Math 215 — HW 12 Print Name ______________________
Fall 2014 Signature ______________________

# Homework # 12

## Section # 5.3

- **1.** Show that if $A$ is both diagonalizable and invertible, then so does $A^T$.
- **2.** If possible diagonalize following matrices:

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{bmatrix}.$$

## Section # 5.4

- **3.** Let $\mathfrak{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ and $\mathfrak{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be bases for vector spaces $V$ and $W$, respectively. Let $T : V \to W$ be a linear transformation: $T(\mathbf{d}_1) = 3\mathbf{b}_1 - 3\mathbf{b}_2$, $T(\mathbf{d}_2) = -2\mathbf{b}_1 + 5\mathbf{b}_2$. Find the matrix for $T$ relative to $\mathfrak{D}$ and $\mathfrak{B}$.
- **4.** Let $\mathfrak{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be bases for vector spaces $V$ and let $T : V \to \mathbb{R}^2$ be a linear transformation: $T(x_1\mathbf{b}_1 + x_2\mathbf{b}_2 + x_3\mathbf{b}_3) = \begin{bmatrix} 2x_1 - 3x_2 + x_3 \\ -2x_1 + 5x_3 \end{bmatrix}$. Find the matrix for $T$ relative to $\mathfrak{B}$ and the standard basis for $\mathbb{R}^2$.
- **5.** Let $T : \mathbb{P}_2 \to \mathbb{P}_4$ be the transformation that maps a polynomial $\mathbf{p}(t)$ into the polynomial $\mathbf{p}(t) + 2t^2\mathbf{p}(t)$.
  a) Find the image of $\mathbf{p}(t) = 3 - 2t + t^2$
  b) Show that $T$ is a linear transformation
  c) Find the matrix for $T$ relative to the bases $\mathfrak{T}_2 = \{1, t, t^2\}$ and $\mathfrak{T}_4 = \{1, t, t^2, t^3, t^4\}$.
- **6.** Find the $\mathfrak{B}$ matrix for the transformation $x \mapsto Ax$, where $\mathfrak{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$

$$A = \begin{bmatrix} -6 & -2 \\ 4 & 0 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

- **7.** Find the $\mathfrak{B}$ matrix for the transformation $x \mapsto Ax$, where $\mathfrak{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$:

$$A = \begin{bmatrix} 6 & -2 & -2 \\ 3 & 1 & -2 \\ 2 & -2 & 2 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}.$$