

**HW5:**

1. **Reaction-diffusion:** Consider reaction-diffusion equation in dimensionless form:

$$\frac{\partial n}{\partial t} - \frac{\partial^2 n}{\partial x^2} = n(1 - n).$$

By introducing variable  $z = x - vt$ , ( $n = n(z)$ ) rewrite it in traveling wave form

$$\frac{d^2 n}{dz^2} + v \frac{dn}{dz} + n(1 - n) = 0$$

- a. Find stationary solutions.  
b. Analyze stability of these stationary solutions for: i)  $0 \leq v < 2$ ; ii)  $v = 2$ ; iii)  $v > 2$ .  
c. Sketch phase portraits for all these cases.
2. **Fourier transform:** Find Fourier transform

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt$$

for the following functions:

a.

$$f(t) = \begin{cases} 0 & \text{if } t < -1, \\ t + 1 & \text{if } -1 \leq t \leq 0, \\ 1 - t & \text{if } 0 \leq t \leq 1, \\ 0 & \text{if } t > 1. \end{cases}$$

b.

$$f(t) = \begin{cases} 0 & \text{if } t < -1, \\ 1 - t^2 & \text{if } -1 \leq t \leq 1, \\ 0 & \text{if } t > 1. \end{cases}$$

c.  $f(t) = \exp(-|t|)$ .