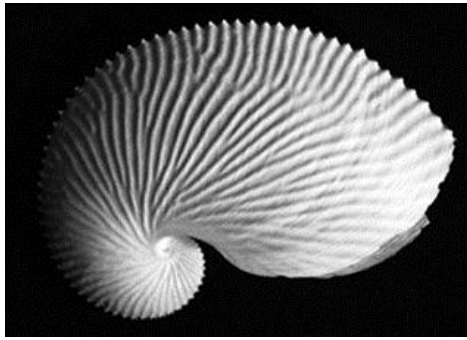

Mathematical Modeling

The Effect of Social Polarization on the Outcome of Information Battles

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Introduction

- *Propaganda Battle*

  - Each member of the society is subject to two competing flows of information

  - These two flows are generated by
    - Two competing parties
    - Each flow consists of propaganda and rumor

  - Way of information displacement
    - Interpersonal communication
    - Media

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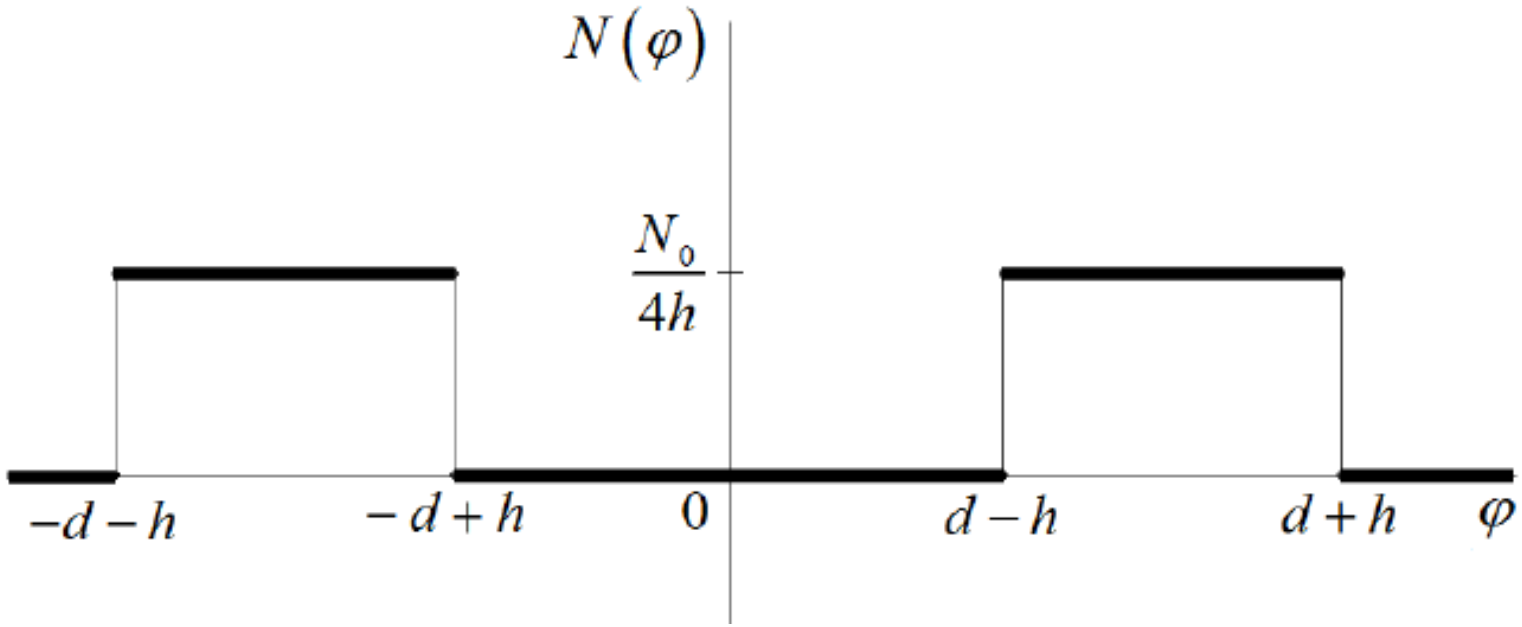
- *Political Polarization*

  - Due to the development of social media and the Internet in general
  - The impact of polarization on political events is widely discussed
  - “**A Polarized Society** “

    - Using a distribution curve with two high horizontal plateaus
    - The distance between the gravity centers of these plateaus is taken as a measure of polarization
    - The process of increasing polarization has the form of a mutual removal of the plateau from each other.

