Dynamics of the Elastic Pendulum; Large Amplitude

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Project Description

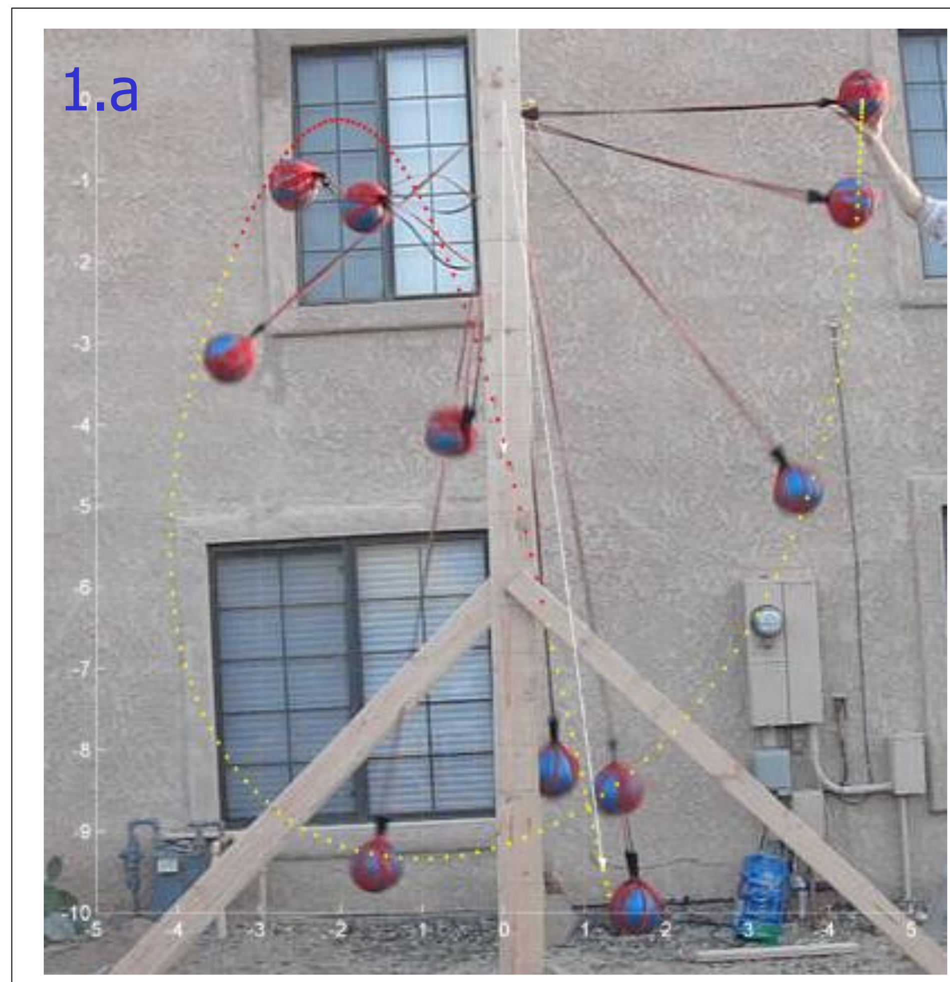
- The elastic pendulum is a complicated system, sensitive to initial conditions, which can be considered chaotic in large amplitudes.
- Using numerical methods the team has shown that the qualitative behavior can be predicted accurately for up to 3 seconds in some regimes of initial conditions.
- Characteristics of an ideal spring-mass have been researched exhaustively by Dr. Peter Lynch with applications to atmospheric phenomena when studied in small amplitude. [1]
- The team set out to study the dynamics of an exercise band swinging a mass that is small in diameter compared to the length of the band.

Scientific Challenges

- There is no analytical solution to this system in either 2- or 3- dimensions. A mathematical model gives a numerical description to a specific spring-mass system.

Potential Applications

- Demonstrating the constraints of the numerical model compared to the physical model could motivate further research in testing regimes of extreme bungee jumping.



