HW3:

1. **Linear Oscillator with damping:** Find solution of the following initial value problem:

$$\ddot{x}(t) + 0.4\ \dot{x} + 4.04\ x(t) = 0, \quad x(0) = 1, \quad \dot{x}(0) = 0.$$

2. **Resonance:** Find solution of the following initial value problem:

$$\ddot{x}(t) + x(t) = \sin(t), \quad x(0) = 0, \quad \dot{x}(0) = 1.$$

3. **Averaging:** Equation

$$\ddot{x}(t) + \varepsilon\,\mathrm{sgn}(\dot{x}) + x(t) = 0, \quad \varepsilon \ll 1$$

describes oscillator with "dry" friction. Here function $\mathrm{sgn}(y)$ is defined as follows:

$$\mathrm{sgn}(y) = \begin{cases} 1 & \text{if } y > 0 \\ 0 & \text{if } y = 0 \\ -1 & \text{if } y < 0 \end{cases}$$

a) Rewrite this equation in terms of phase φ and amplitude I, here $z(t) = I(t)\exp[i\varphi(t)]$, and $z(t) = x(t) + i\dot{x}(t)$.

b) Find averaged equation for $J(t)$ and solve it.

c) Compare behavior of obtained solution and solution of linear equation

$$\ddot{x}(t) + \varepsilon\dot{x}(t) + x(t) = 0.$$

d) Explain the difference in decay rates.

4. **Averaging: Van der Pol oscillator** Equation

$$\ddot{x} + x = \varepsilon(1 - x^2)\dot{x}, \quad \varepsilon \ll 1$$

describes a non-conservative oscillator with nonlinear damping.

a) Rewrite this equation in terms of phase φ and amplitude I, here $z(t) = I(t)\exp[i\varphi(t)]$, and $z = x + i\dot{x}$.

b) Find averaged equation for $J(t)$. Analyze behavior of $J(t)$ near stationary points.