Distribution of Individuals $N(\boldsymbol{\varphi})$



- $\boldsymbol{d}$: $degree\ of\ polarization\ of\ society$($how\ groups\ are\ distant\ from\ each\ other\ in\ attutides$)
- $\frac{\mathbf{1}}{\boldsymbol{h}}$: measure of consolidation of individuals within each group

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Model

- Aim
  - Study how the level of political polarization affects the outcome of the propaganda battle

- Approach
  - Focus on a different aspect of information warfare→ Choose the **position** of individual
  - Model base: Rashevshy's neurological scheme

- Assumption
  - The society is a struggle between two parties **X** and **Y**
  - Each of party has its own media
  - An individual belonging to this society, as each moment of time has a position on the issue in question

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Position Factors

- *Permanent attitude*
  - Individual for each member of the society
  - $\boldsymbol{\varphi} \in (-\infty, \infty)$
  - Fundamental tendency to support one party or another

- *Dynamic component*
  - The information field of society as a whole
  - $\boldsymbol{\psi(t)} \in (-\infty, \infty)$
  - Social environment of the shift of stimuli towards the support of the party X
  - It is affected by the propaganda of both parties through the media and rumors

- Support X: $\boldsymbol{\varphi} + \boldsymbol{\psi(t)} > 0$
- Support Y: $\boldsymbol{\varphi} + \boldsymbol{\psi(t)} < 0$

- $N(\boldsymbol{\varphi})$: Function describing the distribution of individual

  Total number of individuals: $\int_{-\infty}^{\infty} N(\boldsymbol{\varphi}) d\,\boldsymbol{\varphi} = N_0$

$$\boldsymbol{X}(t) = \int_{-\boldsymbol{\psi(t)}}^{\infty} N(\boldsymbol{\varphi}) d\,\boldsymbol{\varphi} \qquad \boldsymbol{Y}(t) = \int_{-\infty}^{-\boldsymbol{\psi(t)}} N(\boldsymbol{\varphi}) d\,\boldsymbol{\varphi}$$

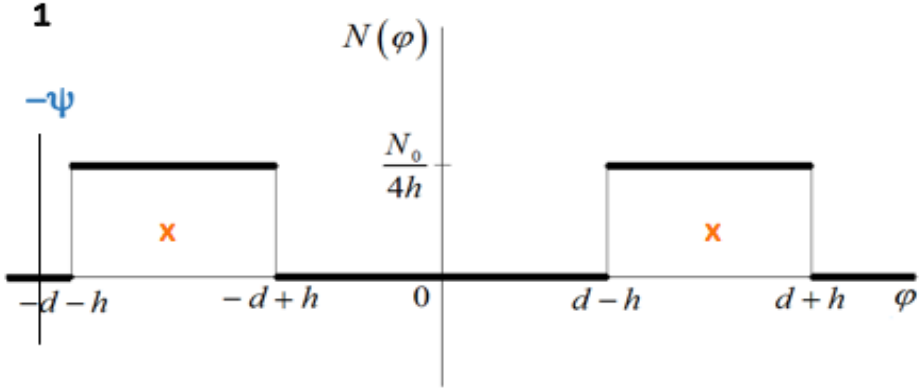
$$\frac{d\psi}{dt} = A\alpha[C(2X(t) - N_0) + b_1 - b_2] - a\psi$$

Initial condition(initial number of supporters for X party): $X(0) = \int_{-\psi(t)}^{\infty} N(\varphi) d\,\varphi$

Positive constant

- C: importance of interpersonal communication
- b_1, b_2: intensity of the media from each party ($b_1 > b_2$)
- $A\alpha$: susceptibility of individuals to stimuli
- a: decay rate

