Homework # 3

Section # 1.4

- Use the definition of $A\vec{x}$ to write matrix equation as a vector equation and vice versa.

1.

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

2.

$$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

- **3.** Determine if the vector $\vec{b}$ is in the span of the columns of the matrix A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

- **4.** Show that

$$\mathbf{R}^2 = span\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right). \tag{0.1}$$

- **5.** Describe the span of given vectors.

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

- Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}.$$

6. How many rows of A contain a pivot position? Does the equation $A\vec{x} = \vec{b}$ have a solution for each $\vec{b}$ in $\mathbf{R}^4$?

7. Can each vector in $\mathbf{R}^4$ be written as a linear combination of the columns of the matrix A? Do the columns of A span $\mathbf{R}^4$?

Section # 1.5

- **7.** Each of the following equations determines a plane in $\mathbf{R}^3$. Do the two planes intersect? If so describe their intersection.

$$\begin{aligned} x_1 + 4x_2 - 5x_3 &= 0 \\ 2x_1 - x_2 + 8x_3 &= 9 \end{aligned}$$

- **8.** Determine if the system has a nontrivial solution.

$$\begin{aligned} 5x_1 - 3x_2 + 2x_3 &= 0 \\ -3x_1 - 4x_2 + 2x_3 &= 0 \end{aligned}$$

- **9.** Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 0 \\ -x_1 + x_2 \qquad\quad &= 0 \end{aligned}$$

- **10.** Describe and compare the solution sets of $x_1 + 5x_2 - 3x_3 = 0$ and $x_1 + 5x_2 - 3x_3 = -2$.

- **11.** Find the parametric equation of the line through $\vec{a}$ parallel to $\vec{b}$.

$$\vec{a} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

- **12.** Construct a 3×3 matrix A such that the vector

$$\vec{a} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

is a solution of $A\vec{x} = 0$.