Homework # 9

Section # 3.3

- **1.** Let S be the parallelogram determined by the vectors $\vec{b}_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$, and let $A = \begin{bmatrix} 6 & -2 \\ -3 & 2 \end{bmatrix}$. Compute the area of the image of S under the mapping $\vec{x} \mapsto A\vec{x}$.
- **2.** Find a formula for the area of the triangle whose vertices are $\vec{0}$, $\vec{v}_1$ and $\vec{v}_2$ in $\mathbf{R}^2$.
- **3.** Let R be the triangle with vertices at (x_1, y_1), (x_2, y_2) and (x_3, y_3). Show that area of triangle S_Δ:

$$S_\Delta = \frac{1}{2} det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}.$$

Section # 4.1

- **4.** Let H be the set of the points inside and on the unit circle in the x, y plane: $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}$. Show that H is not a subspace of $\mathbf{R}^2$.
- Determine if given set is a subspace of $\mathbf{P}_n$ for an appropriate value of n. **Justify answer.**

  5. All polynomials of the form $\mathbf{p}(t) = a + t^2$, where a is in $\mathbf{R}$.

  6. All polynomials of of degree at most 3, with integers as coefficients.

  7. All polynomials in $\mathbf{P}_n$ such that $\mathbf{p}(0) = 0$.
- **8.** Let W be the set of all vectors of the form $\begin{bmatrix} 2s + 4t \\ 2s \\ 2s - 3t \\ 5t \end{bmatrix}$. Show that W is a subspace of $\mathbf{R}^4$.
- **9.** Determine if the set H of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of $M_{2\times 2}$
- **10.** Determine if $\vec{y}$ is in the subspace of $\mathbf{R}^4$ spanned by the columns of A, where

$$\vec{y} = \begin{bmatrix} -4 \\ -8 \\ 6 \\ -5 \end{bmatrix}, \ A = \begin{bmatrix} 3 & -5 & -9 \\ 8 & 7 & -6 \\ -5 & -8 & 3 \\ 2 & -2 & -9 \end{bmatrix}.$$

Section # 4.2

- **11.** Determine if $\vec{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is in $Nul\ A$, where

$$A = \begin{bmatrix} 2 & 6 & 4 \\ -3 & 2 & 5 \\ -5 & -4 & 1 \end{bmatrix}.$$

- **12.** Find A such that the given set is $Col\ A$.

$$\left\{ \begin{bmatrix} 2s+t \\ r-s+2t \\ 3r+s \\ 2r-s-t \end{bmatrix} : r, s, t\ real \right\}.$$

- **13.** Determine whether $\vec{w}$ is in the column space of A, the null space of A, or both, where

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -3 \end{bmatrix}, \quad A = \begin{bmatrix} 7 & 6 & -4 & 1 \\ -5 & -1 & 0 & -2 \\ 9 & -11 & 7 & -3 \\ 19 & -9 & 7 & 1 \end{bmatrix}.$$

Section # 4.3

- **14.** Find baeses for the null spaces of the following matrices:

$$\begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 1 & 4 \\ 3 & -1 & -7 & 3 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & -2 & 1 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & -8 & 0 & 16 \end{bmatrix}.$$

- **15.** Find a basis for the set of vectors in $\mathbf{R}^3$ in the plane $x - 3y + 2z = 0$.

- **16.** Let $\vec{v}_1 = \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 9 \\ -2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 7 \\ 11 \\ 6 \end{bmatrix}$, and also let $H = Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. Verify that $4\vec{v}_1 + 5\vec{v}_2 - 3\vec{v}_3 = \vec{0}$ and find a basis for H.

- **17.** Let $H = Span\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ and $K = Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, where

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ -4 \end{bmatrix},$$

$$\vec{v}_1 = \begin{bmatrix} -2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ -6 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -1 \\ 4 \\ 6 \\ -2 \end{bmatrix},$$

Find bases for H, K, and $H + K$.