Wave Patterns and Their Application to Migraine

Project Description

- The motivation of the model is to describe the spreading depression (SD) in migraines

- Migraine information from [1]:
  - Recurrent throbbing pain in the head-usually on one side
  - Headache attacks associated with nausea, vomiting, sensitivity to light, sound, and even movement
  - Two Types of Migraine: migraines with aura (MA) and migraines without aura (MO)
  - In addition to headache:
    - Aura, which is usually before the headache phase and often lasts less than one hour.
    - If SD occurs in MO, physiological phenomena remain clinically silent.

- From [2], migraines are so debilitating, migraines are ranked at 0.71 on a disability scale from 0.0-1.0 (1.0 being highly disabled).

- The goal is to accurately model general SD that occurs before a migraine.

Scientific Challenges

- Because neurological symptoms last for less than 5 min in MO patients, it is difficult to measure the affected area of the brain in some individuals

Potential Applications

- By describing the behavior of spreading depression in patients who suffer from migraines, a better treatment or cure for migraines could be more easily discovered.

Model Equations

$$\frac{du}{dt} = u \cdot \left(\frac{u_{sat}}{1+v} + u \right) + D \cdot \frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 \frac{du}{dr}\right)$$

$$\frac{dv}{dt} = \varepsilon \cdot (u - \alpha \cdot v + \beta)$$

- Where:
  - $u :=$ Activator
  - $v :=$ Inhibitor
  - $t :=$ time
  - $r =$ radial distance

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Methods

1. Model was modified from that of [2] to more accurately describe physiological phenomenon described by both [1] and [2].

2. Model was numerically solved and graphed using MATLAB (seen in figures 1-3)

Results

1. Previous model was not accurate because activator could become negative

2. From environmental factors, a sudden perturbation in activator occurs (initial condition). At this point, the model becomes applicable. After 2 hours, the model is no longer applicable.
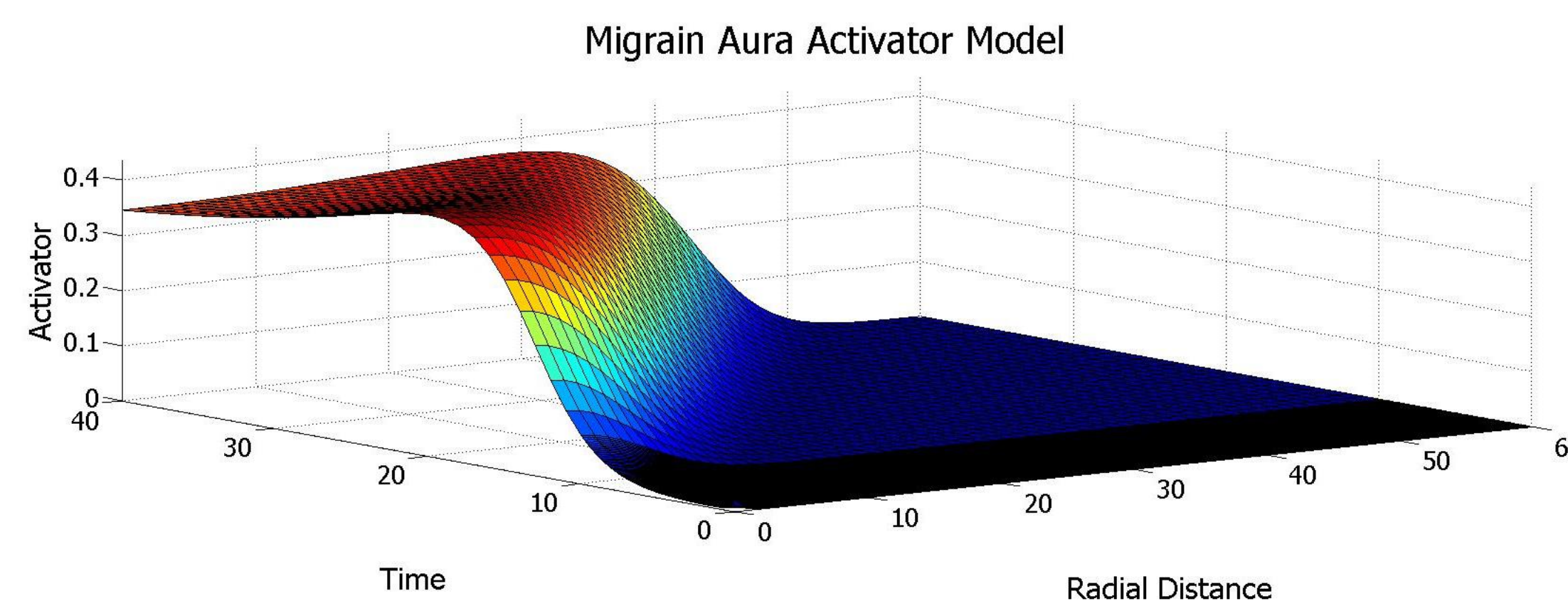

