Homework # 6

Section # 2.2

- **1.** Let A be an invertible $n \times n$ matrix, and let B be an $n \times p$ matrix. Show that equation $AX = B$ has a unique solution $A^{-1}B$.

- **2.** Show that if A is invertible and D satisfies $AD = I$, then $D = A^{-1}$.

- **3.** Suppose $AB = AC$, where B and C are $n \times p$ matrices and A is invertible. Show that $B = C$. Determine if it is true when A is not invertible.

- **4.** Show that if $ad - bc = 0$, then the equation $A\vec{x} = 0$ has more than one solution. Here A is defined as follows:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

- Find the inverses of the matrices if they exists.

  5.
$$\begin{bmatrix} 1 & -3 \\ 4 & -9 \end{bmatrix}$$

  6.
$$\begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$$

- **7.** Let $A = \begin{bmatrix} 1 & -7 & -3 \\ 2 & 15 & 6 \\ 1 & 3 & 2 \end{bmatrix}$. Find the third column of A^{-1} without computing the other columns.

Section # 2.3

- **8.** Determine if $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ is invertible.

- Determine which of the following matrices are invertible. Use as few calculations as possible. Justify your answer.

  9.
$$\begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$$

10.

$$\begin{bmatrix} 3 & 0 & 0 \\ -3 & -4 & 0 \\ 8 & 5 & -3 \end{bmatrix}$$

11.

$$\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

- **12.** Can a square matrix with two identical rows be invertible? Why or why not?
- **13.** Assume that A is $n \times n$ matrix. If the equation $A\vec{x} = \vec{y}$ is inconsistent for some $\vec{y}$ in $\mathbf{R}^n$, what can you say about the equation $A\vec{x} = 0$. Why?
- **14.** If A is an $n \times n$ matrix and transformation $\vec{x} \mapsto A\vec{x}$ is one-to-one, what else can you say about this transformation? Justify your answer.
- Show that T is invertible and find a formula for T^{-1}.

  15. $T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2)$

  16. $T(x_1, x_2) = (2x_1 - 8x_2, -2x_1 + 7x_2)$
- **17.** Explain why the columns of A^2 span $\mathbf{R}^n$ whenever the columns of $n \times n$ matrix A are linearly independent.