Numerical Modeling of Diffusion and Phase Transitions in Heterogeneous Media: Better Hard-Boiled Eggs Through Mathematics

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Project Description

- Model phase transitions for cooking an egg.
- Using the biological aspect of the protein structure within the egg to set the transition point through protein denaturation.
- Due to the time sensitive phase transitions in the egg a model is needed to carry out accurate results.
- Observe phase transitions from liquid to solid in the yolk and no phase transition in the egg whites.

Scientific Challenges

- The significance of developing such a model encapsulates multivariable diffusivity and phase transitioning.
- This model allows for clarity and certainty to back up theory for boiling eggs at various times and temperatures to obtain different results.

Potential Applications

- For those with a peculiar pallet and desire for specific parts of the egg to be cooked (or not) can utilize this model to satisfy their gastronomical requirements
- By using spherical heterogeneous media this numerical model can provide heat transfer behavior along with potential phase transitioning.

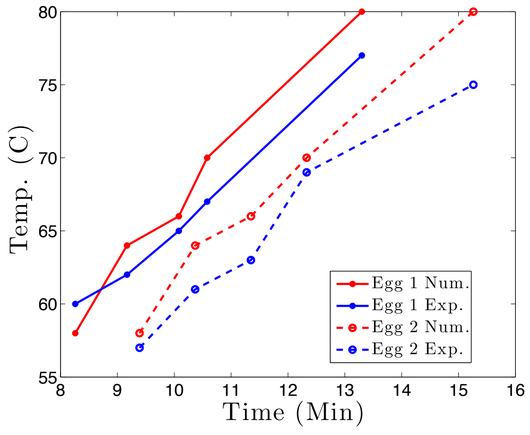

Figure 1: Numerically predicted cooking temperatures(red lines) plotted with experimental temperatures (blue lines) against time. Egg 1 4.584 cm and Egg 2 4.902 cm. The plots suggest that the numerical model over-predicts cooking times by a small amount.

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Figure 2: Egg 4.584 cm cooked after different cooking times. Temperature profile shown in Figure 1.

Methodology

1. Governing Equation:

$$\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(D(r)r^2\frac{\partial u}{\partial r}\right)$$

2. Utilize finite volume approximation through MATLab.
3. Consider the diffusivity to be a piecewise function and calculate it to be the harmonic mean at the interface.
4. Develop a numerical method to model the diffusivity in a non-uniform media and study the effect of changing the parameters.
5. Research values for egg white and yolk diffusion constants [4] along with the denaturation process when heating an egg.
6. Test predictions and compared experimental data to predicted data.

Results

1. Our numerical experiment predicted higher cooking temperatures than were observed in physical experiments (cf. Figure 1).
2. Predictions obtained using our numerical model were verified using physical experiments; we observed a phase transition from liquid to solid in the yolk and no phase transition in the egg whites.
3. Observed analytically predicted scaling of cooking time [1] in both numerical and physical experiments (cf. Figure 3).

Glossary of Technical Terms

Thermal Diffusion: measure of thermal inertia. In a substance with high thermal diffusivity, heat moves rapidly through it because the substance conducts heat quickly relative to its volumetric heat capacity

Conduction: process by which heat or electricity is directly transmitted through a substance when there is a difference of temperature or of electrical potential between adjoining regions, without movement of the material.

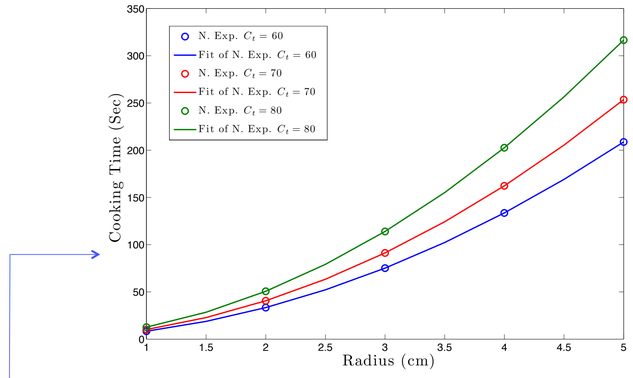

Figure 3: Cooking time plotted against radius for different cooking temperatures. This plot suggests the cooking time scales with the square of the radius.

References

1. Andrey Varlamov and Lev Aslamazov, The Wonders of Physics, World Scientific, 2012
2. Peter Barham The Science of Cooking, Springer, 2001
3. MATLAB and Statistics Toolbox Release 2012b, The MathWorks, Inc., Natick, Massachusetts, United States.
4. Thomas L James and Kenneth T Gillen, Nuclear Magnetic Resonance Relaxation Time and Self-Diffusion Constant of Water in Hen Egg White and Yolk, 1972

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