposite direction of oscillation is more difficult, leading to a shallower potential well in the direction opposite of gravity. There is no bias when β=0 or β=π, as gravity effects both directions equally.

Following, the potential energy was analyzed for various values of λ, graphing the potential energy of the system as a function of θ and β. It was found that the value of λ does not change the bias created by gravity, only the potential barrier between the two stable domains.

Figure 7

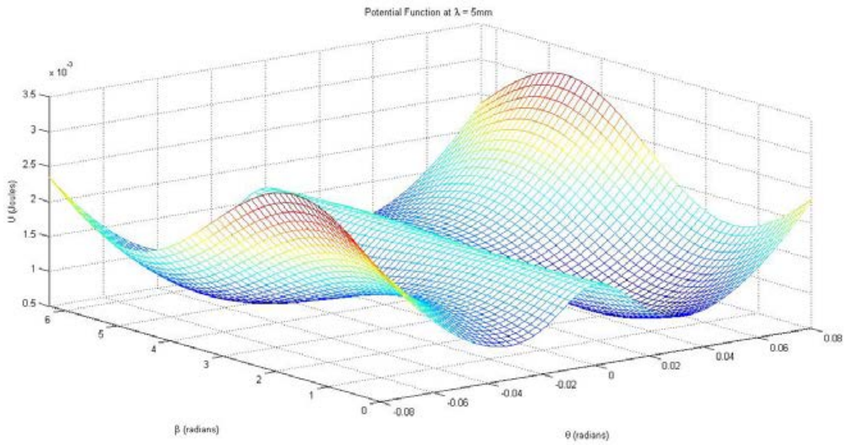


Figure 8

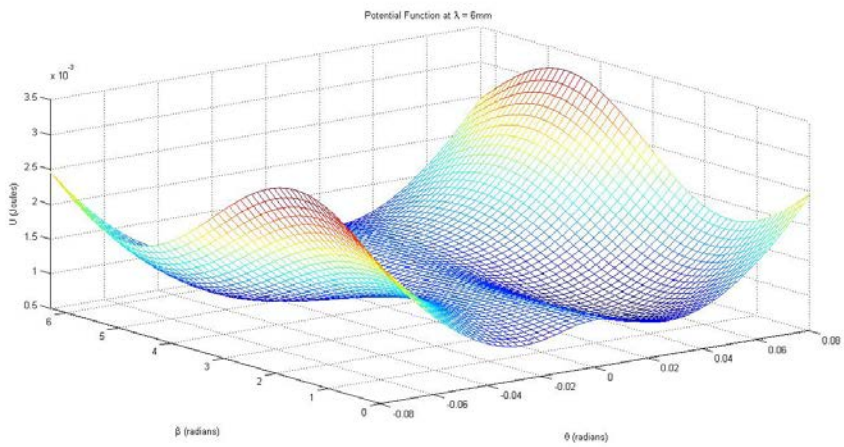
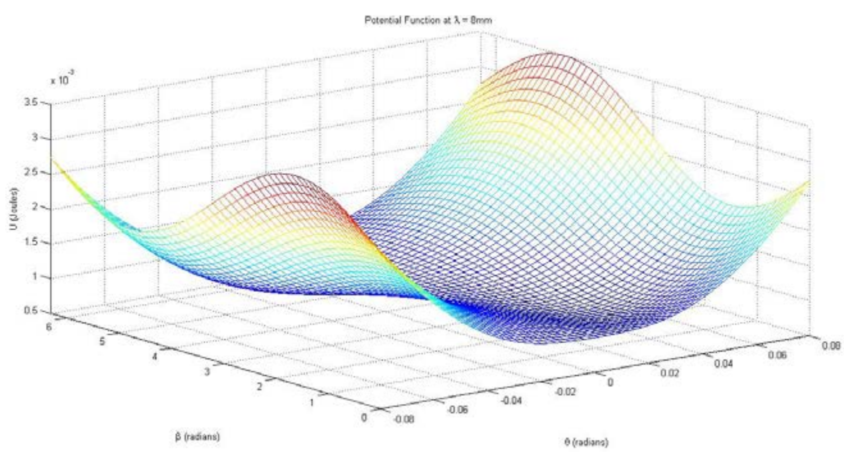


Figure 9



Next, the variation in the potential energy as a function of θ and ψ were found for β=0, $\frac{\pi}{4}$, and $\frac{\pi}{2}$. When there is solely rotation around the y-axis, there is no variation in the potential energy of the oscillator. This is to be expected, as none of the force applied by gravity will be in the direction of oscillation.

Figure 10

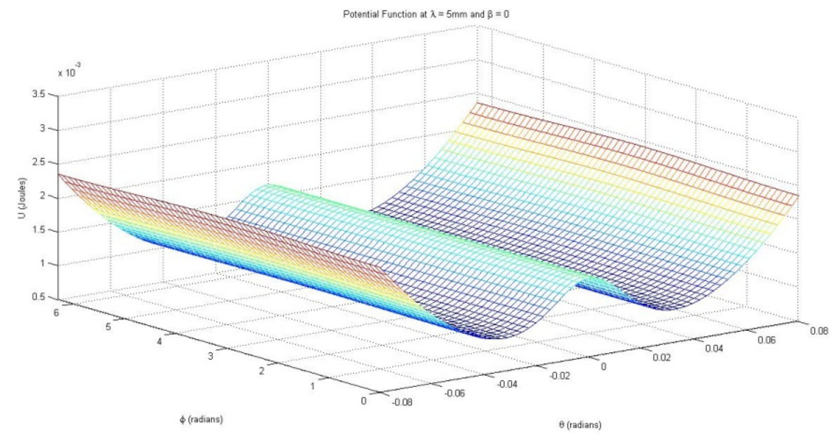


Figure 11

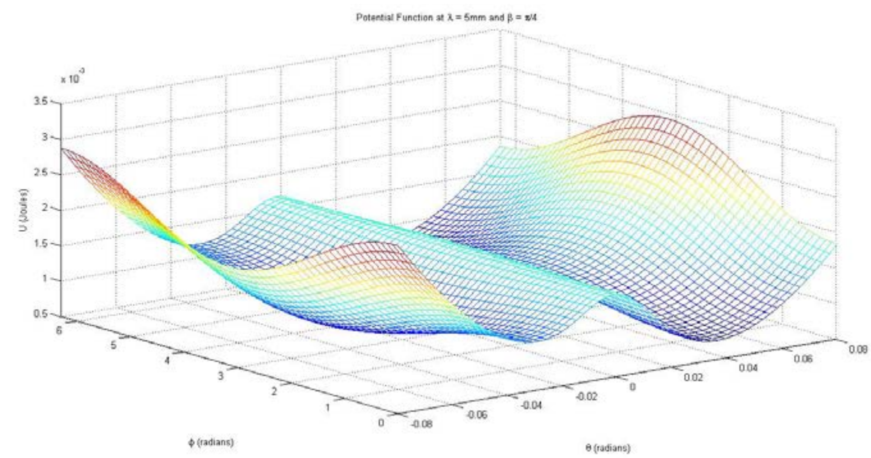


Figure 12