# Homework # 4

## Section # 1.7

- Determine if the columns of the matrix form a linearly independent set. Justify each answer.

  **1.**
  $$\begin{bmatrix} 0 & -3 & 9 \\ 2 & 1 & -7 \\ -1 & 4 & -5 \\ 1 & -4 & -2 \end{bmatrix}$$

  **2.**
  $$\begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 4 & -6 & 2 \\ 0 & 1 & -1 & 3 \end{bmatrix}$$

- Find the values of $h$ for which the vectors are linearly dependent. Justify each answer.

  **3.**
  $$\begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}$$

  **4.**
  $$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

- **5.** How many pivot columns must $6 \times 4$ matrix have if its columns span $\mathbf{R}^4$? Why?

- **6.** Find a nontrivial solution of the equation $A\vec{x} = \vec{0}$,

  $$A = \begin{bmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix}.$$

- **7.** Use as many columns of $A$ as possible to construct matrix $B$ with the property that the equation $B\vec{x} = \vec{0}$ has only the trivial solution. Solve $B\vec{x} = \vec{0}$ to verify your work. Here $A$ is defined as follows:

  $$A = \begin{bmatrix} 3 & -4 & 10 & 7 & -4 \\ -5 & -3 & -7 & -11 & 15 \\ 4 & 3 & 5 & 2 & 1 \\ 8 & -7 & 23 & 4 & 15 \end{bmatrix}.$$

**8.** With $A$ and $B$ as in **exercise 7.** select a column $\vec{v}$ of $A$ that was not used in the construction of $B$ and determine if $\vec{v}$ is in the set spanned by the columns of $B$. Describe your calculations.

**Section # 1.8**

- **9.** Transformation $T$ is defined by $T(\vec{x}) = A\vec{x}$, find a vector $\vec{x}$ whose image under $T$ is $\vec{b}$ and determine if $\vec{x}$ is unique.

$$A = \begin{bmatrix} 1 & 0 & -3 \\ -3 & 1 & 6 \\ 2 & -2 & -1 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$$

- **10.** Let $\vec{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 2 & 10 & -6 \\ 1 & 0 & 2 & -4 \\ 0 & 1 & 2 & 3 \\ 1 & 4 & 10 & 8 \end{bmatrix}$. Is $\vec{b}$ in the range of the linear transformation $\vec{x} \mapsto A\vec{x}$? Why or why not.

- Use a rectangular coordinate system to plot $\vec{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ under given transformation $T$. Make a sketch and describe geometrically what $T$ does to each vector $\vec{x}$ in $\mathbf{R}^2$.

**11.**

$$T(\vec{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

**12.**

$$T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

**13.**

$$T(\vec{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

**Section # 1.9**

- Assume that $T$ is a linear transformation and find the standard matrix of $T$.

**14.** $T : \mathbf{R}^2 \mapsto \mathbf{R}^4$, $T(\vec{e}_1) = (3, 1, 3, 1)$ and $T(\vec{e}_2) = (-5, 2, 0, 0)$, where $\vec{e}_1 = (1, 0)$ and $\vec{e}_2 = (0, 1)$.

**15.** $T : \mathbf{R}^3 \mapsto \mathbf{R}^2$, $T(\vec{e}_1) = (1, 4)$, $T(\vec{e}_2) = (-2, 9)$ and $T(\vec{e}_3) = (-3, 8)$, where $\vec{e}_1 = (1, 0, 0)$, $\vec{e}_2 = (0, 1, 0)$ and $\vec{e}_3 = (0, 0, 1)$.

**16.** $T : \mathbf{R}^2 \mapsto \mathbf{R}^2$, reflects points through the horizontal $x_1-$ axis and then reflects points through the line $x_2 = x_1$.

- Let $T$ be a linear transformation whose standard matrix matrix is given. Decide if $T$ is one-to-one mapping. Justify your answers.

**17.**

$$A = \begin{bmatrix} -5 & 6 & -5 & -6 \\ 8 & 3 & -3 & 8 \\ 2 & 9 & 5 & -12 \\ -3 & 2 & 7 & -12 \end{bmatrix}.$$

**18.**

$$A = \begin{bmatrix} 7 & 5 & 9 & -9 \\ 5 & 6 & 4 & -4 \\ 4 & 8 & 0 & 7 \\ -6 & -6 & 6 & 5 \end{bmatrix}.$$