1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N0/4h
X
X
−d − h
− d + h
0
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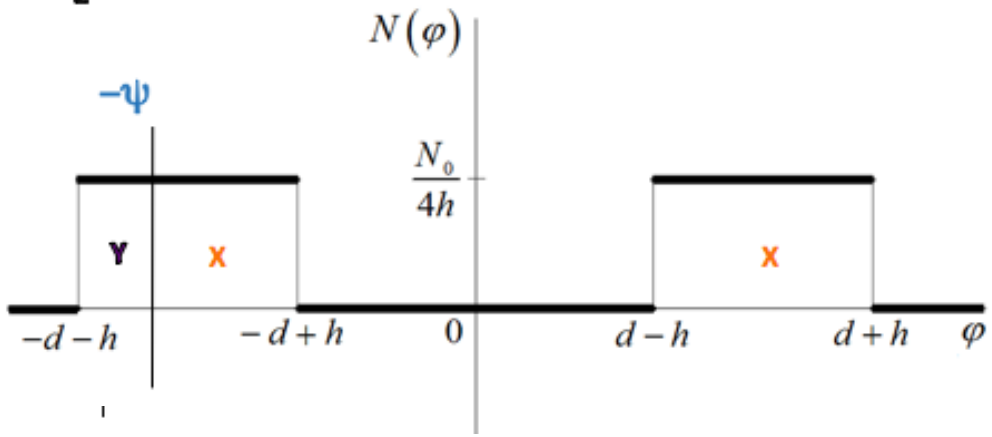
2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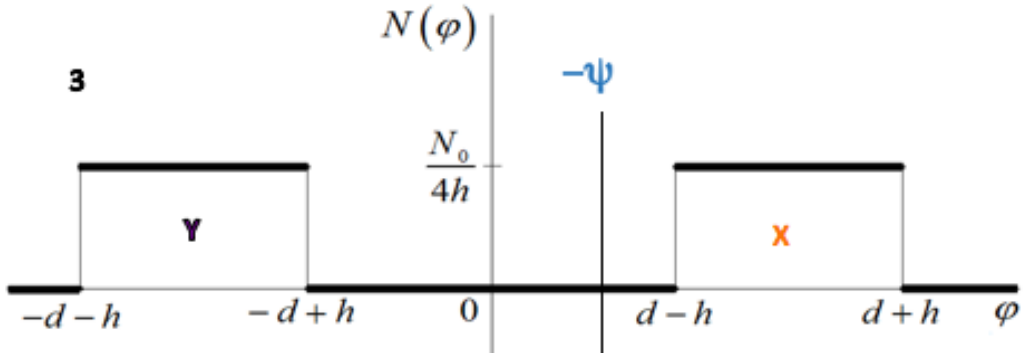
