Homework # 5

Section # 2.1

- Compute the product AB in two ways: (1) by the definition, where $A\vec{b}_1$ and $A\vec{b}_2$ are computed separately, and (2) by the row -column rule for computing AB.

  1.

  $$A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$$

  2.

  $$A = \begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$$

- **3.** Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$ what values of k, if any, will make $AB = BA$?

- **4.** Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & -5 \\ 2 & 1 \end{bmatrix}$. Verify that $AB = AC$ and yet $B \neq C$.

- Matrices A, B, and C are such that the indicated sums and products are defined. Mark each statement "True" or "False" and justify each answer.

  5.

  a) If A and B are 2×2 matrices with columns $\vec{a}_1$, $\vec{a}_2$ and $\vec{b}_1$, $\vec{b}_2$ respectively, then $AB = [\vec{a}_1\vec{b}_1 \;\; \vec{a}_2\vec{b}_2]$.

  b) Each column of AB is a linear combination of the columns of B using weights from the corresponding column of A.

  c) $AB + AC = A(B + C)$

  d) $A^T + B^T = (A + B)^T$

  e) $(AB)^T = A^T B^T$

  6.

  a) The first row of AB is the first row of A multiplied on the right by B.

  b) If A and B are 3×3 matrices and $B = [\vec{b}_1 \; \vec{b}_2 \; \vec{b}_3]$, then $AB = [A\vec{b}_1 + A\vec{b}_2 + A\vec{b}_3]$.

  c) If A is an $n \times n$ matrix, then $(A^2)^T = (A^T)^2$.

  d) $(ABC)^T = C^T A^T B^T$

- **7.** If $A = \begin{bmatrix} 1 & -3 \\ -3 & 5 \end{bmatrix}$ and $AB = \begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$, determine the first and the second columns of B.

- **8.** Let $\vec{u} = \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute $\vec{u}^T\vec{v}$, $\vec{v}^T\vec{u}$, $\vec{u}\vec{v}^T$, and $\vec{v}\vec{u}^T$.

- **9.** Let

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Compute S^k, $k = 2, ...6$.

Section # 2.2

- Find the inverse matrices:

10.

$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$$

11.

$$\begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$$

- **12.** Use the inverse found in **10.** to solve the system:

$$\begin{cases} 8x_1 + 6x_2 = 2 \\ 5x_1 + 4x_2 = -1 \end{cases}.$$

- **13.** Let $A = \begin{bmatrix} 1 & 2 \\ 5 & 12 \end{bmatrix}$, $\vec{b}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\vec{b}_3 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\vec{b}_4 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

  a) Find A^{-1}, and use it to solve the four equations: $A\vec{x} = \vec{b}_i,\ i = 1, 2, 3, 4$.

  b) The four equations in part (a) can be solved by the same set of row operations, since the coefficient matrix is the same in each case. Solve the four equations in part (a) by row reducing augmented matrix $[A\ \vec{b}_1\ \vec{b}_2\ \vec{b}_3\ \vec{b}_4]$.

- **14.** Suppose P is invertible and $A = PBP^{-1}$. Solve for B in terms of A.