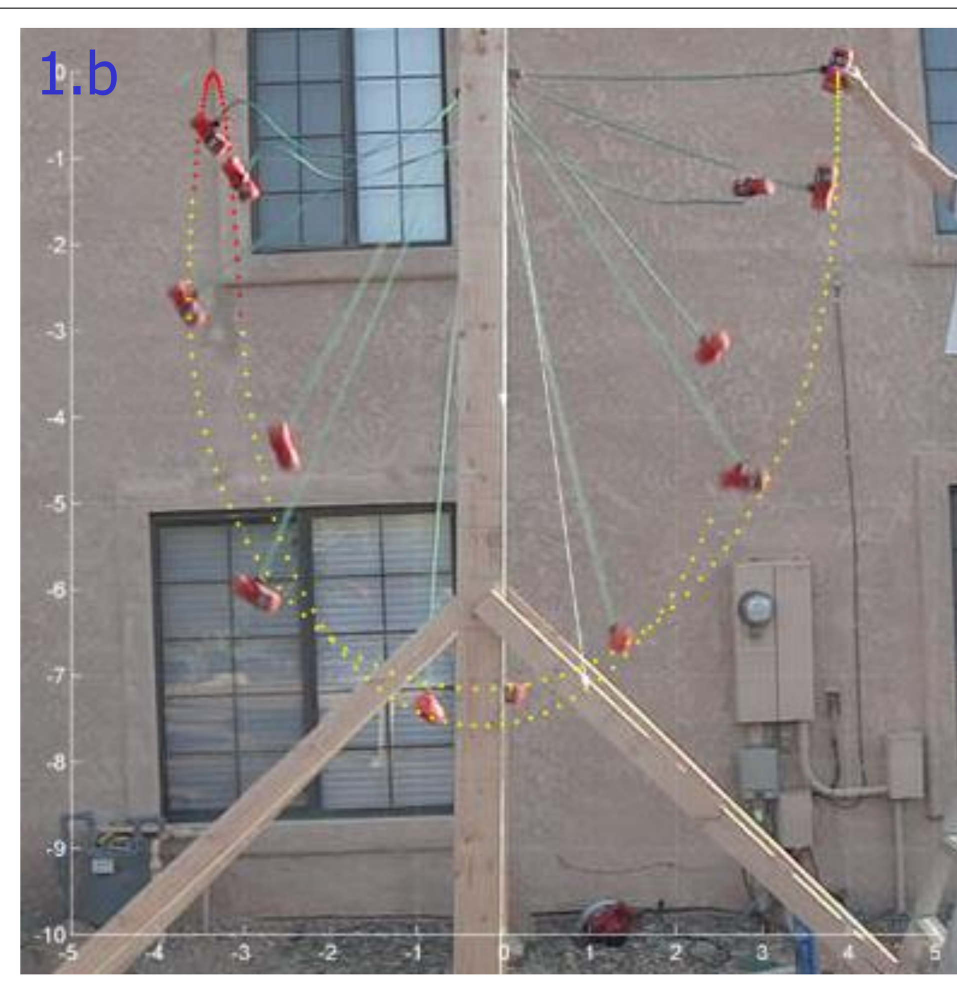


Figure 1a. 10 pound ball attached to elastic band with elastic response coefficients: a = -0.8028 and b=8.5234. Figure 1b. A 3 pound dumbbell attached at center of mass to an elastic band with a= -0.3807 and b= 3.589. For both figures, the MATLAB predicted path is over-laid in yellow (below the apex) and red (above the apex). Weights were released from characteristic length of the band, horizontally.

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Methodology

1. The elastic pendulum was modeled first in MATLAB under different conditions for the spring constant, the length of the spring (l_0), and the mass (m) of the bob.
2. An apparatus was built using elastic "bungee" exercise band as a spring and a heavy ball as the mass. The linear treatment of the elastic response did not model this experiment well.
3. The following system of ODE's were derived from the Lagrangian:

