HW2:

1. Consider two following differential equations:

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx^3(t) = 0.$$
$$m\ddot{y}(t) + \gamma\dot{y}(t) + k\tan[y(t)] = 0.$$

   a) Find the potential V and derive expression for the energy in the form of $E = T + V$ for both cases.

   b) What are dimensions of k and γ in both cases? Here dimensions of mass, time and distance are taken in kilograms, seconds and meters respectively ($[m] = kg$, $[t] = sec$ and $[x] = m$). Function $y(t)$ is dimensionless.

   c) Write these equations in dimensionless form and check if you can "get rid" of both parameters k and γ.

   d) Sketch the phase portrait associated with these differential equations.

2. Show that the potential of the equation

$$\ddot{x}(t) + \frac{1}{2}\sin\frac{x}{2} + \sin x = 0$$

   has the form $V(x) = 2 - \cos(x/2) - \cos x$ which is presented in the following figure:

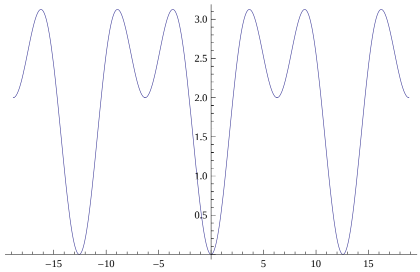

Figure 1: The dependence of V as function of x.

   a) Sketch the phase portrait associated with this equation.

3. The equation of motion for the linear pendulum reads

$$l\ddot{\theta} + \gamma\dot{\theta} + g\theta = 0,$$

   Pendulum length is $l = 10.2\ m$, the gravitational constant is $g \simeq 9.81\ m/sec^2$. Consider two cases: $\gamma = 2\sqrt{gl}$ and $\gamma = 0.2\sqrt{gl}$

   a) Represent the equation in dimensionless form.

b) For each case find solution of the following initial value problem:

$$l\ddot{\theta} + \gamma\dot{\theta} + g\theta = 0,$$
$$\theta(0) = 1, \quad \dot{\theta}(0) = 2.$$

d) Plot graph of both solutions and provide interpretation of results.