

## **Project Description**

- The basis of the project is the analysis of a linear search problem for an object placed from a given probability density function using analytical and numerical methods.
- Traditional Linear Search Problem:
  - An object is placed on a line using a given probability density
  - Using an initial step length, the searcher goes back and forth according to step lengths given by a specific formula
- The goal is to determine the optimal search pattern for a given probability density.
- From the reference paper [1], for a Pareto distribution the optimal plan, the optimal stepping pattern follows a geometric series.
- For the given project, a different distribution (exponential) was chosen for the car placement.

# Scientific Challenges and Applications

- A model is required to determine the optimal search pattern for different probability densities.
- Applications include a simple robot that returns to the origin after each step to report whether it has found the object being searched for or not.
- An additional sight parameter would model such a robot having a camera allowing it to see farther ahead along the path.



# Search for a Parked Car on a Forest Road

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# Methodology

Image Source: Art.com

- 1. In the reference paper [1], an optimal search pattern was determined analytically for the Pareto distribution. Therefore, the first attempt to optimize the problem was determining an analytical solution.
- 2. After calculations, a formula (see glossary) for expectation length was determined. However, for the chosen probability density function, an analytical solution for the expectation length could not be determined.
- 3. Monte Carlo simulations were used to model the searches and determine the optimal search for the chosen probability density.
- 4. An additional sight parameter was added to make the problem more realistic. The sight parameter allows the searcher to look ahead and see if they can spot the car from where they are standing.

# Results

- 1. The graphs to the left show how changes in the sight parameter affect the optimal values of alpha and delta in the geometric series describing step size.
- 2. Using the sight parameter with the Monte Carlo simulations, the average search lengths for different combinations of alpha and delta are shown. The optimal alpha/delta pair occurs at the minimum of the graph, where the expectation length for the search is lowest.
- Preliminary simulations did not include the sight parameter. Because the simulation contained fewer parameters, the results from the Monte Carlo simulations produced more definitive results for fewer trials.



f(x)



# Glossary of Terms

= expectation length, given by: EL

$$L = 2\left[\sum_{n=1}^{\infty} x_n - \sum_{m=2}^{\infty} x_m f(x_{m-1})\right]$$

- = step length, given by the geometric series:  $x = \Delta a^n$
- = geometric constant
- = geometric ratio
- F(x)= probability density function
  - = cumulative distribution function for F(x)
  - = sight parameter, determined by the following distribution



### Monte Carlo Method:

The Monte Carlo Method is used when there is a random event occurring in the simulation. A large number of random samples are taken in order to describe the behavior of the system.

### Example of Monte Carlo Method:

Points are randomly plotted on the diagram shown below. As the number of points plotted increases, the ratio of points inside the circle to total points plotted will converge to a value of  $\pi/4$ .





### **Image Source:**

http://mathfaculty.fullerton.edu/mathews/n2003/montecarlopi/Mon teCarloPiMod/Links/MonteCarloPiMod\_Ink\_2.html

### References

1. Yu Baryshnikov and V Zharnitsky, Search on the brink of *chaos*, Nonlinearity, **25**, 3023–3047 (2012).

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