Stability of Lagrange Points: The James Webb Space Telescope

Project Description

- ❑ Investigates the forces on an object influenced by the gravity of the Sun and the Earth
- ❑ Calculates the stable points of this system, known as Lagrange Points
- ❑ Analyzes stability of the five Lagrange Points
- ❑ Determines time range for stability
- ❑ Predicts behavior of an object near a Lagrange Point

Scientific Challenges

- ❑ There is no general analytical solution to the three-body problem
- ❑ It is impossible to predict the motion of the Sun, Earth, and an object for every case

Potential Applications

- ❑ The James Webb Space Telescope (JWST) will be placed at the second Lagrange Point (L2)
- ❑ JWST, named the successor to the Hubble Space Telescope, will:
  - * Observe infrared light and allows it to see greater distances than Hubble
  - * Gather data about the aftermath of the Big Bang and the formation of stars and galaxies
- ❑ L2 is an ideal place for JWST:
  - * The telescope is able to stay in the same location with minimal position correction
  - * Can observe the entire sky without interference from the sun

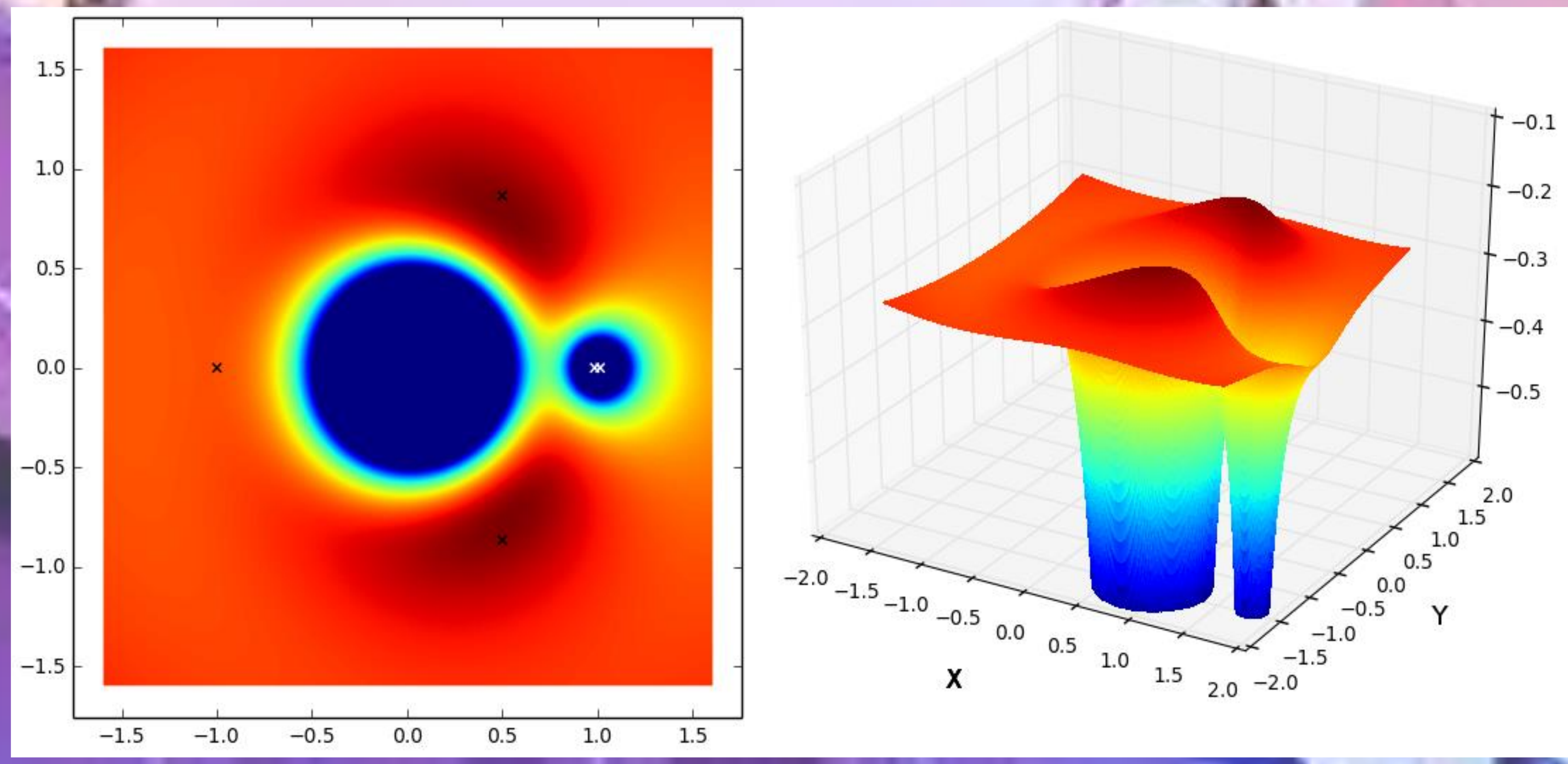

FIGURE 2: Energy Contour plot. Extrema of the generalized potential are marked with x's.

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Method of Solving Analytically

1. Solved for the position of the **Lagrange Points** by implementing the **coriolis force** and the **centrifugal force** in a **co-rotating frame**.
2. Linearized the equations of motion around the equilibrium points to obtain their stabilities.