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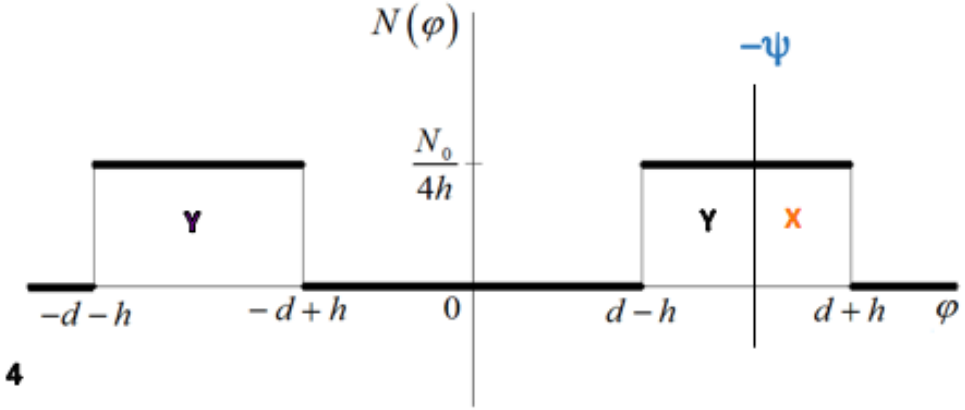
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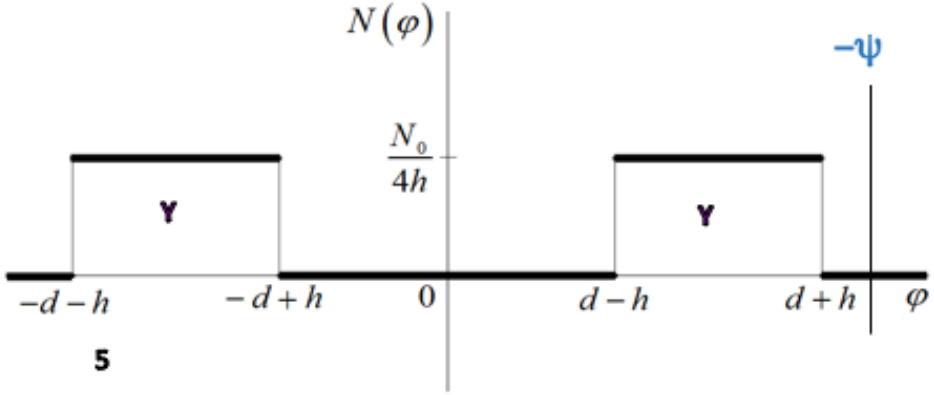
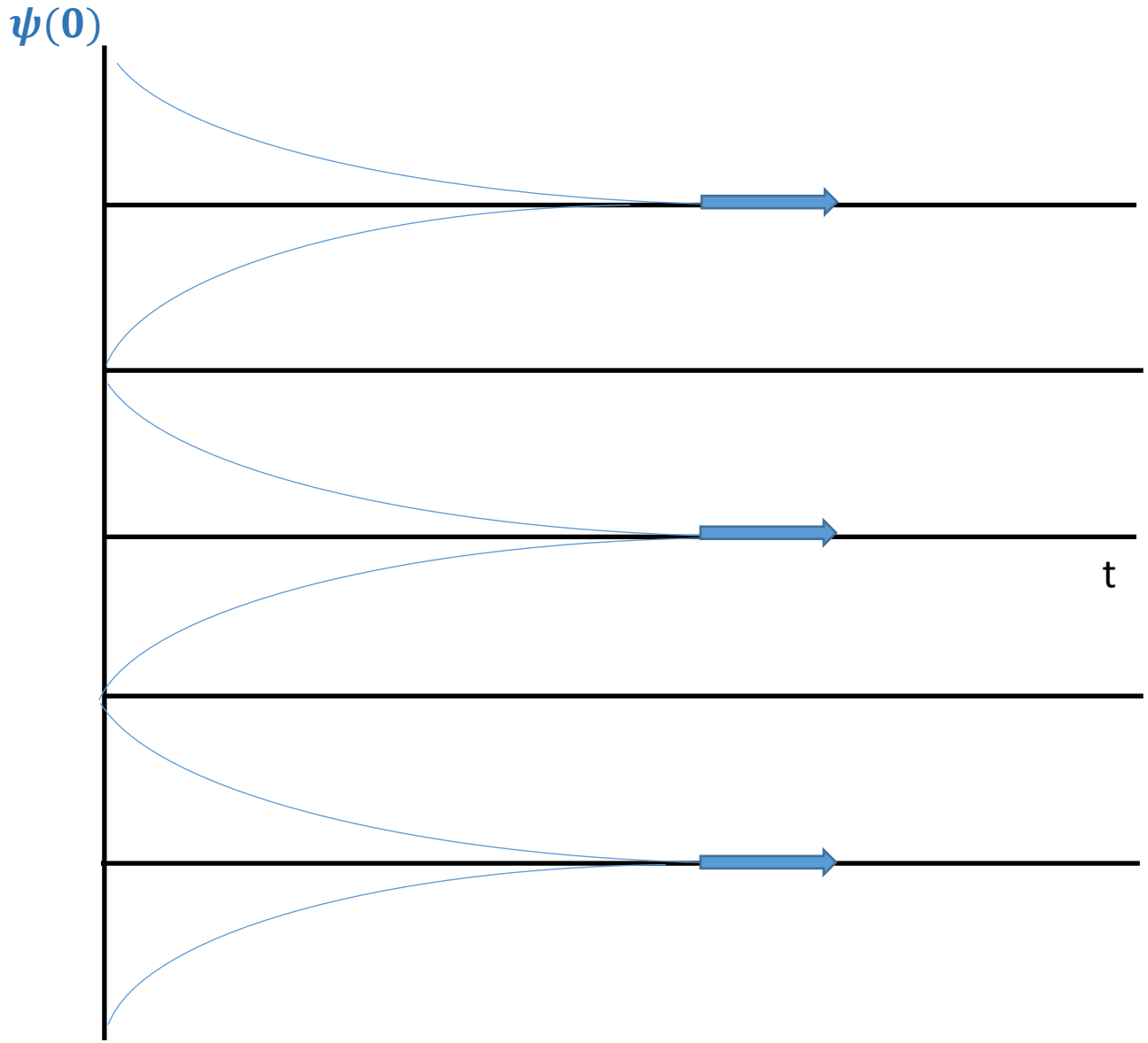

