Homework # 8

Section # 2.9

- **1.** Find the vector $\vec{x}$ determined by the given coordinate vector $[\vec{x}]_{\mathfrak{B}}$ and the given basis $\mathfrak{B}$.

$$\mathfrak{B} = \left\{ \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}, \ [\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

- The vector $\vec{x}$ is in a subspace H with a basis $\mathfrak{B} = \{\vec{b}_1, \vec{b}_2\}$. Find the $\mathfrak{B}$-coordinate vector of $\vec{x}$.

  2. $\vec{b}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \ \vec{b}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$

  3. $\vec{b}_1 = \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}, \ \vec{b}_2 = \begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$

- **4.** The matrix A and an echelon form of A have following forms:

$$A = \begin{bmatrix} 1 & -2 & -1 & 5 & 4 \\ 2 & -1 & 1 & 5 & 6 \\ -2 & 0 & -2 & 1 & -6 \\ 3 & 1 & 4 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & 2 & 0 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

  Find bases for $Col(A)$ and $Nul(A)$ and then state the dimensions of these subspaces.

- **5.** Find a basis for the subspace spanned by the the given vectors. What is the dimension of the subspace?

$$\begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \\ -6 \\ 12 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -3 \\ 7 \end{bmatrix}.$$

- **6.** If the subspace of all solutions of $A\vec{x} = 0$ has a basis consisting of three vectors and if A is a 5×7 matrix, what is the rank of A? Justify your answer.

- **7.** Construct a 3×4 matrix with rank 1. Justify your answer.

- **8.** Let $H = Span\{\vec{v}_1, \vec{v}_2\}$ and $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2\}$. Show that $\vec{x}$ is in H, and find the $\mathfrak{B}$-coordinate vector of $\vec{x}$, when

$$\vec{v}_1 = \begin{bmatrix} 15 \\ -5 \\ 12 \\ 7 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 14 \\ -10 \\ 13 \\ 17 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} 16 \\ 0 \\ 11 \\ -3 \end{bmatrix}.$$

Section # 3.1

- **9.** Compute the determinant using co-factor expansion across the first row.

$$\begin{bmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{bmatrix}.$$

- **10.** Compute the determinant by co-factor expansions. At each step, choose a row or column that involves the least amount of computations.

$$\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix}.$$

- **11.** State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, \ \begin{bmatrix} 3 & 4 \\ 5+3k & 6+4k \end{bmatrix}$$

Section # 3.2

- **12.** Find the determinant by row reduction to echelon form.

$$\begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$$

- **13.** Find the determinant

$$\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix}$$

- **14.** Use the determinant to decide if the set of vectors is linearly independent.

$$\begin{bmatrix} 7 \\ -4 \\ -6 \end{bmatrix}, \ \begin{bmatrix} -8 \\ 5 \\ 7 \end{bmatrix}, \ \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}.$$

Section # 3.3

- **15.** Use Cramer's rule to compute the solution of the following system:

$$\begin{cases} 3x_1 - 2x_2 &= 7 \\ -5x_1 + 6x_2 &= -5. \end{cases}$$

- **16.** Compute the adjugate and find the inverse of the following matrix:

$$\begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}.$$

- **17.** Find the volume of parallelepiped with one vortex at the origin and adjacent vertices at $(1, 0, -2)$, $(1, 2, 4)$ and $(7, 1, 0)$.