Method for Numerical Simulations

1. Used Python and C/C++ with a fourth order Hermite Integrator to develop a 2D program to simulate the Earth-Sun system and Lagrange Points within a co-rotating frame.
2. Imported the equations of motion for the system.
3. Enhanced the program to run in 3D.
4. Within the program an object was placed at known Lagrange Points.
5. Tested the length of time the object will stay in orbit.
6. Recorded the orbital path of the object within the simulation.

Results

Point	Location	Stability	Drift Time (computational)
L1	$\left(R\left[1-\left(\frac{\alpha}{3}\right)^{\frac{1}{3}}\right],0\right)$	Saddle	24.9 days
L2	$\left(R\left[1+\left(\frac{\alpha}{3}\right)^{\frac{1}{3}}\right],0\right)$	Saddle	24.4 days
L3	$\left(-R\left[1-\left(\frac{5\alpha}{12}\right)^{\frac{1}{3}}\right],0\right)$	Saddle	43.12 years
L4	$\left(\frac{R}{2}\left(\frac{M_1-M_2}{M_1+M_2}\right),\frac{\sqrt{3}}{2}R\right)$	Stable Due to coriolis force	Not Calculated Due to stability
L5	$\left(\frac{R}{2}\left(\frac{M_1-M_2}{M_1+M_2}\right),-\frac{\sqrt{3}}{2}R\right)$	Stable Due to coriolis force	Not Calculated Due to stability

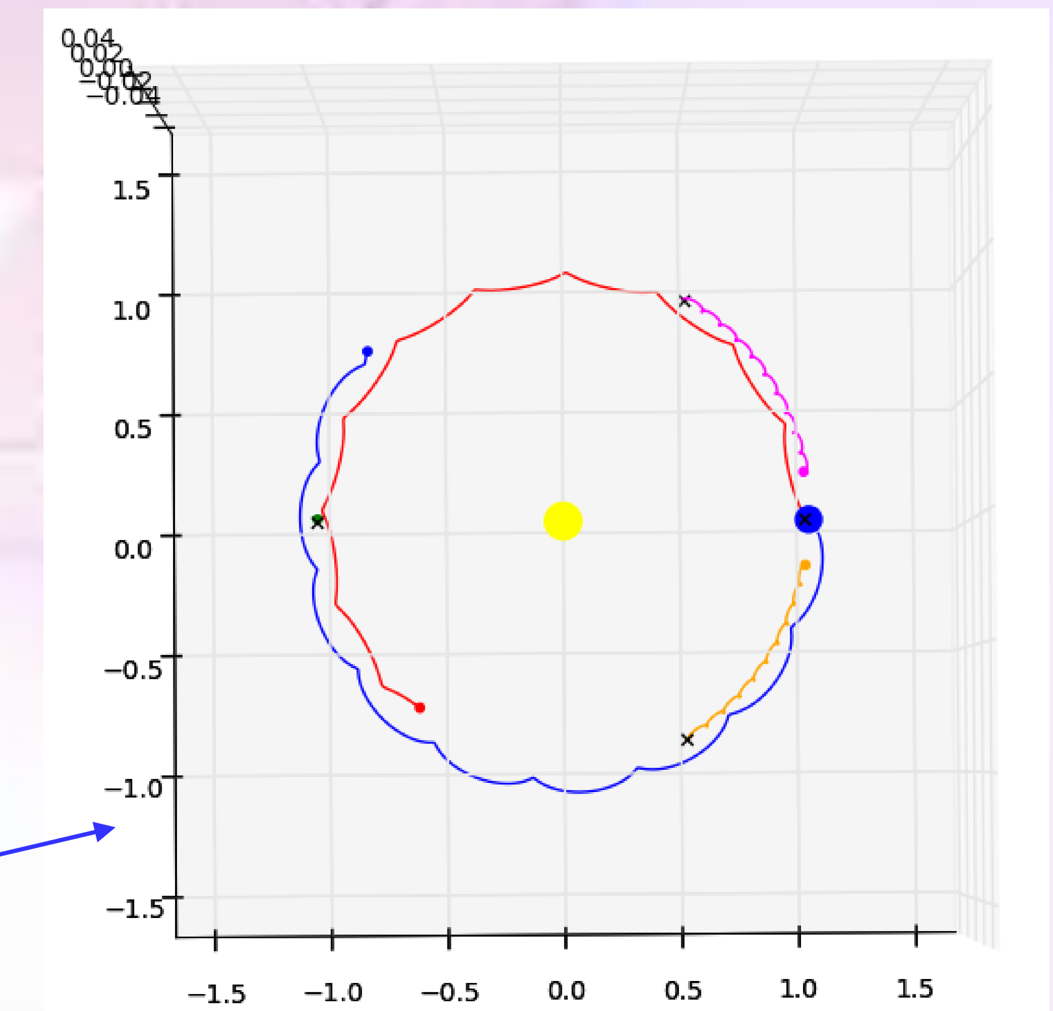
$$\alpha = \frac{M_2}{M_1+M_2}$$

Glossary of Technical Terms

Lagrange Point: A point where the forces balance out between two bodies
Co-rotating frame: Rotates at the same speed as the Earth in order to make the system appear stationary
Coriolis force: Accounts for the curved motion of a body that appears as a straight motion in a co-rotating frame
Centrifugal force: A fictitious force that draws an orbiting body away from the center of rotation

FIGURE 1: Orbits of Lagrange Points in a co-rotating frame after 10 years.

L1 L2 L3 L4 L5



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