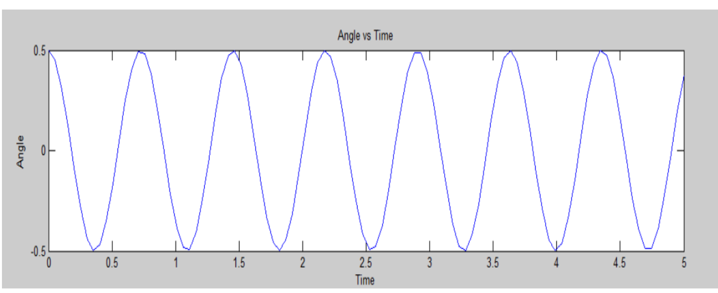
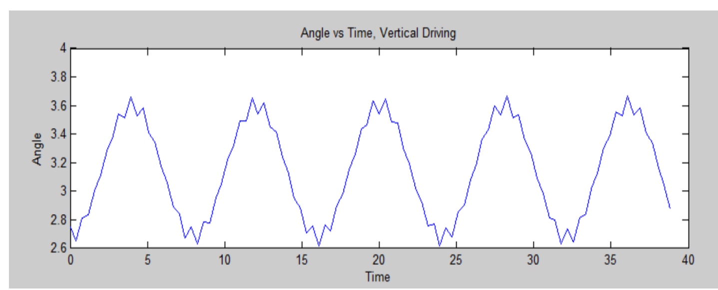
Pendulum with Vibrating Base

Project Description

Simple vs. Inverted Pendulum

People maybe well acquainted with simple pendulum problems. It's stable downward vertically, and unstable at inverted position. However, when adding a vibrating base on the pivot of the simple pendulum, the system seems to be stable at the inverted position. Simple pendulum swings in a smooth motion and is often models as $\frac{d^2\theta}{dt^2} + glsin\theta = 0$. With a vibrating base, the motion is no longer smooth. It creates small but rapid oscillations on top of the swing, which adds difficulties in modeling.





HISTORY

This subject, pendulum with vibrating base, is in fact well explored by scientists. In 1958, a Russian physicist Pyotr Kapitza successfully analyzed the stability of this system by separating the motions into fast and slow, and introduced a new concept called Effective Potential.

Objectives

1. Derive the Lagrangian for vertical, horizontal, and arbitrary angels
2. Find the Effective Potentials using the Average technique
3. Analyze the stability at each of the stationary positions of all three cases
4. Compare our theoretical results with actual lab experiments

Experimental Model

Methodology

1. **Measure the dimensions of the pendulum and find the center of mass.**
2. **Using a high speed camera, determine the minimum frequency of stability for the vertical, horizontal, and arbitrary case.**
3. **Compare experimental results with theoretical expectations.**
4. **Determine error.**

Results and Comparison

Raw Data Table	Measurement
Length of Pendulum (m)	.187
Diameter of Pendulum (m)	0.009525
Amplitude (m)	0.020
Minimum	0.010
Maximum	0.030
Frequency (rad/s)	275.62
Angle of Base	51°
Momentum of Inertia	$I = \frac{1}{3}ml^2$

The theoretical model we used is for a simple pendulum. The pendulum used in the experiment was a physical pendulum. To develop a more accurate model, we account for the moment of inertia of the rod, which is summarized in the table below under the "Corrected Theoretical" column

Measurement	Theoretical	Experimental	Corrected Theoretical
Vertical Stability Angle	180°	180°	180°
Critical Angle	97°	113°	98°
Horizontal Stability Angle	83°	76°	82°
Arbitrary Stability Angle	136°	109°	135°

Team Members

Thomas Bello
Emily Huang
Fabian Lopez
Kellin Rumsey
Tao Tao

Theoretical Models

Methodology

1. The Pendulum is treated as a point mass at its center.
2. The equations of motion for the vertical, horizontal, and arbitrary cases are determined by finding the Lagrangian for each case.
3. Determine stability conditions.
4. Using averaging, determine the Effective Potential.
5. Use the Effective Potential to determine ranges of stability.

Results

- Vertical Equations

$$L = \frac{1}{2}l\dot{\theta}^2 + \dot{\theta}\sin(\theta)\,d_0\omega^2\sin(\omega t) - gcos(\theta)$$

$$\ddot{\theta} + \left(\frac{d_0\omega^2}{l}\cos(\omega t) - \frac{g}{l}\right)\sin(\theta) = 0$$

$$U_{eff} = -\frac{g}{l}\cos(\theta) + \frac{1}{4}\frac{d_0^2\omega^2}{l^2}\sin^2(\theta)$$

- Horizontal Equations

$$L = \frac{1}{2}l\dot{\theta}^2 + \dot{\theta}\ \sin(\theta)\,d_0\omega^2\cos(\omega t) + gcos(\theta)$$

$$\ddot{\theta} - \frac{d_0\omega^2}{l}\cos(\theta)\cos(\omega t) + \frac{g}{l}\sin(\theta) = 0$$

$$U_{eff} = -\frac{g}{l}\cos(\theta) - \frac{1}{4}\frac{d_0^2\omega^2}{l^2}\sin^2(\theta)$$

