MATH 215

Homework # 1

- Determine which equations are linear equations in the variables x, y and z. If any equation is not linear, explain why not.

  1. $x^2 + y^2 + z^2 = 1$

  2. $2x - 3y - 5z = 0$

- Find the solution set of each equation.

  3. $3x - 6y = 0$

  4. $x + 2y + 3z = 4$.

- **5.** Solve the given system by back substitution

$$\begin{aligned} 2u - 3v &= 5 \\ 2v &= 6 \end{aligned}$$

- **6.** Find the augmented matrix of the following linear system

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + \ x_3 &= 0 \\ -x_1 + 2x_2 - 2x_3 &= 0 \end{aligned}$$

- **7.** Find a system of linear equations that has the given matrix as its augmented matrix

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}.$$

  Solve this system of equations.

- **8.** Determine whether the given matrix is in row echelon form. If it is, state whether it is also in reduced row echelon form.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- **9.** Use elementary row operations to reduce the given matrix to (a) row echelon form and (b) reduced row echelon form.

$$\begin{bmatrix} -2 & -4 & 7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{bmatrix}$$

- **10.** Show that the given matrices are row equivalent and find a sequence of elementary row operations that will convert A into B.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$$

- **11.** What is wrong with the following "proof" that every matrix with at least two rows is row equivalent to a matrix with a zero row?

  Perform $R_2 + R_1$ and $R_1 + R_2$. Now rows 1 and 2 are identical. Now perform $R_2 - R_1$ to obtain a row of zeros in the second row.

- **12.** For what value(s) of k, if any, will the system have (a) no solutions, (b) a unique solution, and (c) infinitely many solutions?

$$\begin{aligned} x + ky &= 1 \\ kx + y &= 1 \end{aligned}$$