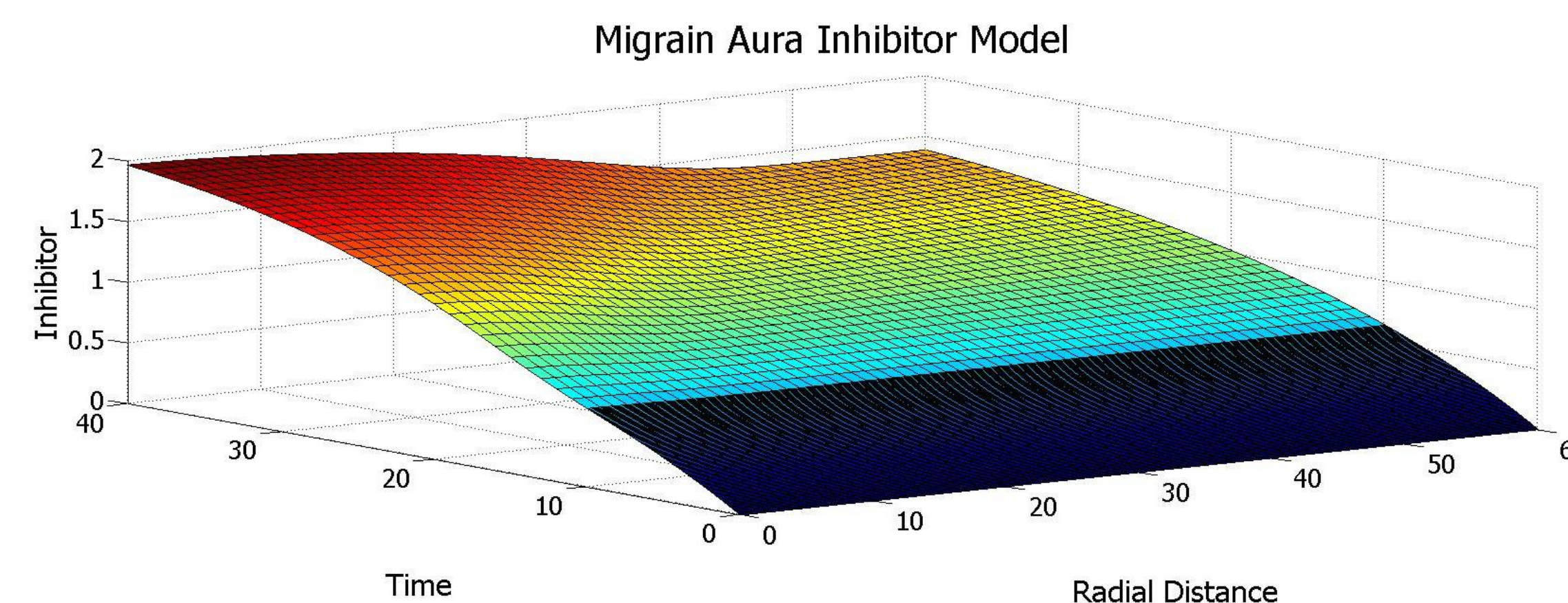Figure 1: Results for Activator from New Model with small time scale

