Universal Properties of Mythological Networks
Midterm report: Math 485

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Abstract:

In the past few decades, network theory, as found in computer science and statistics, has increasingly been applied to social networks as well. These networks are made up of people or groups who interact with others in statistically interesting ways. Carron and Kenna, the authors of the original paper Universal Properties of Mythological Networks (1), used statistical metrics of network theory to determine the plausibility of networks found in epic literature. Through the use of universal properties, the authors determined the veracity of three epics: Táin, The Iliad, and Beowulf. Based on the characteristics of the social networks found in these three tales, Carron and Kenna found that The Iliad was the most similar to a real life network, while Beowulf had some bits of fiction along with realistic network composition. Táin was found to be the most fictional of all three networks.

Introduction:

The field of comparative mythology has been studied in depth by academics for centuries. In recent decades, however, network theory has increasingly been applied to mythology. This paper, authored by Carron and Kenna (1), explores one application of this. The authors begin by studying the properties of several fictional networks: Victor Hugo's *Les Miserables*, the Marvel Universe, The Fellowship of the Ring, Harry Potter, and others. Next, the authors study the properties of several nonfictional networks, which include real social networks, company directors, jazz musicians, and others. These two sets of networks make up the extreme ends of the scale. The purpose of the paper is to analyze the properties of three epics, Táin, The Iliad, and Beowulf, and to determine where they fall on the scale of purely fictional to purely nonfictional. By using statistics and universal properties of networks, the authors will be able to determine which of the epics is based on the structures of real networks, and which are purely fictional.

Methods and Tools:

Within Network Theory, methods of statistical analysis are used to classify and compare various networks quantitatively. These connectivity descriptors are calculated typically from nodal degrees, the number of neighbors each element of the network links to. Degree distributions are the probabilistic histograms used when considering the degrees of a network as a whole; these distribution functions yield the probability of a random element having each degree value. Degree distributions functions often behave following a power-law dependency on node degrees: $p(k) \sim k^{-\gamma}$, where $p(k)$ is the probability of a node having degree, k and γ is a positive, constant parameter of the network (1).

Scale-free networks are a category of complex networks, frequently occurring in many areas of study and defined by this type of power-law degree distribution, specifically where γ falls between two and three; this distribution is indicative of large, thoroughly connected hub nodes supported by lesser degree node connections (5). Scale-free networks are of interest in that they are characterized as robust networks with small world efficiency. The robustness of a network refers to its resilience, a measure of the extent to which a network remains connected after the removal of random nodes (1). While, small world efficiency is the idea that a relatively short distance must be traveled between any given nodes, resulting in a compact system diameter or width. Graphically, scale-free networks demonstrate giant components formed by influential nodes connecting many other smaller degree nodes, accounting for the robustness of the network (1).

A measure of how influential a node is may be quantified by its Betweenness Centrality, a statistical measure of flow or paths through a network. A geodesic is a path of the shortest length between two elements of a network, so the proportion of all geodesics, σ_l through a point relative to the total number of geodesics possible, σ measures how central to the network the point is (1). This calculation is normalized to account for the possibility that all geodesics pass through a certain node and is calculated by the following equation, where centrality of node-l is denoted, g_l (1):

$$g_l = \frac{2\sum_{i \neq j} \frac{\sigma_l}{\sigma}}{(N-1)(N-2)}$$

Additionally, network nodes may be characterized by their clustering coefficient, a measure of the density of connections indirectly surrounding them. The clustering coefficient of an individual node is calculated as the percentage of possible connections between immediate neighbors, actually realized within the network (1). Taken as an average over an entire network, the clustering coefficient may be used as a measure of transitivity and yields the probability that any two neighbor-nodes are connected (1). As a percentage, the coefficient will fall between zero and one, those closest to the latter indicating networks that are "cliquey" or strongly grouped. A specific sub-branch of scale-free networks, known as hierarchical networks, are known to exhibit clustering coefficients in this manner (1).

Among network descriptions, the term hierarchical is applied to those in which modular structures form largely isolated groups. Networks containing these node communities are classified as having clustering coefficients that follow a power-law dependency on the inverse of its node degrees (1). Easier demonstrated graphically, hierarchical networks display predominately low-degree nodes organized into communities by few structural high-degree vertices; this results in dense sub-graphs of low connectivity points.

Assortativity is a preference for a network's nodes to attach to others that are similar in some way. In many observable networks, it is easy to find correlations between nodes with similar degrees (2). Highly connected nodes tend to be connected with other high degree nodes. This tendency is known as assortativity. Assortativity coefficient is a measure of assortativity. It is the Pearson correlation coefficient of degree between pairs of linked nodes (2). Positive values of r indicate a correlation between nodes of similar degree, while negative values indicate relationship between nodes different degree. The assortativity coefficient is given by

$$r = \frac{\sum_{jk} jk(e_{jk} - q_j q_k)}{\sigma_q^2}$$

where j and k refer to two nodes and e_{jk} is the joint distribution of remaining degree of those two nodes q_j and q_k are the distribution of remaining degree for node j and node k. σ_q^2 is the variance of remaining degree for node q (2).

Closed triads are often found in scale free networks. A triad is a group of three nodes. A closed triad means every node in the triad has degree two so every node in the triad has a connection.

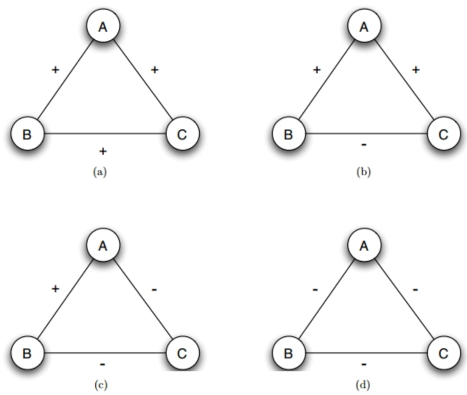

Figure 1. Closed triads.

