Math 129-8H Written Homework #2

Due September 10, in class.

1. Compute the following integrals:
   (a) \( \int x^2 e^{x^3} \, dx \)
   (b) \( \int_0^\pi e^x \sin e^x \, dx \)

2. Compute the following integrals:
   (a) \( \int x^2 \cos 3x \, dx \)
   (b) \( \int_0^e x^2 \ln (3x) \, dx \)

3. Some functions are defined as integrals. Consider the functions \( F_k \) defined by
   \[
   F_k(x) = \int_0^x \sqrt{1 - k^2 \sin^2 t} \, dt.
   \]
   These functions are called elliptic functions of the second kind and can be thought of as just some other set of functions like \( e^x, \sin x, \) etc. In fact, you can find tables of values for these functions in standard mathematical reference books. Notice that \( F_k(0) = 0 \). Find the following integrals in terms of the functions \( F_k \) by using integration by parts or substitution. Answers should be a number like \( 3e^2 + F_1(2) - \frac{1}{2} F_1(\pi) \) or something like that.
   (a) \( \int_0^1 \sqrt{1 - \sin^2 t} \, dt \)
   (b) \( \int_0^1 t \sqrt{1 - \sin^2 (t^2)} \, dt \)
   (c) \( \int_0^1 \frac{t \sin t \cos t}{\sqrt{1 - \sin^2 t}} \, dt \) (Hint: consider the derivative of \( \sqrt{1 - \sin^2 t} \) and then do integration by parts).