MATH 215

Homework # 2

- Find the general solution of the systems whose augmented matrices have the following form:

    1.
    $$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix},$$

    2.
    $$\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix},$$

    3.
    $$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

- **4.** Determine the value of h such that the matrix is the augmented matrix of a consistent linear system.

    $$\begin{bmatrix} 1 & -1 & 4 \\ -2 & 3 & h \end{bmatrix}.$$

- **5.** Suppose a system of linear equations has 3×5 augmented matrix whose fifth column is not a pivot column. Is the system consistent? Why (or why not)?

- **6.** Compute $\vec{u} + \vec{v}$ and $\vec{u} - 2\vec{v}$.

    $$\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

- **7.** Write a system of equations that is equivalent to the given vector equation

    $$x_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- **8.** Write a vector equation that is equivalent to the given system of equations

    $$\begin{aligned} 3x_1 - 2x_2 + 4x_3 &= 3 \\ -2x_1 - 7x_2 + 5x_3 &= 1 \\ 5x_1 + 4x_2 - 3x_3 &= 2 \end{aligned} \tag{0.1}$$

1

- **9.** Determine if $\vec{b}$ is a linear combination of $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$.

$$\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

- **10.** Determine if $\vec{b}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

- **11.** List five vectors in $\mathtt{Span}\{\vec{v}_1, \vec{v}_2\}$. For each vector, show the weights on $\vec{v}_1$ and $\vec{v}_2$ used to generate the vector and list the three entries of the vector. Do not make a sketch.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}.$$

- **12.** Let $A = \begin{bmatrix} 1 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}$ and let W be the set of all linear combinations of the columns of A.

a. Is $\vec{b}$ in W?

b. Show that the second column of A is in W.