Homework # 10

Section # 4.4

- **1.** Find the coordinate vector $[\vec{x}]_{\mathfrak{B}}$ of $\vec{x}$ relative to the given basis $\mathfrak{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$,

$$\vec{b}_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \ \vec{b}_2 = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, \ \vec{b}_3 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}.$$

- **2.** Find the change of coordinate matrix from $\mathfrak{B}$ to the standard basis in $\mathbf{R}^n$.

$$\mathfrak{B} = \left\{ \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \ \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}, \ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}.$$

- Use the inverse matrix to find $[\vec{x}]_{\mathfrak{B}}$ for the given $\vec{x}$ and $\mathfrak{B}$.

  3.
$$\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \ \begin{bmatrix} -3 \\ 5 \end{bmatrix} \right\}, \ \vec{x} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}.$$

  4.
$$\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, \ \vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

- **5.** The set $\mathfrak{B} = \{1 - t^2, t - t^2, 2 - t + t^2\}$ is a basis for $\mathbf{P}_2$. Find the the coordinate vector $\mathbf{p}(t) = 1 + 3t - 6t^2$ relative to B.

- **6.** The vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ span $\mathbf{R}^2$ but do not form a basis. Find two different ways to express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

- **7.** Let $\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \end{bmatrix} \right\}$. Since the coordinate mapping determined by $\mathfrak{B}$ is the linear transformation from $\mathbf{R}^2$ into $\mathbf{R}^2$ this mapping must be implemented by some 2×2 matrix A. Find it.

- **8.** Let $\mathbf{p}_1(t) = 1 + t^2, \mathbf{p}_2(t) = 1 - 3t^2, \mathbf{p}_3(t) = 1 + t - 3t^2$.
  (a) Use coordinate vectors to show that these polynomials form basis for $\mathbf{P}_2$.
  (b) Consider basis $\mathfrak{B} = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ for $\mathbf{P}_2$. Find $\mathbf{q}$ in $\mathbf{P}_2$ given that $[\mathbf{q}]_B = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

Section # 4.5

- **9.** For the subspace $H = \left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix} : a, b, c \text{ in } \mathbf{R}, \right\}$ find a basis and state the dimension.

- **10.** Determine the dimension of $Nul\ A$ and $Col\ A$ for the following matrix:

$$A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

11. The first four Hermite polynomials are $1, 2t, -2+4t^2$ and $-12t+8t^3$. Show that the first four Hermite polynomials form a basis of $\mathbf{P}_3$.

Section # 4.6

- **12.** If 4×7 matrix A has rank 3, find $dim(Nul\ A)$, $dim(Row\ A)$ and $rank\ A^T$.

- **13.** If the null space of an 8×5 matrix A is 5-dimensional, what is the dimension of the row space of A?

- **13.** Determine whether $\vec{w}$ is in the column space of A, the null space of A, or both, where

- **14.** Verify that $\text{rank}(\vec{u}\vec{v}^T) \leq 1$ if $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Section # 4.7

- **15.** Let $\mathfrak{A} = \{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ and $\mathfrak{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ be bases for a vector space V, and suppose $\vec{a}_1 = 4\vec{b}_1 - \vec{b}_2$, $\vec{a}_2 = -\vec{b}_1 + \vec{b}_2 + \vec{b}_3$ and $\vec{a}_3 = \vec{b}_2 - 2\vec{b}_3$
a) Find change of coordinate matrix from $\mathfrak{A}$ to $\mathfrak{B}$.
b) Find $[\vec{x}]_{\mathfrak{B}}$ for $\vec{x} = 3\vec{a}_1 + 4\vec{a}_2 + \vec{a}_3$

- Let $\mathfrak{B}$ and $\mathfrak{C}$ be bases for $\mathbf{R}^2$. Find the change of coordinate matrix from $\mathfrak{B}$ to $\mathfrak{C}$ and the change of coordinate matrix from $\mathfrak{C}$ to $\mathfrak{B}$.

16.

$$\vec{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \vec{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \vec{c}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}.$$

- **17.**

$$\vec{b}_1 = \begin{bmatrix} 6 \\ -12 \end{bmatrix}, \ \vec{b}_2 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \ \vec{c}_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \ \vec{c}_2 = \begin{bmatrix} 3 \\ 9 \end{bmatrix}.$$

- **18.** Let

$$P = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -5 & 0 \\ 4 & 6 & 1 \end{bmatrix}, \ \vec{v}_1 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} -8 \\ 5 \\ 2 \end{bmatrix}, \ \vec{v}_3 = \begin{bmatrix} -7 \\ 2 \\ 6 \end{bmatrix}.$$

Find a basis $\mathfrak{U} = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ such that P is the change of coordinate matrix from $\mathfrak{U} = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ to the basis $\mathfrak{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.