- Arbitrary Equations

$$L = \frac{1}{2}l\dot{\theta}^2 + \dot{\theta}\sin(\theta - \rho)\,d_0\omega^2\cos(\omega t) + gcos(\theta)$$

$$\ddot{\theta} - \frac{d_0\omega^2}{l}\cos(\theta - \rho)\cos(\omega t) + \frac{g}{l}\sin(\theta) = 0$$

$$U_{eff} = -\frac{g}{l}\cos(\theta) - \frac{1}{4}\frac{d_0^2\omega^2}{l^2}\sin^2(\theta - \rho)$$

Error Analysis

Error Analysis	Measured Value	Absolute Error	Percent Error
Length of Pendulum (l)	l = 0.187 meters	δl = 0.001 meters	.54%
Amplitude of Base (d_0)	d_0 = 0.020 meters	δd_0 = 0.001 meters	5.0%
Period for 60 Oscillations (T*)	T* = 1.35 seconds	δT* = 0.05 seconds	3.7%

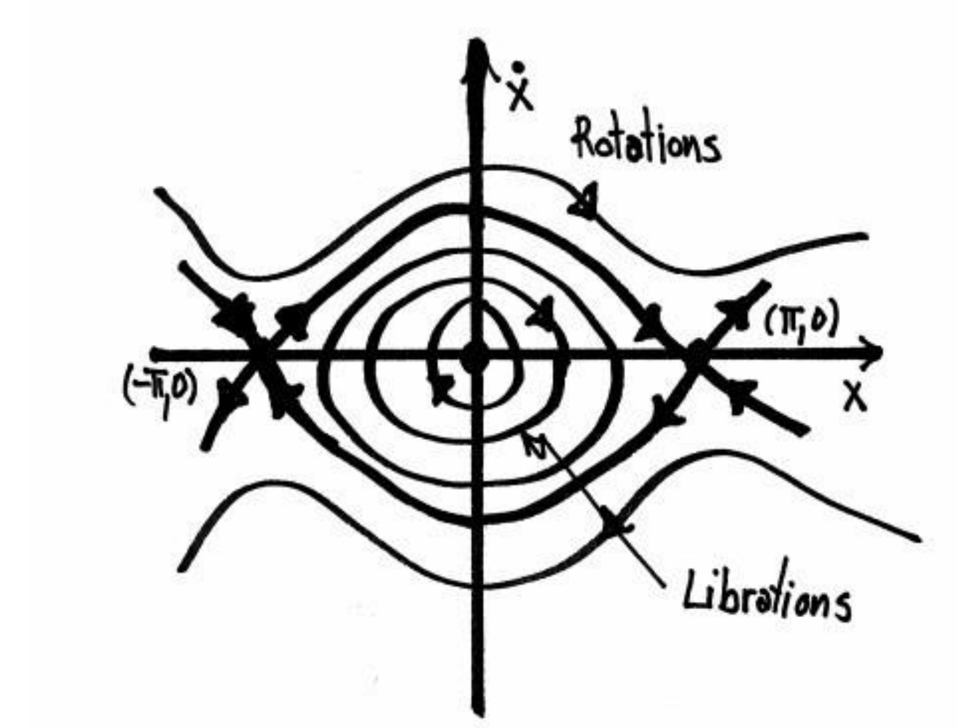
Note that angular frequency could not be directly measured. T is the length of time required for the base of the pendulum to make 60 oscillations. T* can be related to Angular Frequency in the following way: $\omega = 2\pi * \frac{60}{T*}$

$$\delta\theta = \sqrt{\left(\frac{\partial\theta}{\partial l}\delta l\right)^2 + \left(\frac{\partial\theta}{\partial T^*}\delta T^*\right)^2 + \left(\frac{\partial\theta}{\partial d_0}\delta d_0\right)^2}$$

θ_c = 97° ± 0.86 ° = 97° ± 0.88%

θ_s = 83° ± 0.86 ° = 83° ± 1.04%

Energy Manifold



Theoretical Definitions

$$Lagrangian\ (L) = Kinetic\ Energy - Potential\ Energy$$

$$Euler - Lagrange\ Equation\ of\ Motion: \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0$$

$$Stability\ Condition:\ \omega > \sqrt{\frac{2gl}{d_0^2}},\ Critical\ Angle:\ \theta_c = \cos^{-1}\left(-\frac{2gl}{d_0^2\omega^2}\right)$$

Potential Applications

A Segway is an example of an Inverted Pendulum with an oscillatory base, while Magnetic Levitation is a system that uses the interaction of two oscillatory forces with different speeds.

Reference

1) Landau L. D. & Lifshitz E. M., *Mechanics*, **Second Edition: Volume 1**, (Course of Theoretical Physics), (Oxford ; New York : Pergamon Press, 1976), pp 665-70. Oscillations of systems with more than one degree of freedom.

Acknowledgement

This project was mentored by Prof. Ildar Gabitov, whose help is acknowledged with great appreciation. We would also like to acknowledge support from the Bryant Bannister Tree-Ring Lab for access to their equipment. Support from a University of Arizona TRIF (Technology Research Initiative Fund) grant to J. Lega is also gratefully acknowledged.