N(φ)
−ψ
N0/4h
Y
Y
−d − h
− d + h
0
d − h
d + h
φ

$$\frac{d\boldsymbol{\psi}}{dt} = A\alpha\left[C\left(2\int_{-\psi(t)}^{\infty} N(\boldsymbol{\varphi})d\,\boldsymbol{\varphi} - N_0\right) + b_1 - b_2\right] - a\boldsymbol{\psi}$$

Equilibrium: set $\frac{d\boldsymbol{\psi}}{dt} = 0$

$$P = \frac{A\alpha(b_1 - b_2)}{a} \qquad Q = \frac{A\alpha C N_0}{a}$$

$$\boldsymbol{\psi^1} = P + Q > 0, \quad \boldsymbol{\psi^2} = \frac{Q(-d-h)+2hP}{2h-Q} > 0, \quad \boldsymbol{\psi^3} = P > 0, \quad \boldsymbol{\psi^4} = \frac{Q(d-h)+2hP}{2h-Q} < 0, \quad \boldsymbol{\psi^5} = -Q + P < 0$$



$$\psi^1 = P + Q$$

$$\psi^2 = \frac{Q(-d-h)+2hP}{2h-Q}$$

$$\psi^3 = P$$

$$\psi^4 = \frac{Q(d-h)+2hP}{2h-Q}$$

$$\psi^5 = -Q + P$$

A sociological interpretation of the results

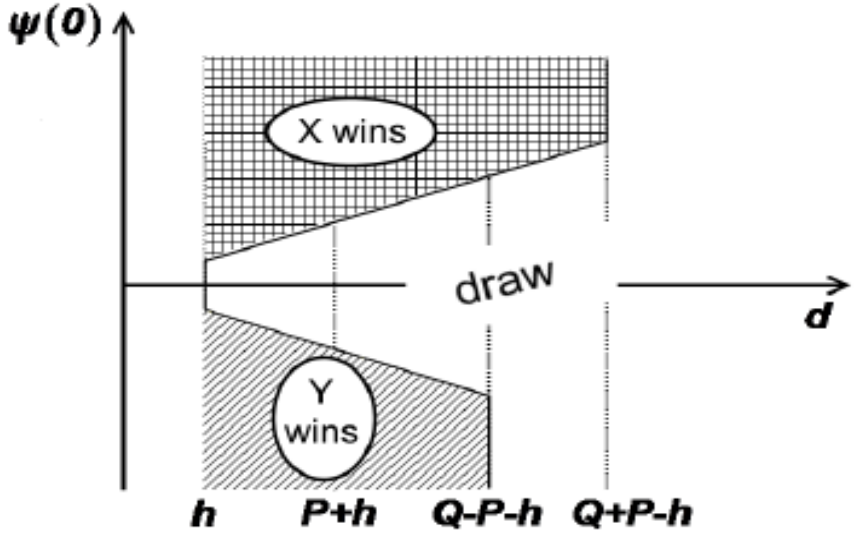
From the final result for one thing we will three possible cases

- ***X win***
- ***Y win***
- ***Draw***

From Statistical point of view

- X have certain percentage of chance to win
- Y have certain percentage of chance to win
- The third case, it will have certain percentage of chance to become a draw.

Assumption : $h < \frac{Q}{2} - P$



- $\boldsymbol{h < d < Q - P - h}$: X and Y and the same percentages to win, but it also could be draw.
- $\boldsymbol{Q - P - h < d < Q + P - h}$: Y will lose any chance to win. The result will either be X win or draw.
- $\boldsymbol{Q + P - h < d}$: The final result will always be draw.
- $\boldsymbol{d < h}$: Overlaps

There are more media to support the Party X, so the area for X wins is bigger than Y wins.

References

- **Thanks to our mentor Jonathan. David Taylor**
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