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{a}{3}(r - l_0)^3 - \frac{b}{2}(r - l_0)^2 - mgz$$

$$\begin{cases} \ddot{x} = \frac{a}{m} \cdot \frac{(r - l_0)^2}{r} x + \frac{b}{m} \cdot \frac{r - l_0}{r} x \\ \ddot{y} = \frac{a}{m} \cdot \frac{(r - l_0)^2}{r} y + \frac{b}{m} \cdot \frac{r - l_0}{r} y \\ \ddot{z} = \frac{a}{m} \cdot \frac{(r - l_0)^2}{r} z + \frac{b}{m} \cdot \frac{r - l_0}{r} z + g \end{cases}$$

4. Using a high speed camera, images of the mass-spring system in motion were caught at 0.25 second intervals.
5. The MATLAB graph was over-laid with the corresponding trial. (Figs. 1a, 1b)
6. This technique was repeated using a long-exposure photograph of the system, this time with a light attached to the mass to illuminate the entire path. (Fig. 2)

Results

1. Data at 0.25 second intervals (from the high-speed video of our spring-mass system) was plotted and fit to a Fourier series
2. Data (in many cases) reliably fit to a function using a finite Fourier expansion for the regime 0 to 5 seconds. (Fig. 3)

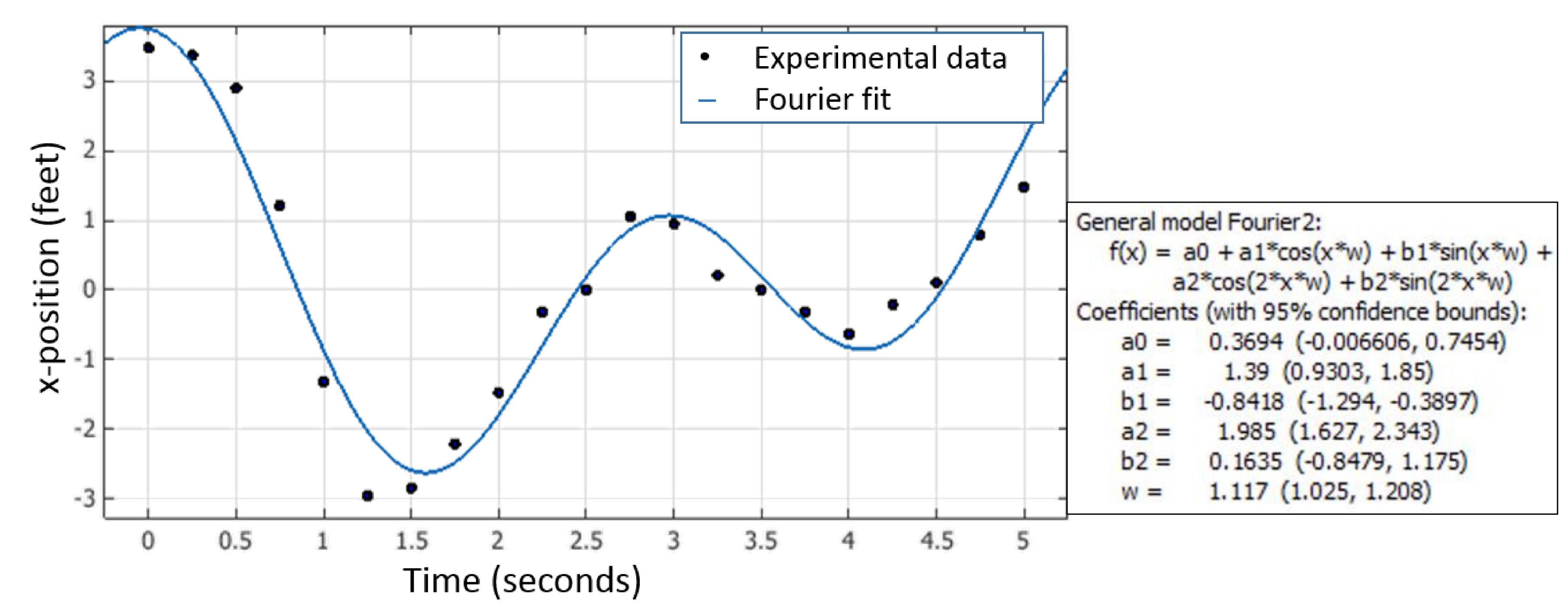

Figure 3. Fourier series fit of experimental data

Glossary of Technical Terms

Elastic response coefficents: Coefficients "a" and "b" of elastic force to extension relationship of the form $F = a(r - l_0)^2 + b(r - l_0)$

Ideal Spring-Mass System: Non-realistic spring: spring does not bend, spring behaves linearly, spring loses no energy, mass is considered a point mass, etc.