In a network, closed triad with odd number of negative edge is known as structural balance (3). We take a closed triad using friendly and hostile as edges as our example. Positive edge means friendly and negative edge means hostile. In figure 1(a), nodes A, B, C are mutual friends. The structure of the closed triad is balanced and there are zero negative edges. In figure 1(b), A is friends with B and C, but B and C don't get along with each other. In such case, A tends to make B and C become friends or A will have to choose side with either B or C. The structure of the closed triad is not balanced and there is one negative edge. In figure 1(c), A and B are friends with C as a mutual enemy. The structure of the closed triad is balanced and there are two negative edges. Finally, in figure 1(d), A, B, C are mutual enemies. In such case, two of the three will be motivated to team up against the third, since "the enemy of my enemy is my friend". The structure of the closed triad is not balanced and there are three negative edges. Overall, this propensity to disfavor odd number of hostile links in a closed triad is known as structural balance (4).

There are two types of attacks in network theory: target attack and random attack. Target attack means removing nodes with high degrees and random attack means removing nodes randomly. For example, removing 5 nodes with the highest degree is a target attack and removing fifty nodes randomly is a random attack. There are two properties of the network within attacks: robust and vulnerable. While attacking the network, if the network remains unaffected, we say the network is robust. On the other hand, if the network breaks down, we say the network is vulnerable.

Results:

Carron and Kenna determined that all seven networks studied were similar to real social networks in that they are small world, highly clustered, hierarchical and resilient to random attack. Based upon the seven stories examined and various other networks they were provided from outside sources, Carron and Kenna were able to devise a table that showed the universal properties of fictional and social networks (shown below).

	Social	Myth (friendly)	Fiction
Small world	Yes	Yes	Yes
Hierarchy	Yes	Yes	Yes
$p(k)$	Power law	Power law	Exp.
Scale free	Yes	Yes	No
G_c	$< 90\%$	$< 90\%$	$> 90\%$
TA	Vulnerable	Vulnerable	Robust
RA	Robust	Robust	Robust
Assortative	Yes	Yes	No

Figure 2. Summary of universal properties belonging to Social and Fictional networks (1).

They found that social networks have power law distributions, were scale free, had smaller giant components, were vulnerable to targeted attack and were assortative, whereas fictional networks have a exponential degree distribution, are not scale free, have larger giant components, are robust under targeted attack and random attack and are disassortative.

Now that it was possible to distinguish the difference between a social network and a fictional network, Carron and Kenna could closely look at the properties of the Iliad, Beowulf and Táin to determine which type of network each epic was. They found that the Iliad had properties most similar to those of a social network. Beowulf was determined to be a mix of reality and fiction. It is small world, like a social network; however, it is disassortative like a fictional network. Carron and Kenna went on to examine various ways to manipulate the data in order to get the distribution of the Beowulf network similar to that of a real social network. They found that if the main character is removed the network becomes assortative, like a social network. Thus, they determined that although the epic as portrayed is not a real social network, it has an assortative foundation, rendering it possible that some of the story is based upon real people and real events. Táin was found to be the most fictitious of the three epics. Again, Carron and Kenna manipulated the Táin network to see if they could make it closely resemble a social network. They compared the distribution of Táin to that of Beowulf and found that they had some similarities. Carron and Kenna noticed that if the top six vertices of the Táin network are removed, the distribution of the network is very similar to that of Beowulf. So, they removed the weak social links connected to those characters. After doing so, the Táin distribution became similar to that of the Iliad.

Figure 3. The degree distribution of (a) Beowulf and Táin and (c) truncated power laws for Táin and the Iliad (1).

What Carron and Kenna derived from this finding is that the artificiality of the Táin network is closely associated with these six characters, and therefore they are most likely amalgams (mixtures of characters that got fused as the story was passed down from generation to generation).

A summary of Carron and Kenna's findings is shown below:

	Social	**Fiction**	**Iliad**	**Beowulf**	**Tain**
Small world	Yes	Yes	Yes	Yes	Yes
Hierarchy	Yes	Yes	Yes	Yes	Yes
p(*k*)	Power law	Exponential	Power Law	Power law	Exponential
Scale Free	Yes	No	Yes	Yes	Yes
Gc	<90%	>90%	>90%	<90%	>90%
TA	Vulnerable	Robust	Vulnerable	Vulnerable	Robust
RA	Robust	Robust	Robust	Robust	Robust
Assortative	Yes	No	Yes	No	No

Figure 4. A summary of Carron and Kenna's findings (1).

Conclusions:

Carron and Kenna chose three epic mythologies, the Iliad, Beowulf and Táin, to analyze using network theory. They first set out to establish the networks of each epic and then compared the distributions of those networks to four known fictitious networks; the Marvel Universe, Victor Hugo's *Les Miserables*, The Fellowship of the Ring, and Harry Potter. By examining universal properties, such as assortativity, small worldedness, degree distribution and hierarchy, Carron and Kenna discovered that all three networks had similarities to real social networks. It was determined that the Iliad is the most similar to a real social network, Beowulf has properties of fictitious and social networks, and Táin was the most fictitious of the three networks.

Based upon the data collected, the results determined and the universal properties shown by Carron and Kenna, our group will be looking at various TV shows to determine the networks established by character-to-character interaction. This interaction is established when one character communicates with another character. By examining these connections, we will be able to determine the interconnectedness of the TV show networks and also determine if the story lines match those of actual social networks. We will be using the universal properties presented in the paper by Carron and Kenna to analyze all of the networks for the various TV shows. Also, we will try to find unknown universal properties that hold for these TV shows.

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