Figure 2: Results for Inhibitor from New Model with small time scale
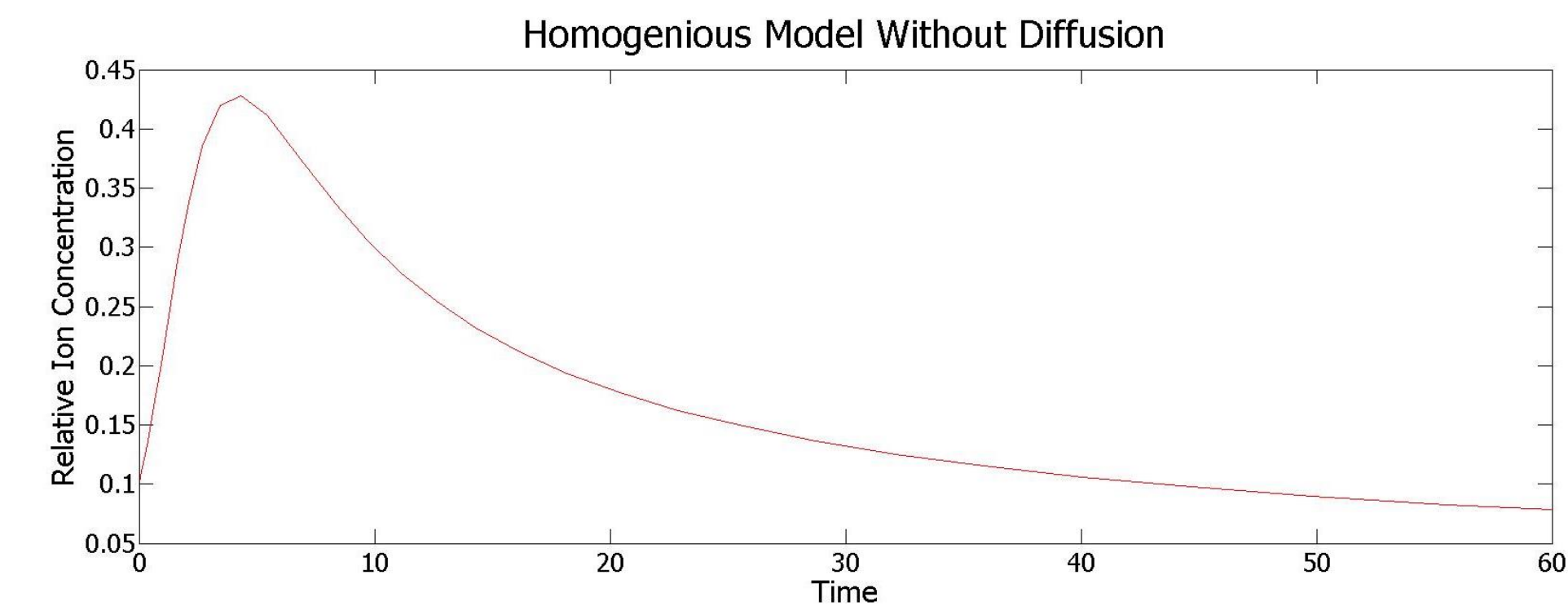

Figure 3: Results for Activator from New Model with large time scale

Glossary of Technical Terms

Spreading Depression: A massive but temporary perturbation of ion homeostasis due to seizure-like discharges of Neurons which causes aura.
Aura: Neurological symptoms such as visual hallucinations that occur in addition to headaches.
Clinically Silent: Symptoms must last less than 5 min.
Activator: Ion (such as potassium) concentration.
Inhibitor: Body's response to increased level of ion concentration.

$$\frac{\partial u}{\partial t} = u - \frac{1}{3}u^3 - v + D\nabla^2 u,$$

$$\frac{\partial v}{\partial t} = \varepsilon \left(u + \beta + K \int H(u)\, \mathrm{d}x\mathrm{d}y \right),$$

References

1. Markus A. Dahlem · Thomas M. Isele, Transient Localized Wave Patterns and Their Application to Migraine, Journal of Mathematical Neuroscience (2013)

2. Markus A. Dahlem, Migraine generator network and spreading depression dynamics as neuromodulation targets in episodic migraine, Chaos: An Interdisciplinary Journal of Nonlinear Science 23, 046101 (2013)

Acknowledgments