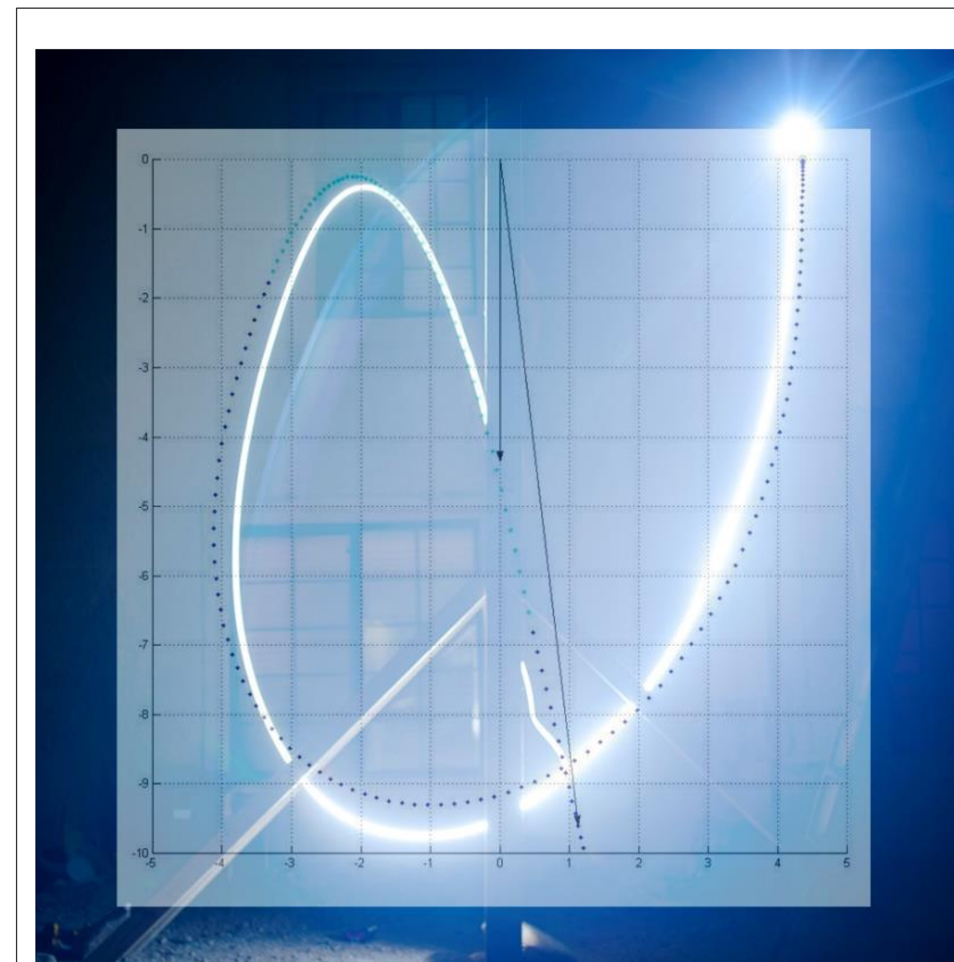

Figure 2. Long exposure shot showing the path taken by the system described in Fig. 1.a.

References

1. Holm, Darryl D. and Peter Lynch, 2002: *Stepwise Precession of the Resonant Swinging Spring*, SIAM Journal on Applied Dynamical Systems, 1, 44-64
2. Lynch, Peter, and Conor Houghton, 2003: Pulsation and Precession of the Resonant Swinging Spring, Physica D Nonlinear Phenomena
3. Lynch, Peter, 2002:. Intl. J. *Resonant Motions of the Three-dimensional Elastic Pendulum* Nonlin. Mech., 37, 345-367

Acknowledgments

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