Homework # 7

Section # 2.3

- **1.** Let A and B be $n \times n$ matrices. Show that if AB is invertible so is B.
- T is a linear transformation from $\mathbf{R}^2$ into $\mathbf{R}^2$. Show that T is invertible and find formula for T^{-1}.

  2. $T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2)$

  3. $T(x_1, x_2) = (2x_1 - 8x_2, -2x_1 + 7x_2)$

Section # 2.4

- Find formulas for X, Y, and Z in terms of A, B, and C. Justify your calculations.

  4.

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}.$$

  5.

$$\begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} X & Y & Z \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}.$$

- **6.** The inverse of

$$\begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ B & D & I \end{bmatrix} \text{ is } \begin{bmatrix} I & 0 & 0 \\ P & I & 0 \\ Q & R & I \end{bmatrix}$$

  Find P, Q, and R.

- **7.**

  a. Verify that $A^2 = I$ when $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$.

  b. Use partitioned matrices to show that $M^2 = I$ when $A^2 = I$ when

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}.$$

- **8.** Without using row reduction, find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 5 & 6 \end{bmatrix}$$

Section # 2.5

- **9.** Solve the equations using LU factorization given for A.

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} \begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

- Find LU factorization of the following matrices (with L unit lower triangular).

10.

$$\begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}$$

11.

$$\begin{bmatrix} 2 & 3 & 2 \\ 4 & 13 & 9 \\ -6 & 5 & 4 \end{bmatrix}$$

12.

$$\begin{bmatrix} 2 & 0 & 5 & 2 \\ -6 & 3 & -13 & -3 \\ 4 & 9 & 16 & 17 \end{bmatrix}$$

Section # 2.8

- **13.** Let $A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$. Is $\vec{u}$ in $Nul(A)$? Justify each answer.

- **14.** Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, find a vector in $Nul(A)$ and a vector in $Col(A)$.

- **15.** Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -3 \\ -3 \\ 10 \end{bmatrix}$. Determine if $\vec{w}$ is in the subspace of $\mathbf{R}^3$ generated by $\vec{v}_1$ and $\vec{v}_2$.

- **16.** For A given below find a nonzero vector in $Nul(A)$ and a nonzero vector in $Col(A)$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ -5 & -1 & 0 \\ 2 & 7 & 11 \\ 3 & 3 & 4 \end{bmatrix}$$

- Determine which sets are bases for $\mathbf{R}^2$ or $\mathbf{R}^3$. Justify each answer.

17. $\begin{bmatrix} -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \end{bmatrix}$

18. $\begin{bmatrix} 1 \\ -6 \\ -7 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 9 \end{bmatrix}$.